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New pairwise fuzzy topology through fuzzy ideal

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Abstract
The aim of this paper is introducing pairwise fuzzy topology on fuzzy bitopological spaces in Lowen's sense. We define the concept of fuzzy local function in pairwise fuzzy topological spaces and study some of its properties. Finally we find a new pairwise fuzzy topology from old one through fuzzy ideals.

Keywords: Fuzzy bitopological space, pairwise fuzzy topology, fizzy local function, induced pairwise fuzzy topology

Introduction

Zadeh [9] introduced the concept of fuzzy sets and operations on fuzzy sets. Zadeh defined as follows: let Y be a non-empty set. A fuzzy set on Y is a function with domain Y and values in I = [0, 1]. Lowen [4] defined the notion of fuzzy topology. Mahomoud [5] introduced and studies about the notions of fuzzy ideal and fuzzy local functions. Nouh [7] started the study of fuzzy bitopological spaces. A fuzzy bitopological space [2] (shortly fbts) is a triple (X, τ_1, τ_2) where X is a non-empty set, $\tau_1$and τ_2 are any two fuzzy topologies on X. In this paper we follow the definition of fuzzy topology in Lowen's [4] sense.

2. Preliminaries

Before entering into our work we need the following definitions and examples.

2.1. Definition [8] *If A is a fuzzy set of X, then the support of A is defined as* $S(A) = \{x \in X \,/ A(x) > 0\}$. *Let X be a non-empty set. A fuzzy set B is said to be finite fuzzy set of X if and only if* $S(B)$ *is a finite set.*

2.2. Definition [8] *A non-empty collection* $\mathcal{I}$ *of fuzzy sets of X is said to be a fuzzy ideal on X,if* $A, B \in \mathcal{I} \Rightarrow A \vee B \in \mathcal{I}$ *and* $A \in \mathcal{I}, B \leq A => B \in \mathcal{I}$.

2.3. Definition [8] *Let A and B are two fuzzy sets of X. Then A 'intersection' B is defined as* $(A \cap B)(x) = max\{0, A(x) + B(x) - 1\}$ *for all* $x \in X$.

2.4. Definition [8] *Let (X, τ) be a fuzzy topological space. The cl(A), the closure of a fuzzy set A is a fuzzy set defined by -cl(A)(x) =* $\vee\{\lambda / B \in \tau, B(x) > 1 - \lambda \Rightarrow A \cap B \neq \bar{0}\}$ *for all x ∈ X.*

2.5. Definition [4] *Let (X,τ) be a fuzzy topological space. The interior* A^0 *of a fuzzy set A of X is defined as* $A^0 = \vee\{B: B \leq A, B \in \tau\}$.
Now we write the example 2.3.3 in [8] with small modification as follows:

2.6. Example Let X = I = [0, 1]. To each $\alpha_1, \alpha_2 \in$ I with $0 \leq \alpha_1 \leq \alpha_2 \leq 1$,
(i) let f_{α_1,α_2} a function defined by

$$f_{\alpha_1,\alpha_2}(x) = \begin{cases} \alpha_1 & if\ 0 \leq x \leq \alpha_1 \\ x & if\ \alpha_1 \leq x \leq \alpha_2 \\ \alpha_2 & if\ \alpha_2 \leq x \leq 1 \end{cases}$$

Then $\tau_1 = \{f_{\alpha_1,\alpha_2} \,/\, 0 \le \alpha_1 \le \alpha_2 \le 1\}$ is a fuzzy topology on X.
Similarly

(ii) let $\tau_2 = \{f_{\beta_1,\beta_2} \,/\, 0 \le \beta_1 \le \beta_2 \le 1\}$ defined by

$$f_{\beta_1,\beta_2}(x) = \begin{cases} \beta_2 \; if \; 0 \le x \le \beta_1 \\ (\beta_1+\beta_2) - x \; if \; \beta_1 \le x \le \beta_2 \\ \beta_1 \; if \; \beta_2 \le x \le 1 \end{cases}$$ is also a fuzzy topology on X.

2.7. Definition [8] *Let (X,τ) be a fuzzy topological space with fuzzy ideal $\mathcal{I}$ on X. Let A be a fuzzy set on X. The fuzzy local function A* is defined by* $A^*(x) = \vee\left\{\lambda \frac{}{B} \in \tau, B(x) > 1 - \lambda \Rightarrow A \cap B \notin \mathcal{I}\right\}$ *for all* $x \in X$.

2.8. Definition [8] *Let (X, τ) be a fuzzy topological space with fuzzy ideal $\mathcal{I}$ on X. We define a fuzzy topology τ^* $(\mathcal{I})$ induced by τ and $\mathcal{I}$ as $\tau^*(\mathcal{I}) = \{U \in I^X : \psi(1-U) = 1-U\}$ where $\psi(A) = A \vee A^* : \psi(A)$ is denoted by cl* (A).*

2.9. Remark [1] Let (X, τ_1, τ_2) be a fuzzy bitopological space and μ be a fuzzy set on X. τ_i-cl(μ),τ_i-int(μ),i∈{1, 2} denote closure and interior of μ. In this paper we refer τ_i-cl(μ) by $\mathrm{cl}(\mu)_i$ and τ_i-int(μ) by μ_i^0.

2.10. Lemma [8] If *A, B and C are fuzzy sets on X then* $C \cap (A \vee B) = (C \cap A) \vee (C \cap B)$.

3. Pairwise fuzzy topological space

3.1. Definition *A fuzzy subset g of a fuzzy bitopological space (X, τ_1, τ_2) is called a pairwise fuzzy open set if $g = f_1 \vee f_2$ where $f_1 \in \tau_1$ and $f_2 \in \tau_2$ .The complement of a pairwise fuzzy open set is called as pairwise fuzzy closed set.*
In this paper the collection of all pairwise fuzzy open sets on X is denoted by $\tau_{1,2}$. We refer the collection $\tau_{1,2}$ as pairwise fuzzy topology on X and the pair (X, $\tau_{1,2}$) is called as pairwise fuzzy topological space (shortly pfts).
The following example asserts that $\tau_{1,2}$ need not be a fuzzy topology on X.

3.2. Example Let X = I = [0, 1]. Consider the fuzzy bitopological space (X, τ_1, τ_2) where τ_1 and τ_2 are the two fuzzy topologies given in example 2.6.
Let $f_1 \in \tau_1$ and $f_2 \in \tau_2$.Then consider g (x) = $(f_1 \vee f_2)$ (x).

$$\text{Then } g\,(x) = \begin{cases} \alpha_1 \vee \beta_2 \; \le \alpha_1 \vee \beta_1 \; if \; 0 \le x \\ x \vee [(\beta_1+\beta_2) - x] \le \alpha_2 \vee \beta_2 \; if \; \alpha_1 \vee \beta_1 \le x \\ \alpha_2 \vee \beta_1 \le 1 \; if \; \alpha_2 \vee \beta_2 \le x \end{cases}$$

∴ $\tau_{1,2} = \tau_1 \cup \tau_2 \cup$ {all functions of the form g(x)}. Now we show that $\tau_{1,2}$ is not a fuzzy topology on X. For this let $f_1 = f_{\frac{1}{4},\frac{7}{8}}$ and $f_2 = f_{\frac{3}{8},\frac{3}{4}}$

Let h(x) = $(f_1 \vee f_2)$ (x) .

$$\text{Then } h(x) = \begin{cases} \frac{3}{4} \; if \; 0 \le x \le \frac{3}{8} \\ x \vee (\frac{9}{8} - x) \; if \; \frac{3}{8} \le x \le \frac{7}{8} \\ \frac{7}{8} \; if \; \frac{7}{8} \le x \le 1 \end{cases}$$

Take $\bar{\frac{5}{8}} \in \tau_{1,2}$ and the above h(x).

$$\text{Now } \left(\bar{\frac{5}{8}} \vee h\right)(x) = \begin{cases} \frac{3}{4} \; if \; 0 \le x \le \frac{3}{8} \\ \frac{9}{8} - x \; if \; \frac{3}{8} \le x \le \frac{1}{2} \\ \frac{5}{8} \; if \; \frac{1}{2} \le x \le \frac{5}{8} \\ x \; if \; \frac{5}{8} \le x \le \frac{7}{8} \\ \frac{7}{8} \; if \; \frac{7}{8} \le x \le 1 \end{cases}$$

Note that $(\bar{\frac{5}{8}} \vee h)$(x) is not of the form g(x). ∴ $(\bar{\frac{5}{8}} \vee h) \notin \tau_{1,2}$. Hence $\tau_{1,2}$ is not a fuzzy topology on X.

3.3. Remark

1. If $\tau_1 \subseteq \tau_2$ then $\tau_{1,2} = \tau_2$. Similarly if $\tau_2 \subseteq \tau_1$ then $\tau_{1,2} = \tau_1$.
2. For each $\alpha \in [0,1], \bar{\alpha} \in \tau_{1,2}$.
3. For a given pfts (X, $\tau_{1,2}$) there must be a fbts (X, τ_1, τ_2).

3.4. ***Definitions*** *Let (X, τ_1, τ_2) be a fbts with fuzzy ideal $\mathcal{I}$ on X. Then for each $i \in \{1,2\}$*
1.$cl(A)_i(x) = \vee\{\lambda \,/\, B \in \tau_{i,} B(x) > 1-\lambda \Rightarrow A \cap B \neq \bar{0}\}$ *for all $x \in X$.*
2.$cl(A)_{1,2}(x) = \vee\{\lambda \,/\, B \in \tau_{1,2,} B(x) > 1-\lambda \Rightarrow A \cap B \neq \bar{0}\}$ *for all $x \in X$.*
3. $A_i^*(x) = \vee\{\lambda \,/\, B \in \tau_{i,} B(x) > 1-\lambda \Rightarrow A \cap B \notin \mathcal{I}\}$ *for all $x \in X$.*
4. $A_{1,2}^*(x) = \vee\{\lambda \,/\, B \in \tau_{1,2,} B(x) > 1-\lambda \Rightarrow A \cap B \notin \mathcal{I}\}$ *for all $x \in X$.*

3.5. Example Let (X, τ_1, τ_2) be a fbts with fuzzy ideal $\mathcal{I}$ on X, where τ_1 = indiscrete fuzzy topology = $\{\bar{\alpha} \,/\, 0 \le \alpha \le 1$ and τ_2 = discrete fuzzy topology =$\{ f / f : X \to [0,1]\}$. Then for all $\alpha \in (0,1], \bar{\alpha}^*_{1,2} = \bar{\alpha}$.
$\bar{\alpha}^*_{1,2}$(x) = $\vee\{\lambda \,/\, \bar{\beta} \in \tau_{1,2}$ with $\bar{\beta}$ (x) $> 1-\lambda \Rightarrow \bar{\alpha}$ (x) +$\bar{\beta}$ (x) -1$> 0\}$
=$\vee\{\lambda \,/\, \bar{\beta} \in \tau_{1,2,}$ with $\bar{\beta}$ (x) $> 1-\lambda \Rightarrow \bar{\beta}$ (x) $> 1-\bar{\alpha}$ (x)$\}$
= $\vee\{\lambda \,/\, \lambda \le \alpha\}$
= $\bar{\alpha}$ (x)

3.6. Theorem *Let (X, τ_1, τ_2) be a fbts with fuzzy ideal $\mathcal{I}$ on X.*
Let $A, B \in I^X$, then for all $x \in X$ we have
$(i)\ A_{1,2}^*(x) \le A_1^*(x) \vee A_2^*(x)$.
(ii) *if* $A \le B$ *then* $A_{1,2}^*(x) \le B_1^*(x) \vee B_2^*(x)$.
$(iii)\ A_{1,2}^*(x) \le cl(A)_{1,2}(x) \le (cl(A)_1 \vee cl(A)_2)(x)$.
$(iv)\ (A_{1,2}^*)^* \le A_{1,2}^*(x)$.

Proof
(i) *Let* $(A_{1,2}^*)^*$(x) = μ. Then for all C$\in \tau_{1,2}$ With C(x)$> 1-\mu \Rightarrow C \cap A \notin \mathcal{I}$. C $\in \tau_{1,2} \Rightarrow$ there exists $C_1 \in \tau_1$ and $C_2 \in \tau_2$ with C= $C_1 \vee C_2$. That is $(C_1 \vee C_2)$(x)$> 1-\mu \Rightarrow (C_1 \vee C_2) \cap A \notin \mathcal{I}$. Note that $(C_1 \vee C_2) \cap A = (C_1 \cap A) \vee (C_2 \cap A) \notin \mathcal{I}$. If both $(C_1 \cap A) \in \mathcal{I}$ and $(C_2 \cap A) \in \mathcal{I}$ then $(C_1 \cap A) \vee (C_2 \cap A) \in \mathcal{I}$. Which is a contradiction.
$\therefore$ either $(C_1 \cap A) \notin \mathcal{I}$ or $(C_2 \cap A) \notin \mathcal{I}$. Assume that $(C_1 \cap A) \notin \mathcal{I}$. Then $(C_1 \cap A) \neq \bar{0}$, there exists y $\in$ X such that C_1(y)+ A(y) -1 > 0. If C_1(x)$> 1-\mu$ with $(C_1 \cap A) \notin \mathcal{I}$, then $\mu \le A_1^*$ (x). $\therefore$ either $\mu \le A_1^*$ (x) or $\mu \le A_2^*$ (x). That is $A_{1,2}^*(x) \le A_1^*$ (x)$\vee A_2^*$ (x).

(ii) let $A \le B$. If C_1(x)$> 1-\mu$ with $(C_1 \cap A) \notin \mathcal{I}$,then$(C_1 \cap B) \notin \mathcal{I}$,(Since $C_1 \cap A) \le C_1 \cap B$). Therefore $\mu \le B_1^*$(x) or $\mu \le B_2^*$(x).Hence$A_{1,2}^*$(x) $\le B_1^*$(x) $\vee B_2^*$(x).

(iii) *Let* $\mu = A_{1,2}^*(x)$. Then for all C $\in \tau_{1,2}$ with C(x)$> 1-\mu$ with $(C \cap A) \notin \mathcal{I}$ implies that C$\cap A \neq \bar{0}$. That is $\mu \le cl(A)_{1,2}$(x). Therefore $A_{1,2}^*$(x) $\le cl(A)_{1,2}$(x).
Let $v = cl(A)_{1,2}$(x). If C = $C_1 \vee C_2$ then$(C_1 \vee C_2) \cap A \neq \bar{0}$, there exists y $\in X$ with $(C_1 \vee C_2)$ (y) +A(y)-1> 0. So C_1(y) +A(y)-1> 0 or C_2(y) +A(y)-1> 0. That is $C_1 \cap A \neq \bar{0}$ or $C_2 \cap A \neq \bar{0}$. If $C_1 \cap A \neq \bar{0}$
then $v \le cl(A)_1$(x) . Therefore either $v \le cl(A)_1$(x)or$v \le cl(A)_2$(x). Thus $A_{1,2}^*(x) \le cl(A)_{1,2}(x)\le (cl(A)_1 \vee cl(A)_2)$(x).

(iv) *Let* $\mu = (A_{1,2}^*)^*$(x). Then for $B \in \tau_{1,2}$ with B(x)$> 1-\mu$, *we have* $B \cap A_{1,2}^* \notin \mathcal{I}$. This implies that $B \cap A_{1,2}^* \neq \bar{0}$. Let $y \in X$ such that B(y) +$A_{1,2}^*(y) > 1$. Let λ =$A_{1,2}^*(y)$. Then $A_{1,2}^*(y) \neq 0$ and B(y) $> 1-\lambda$. So $B \cap A \notin \mathcal{I}$. Thus if $(A_{1,2}^*)^*$(x) = μ then B $\in \tau_{1,2}$ with B(x)$> 1-\mu \Rightarrow A \cap B \notin \mathcal{I}$. Therefore $\mu \le A_{1,2}^*$(x). Hence $(A_{1,2}^*)^*$(x)$\le A_{1,2}^*$(x).

3.7. Example Let $\mathbb{N}$ be the set of all Natural numbers. Consider the fbts $(\mathbb{N}, \tau_1, \tau_2)$ with indiscrete fuzzy topology $\tau_1 = \{\bar{\alpha} / 0 \le \alpha \le 1\}, \tau_2 = \{$f : $\mathbb{N} \to [0,1]$ / for each n$\in \mathbb{N}$, f(2n) = 0 iff f(2n-1) = 0$\}$, then for all $\alpha \in [0,1]$, $(\bar{\alpha}_{1,2})^* \le \bar{\alpha}$.
We know that by above theorem
$(\bar{\alpha}_{1,2})^* \le (\bar{\alpha}_1)^* \vee (\bar{\alpha}_2)^*$ where $\bar{\alpha}_1 \in \tau_1$ and $\bar{\alpha}_2 \in \tau_2$. If $(\mathbb{N}, \tau_1)$ be a fuzzy topological space, then $(\bar{\alpha}_1)^*$(x)= $\vee\{\lambda \,/\, \bar{\beta} \in \tau_1$ with $\bar{\beta}$ (x)$> 1-\lambda \Rightarrow \bar{\alpha}_1$(x)+$\bar{\beta}$ (x) -1$> 0\}$
= $\vee\{\lambda \,/\, \bar{\beta} \in \tau_1$ with $\bar{\beta}$ (x)$> 1-\lambda \Rightarrow \bar{\beta}$ (x)$> 1-\bar{\alpha}_1$(x)$\}$
= $\vee\{\lambda \,/\, \lambda \le \alpha_1\}$
=$\bar{\alpha}_1$(x).
That is $(\bar{\alpha}_1)^* = (\bar{\alpha}_1)$………..(i)
Consider the fuzzy topology τ_2 on $\mathbb{N}$ and the fuzzy ideal $\mathcal{I}_f$ of finite fuzzy sets in $\mathbb{N}$, then we show that for any $\alpha_2 \in \mathbb{N}$, $(\bar{\alpha}_2)^*(\mathcal{I}_f) = \bar{0}$. Let x$\in \mathbb{N}$. If $(\bar{\alpha}_2)^*$(x)$\neq 0$, then $(\bar{\alpha}_2)^*$(x) $= \mu$ for some $0 < \mu \le 1$. By definition of $(\bar{\alpha}_2)^*$, If $B \in \tau_2$ with B(x)$> 1-\mu$ then $\bar{\alpha}_2 \cap B \notin \mathcal{I}_f$. That is S $(\bar{\alpha}_2 \cap B)$ is not a finite subset of $\mathbb{N}$.
Hence { y$\in \mathbb{N}$ / $\bar{\alpha}_2$(y) + B(y) > 1} is not a finite subset of $\mathbb{N}$. But this is not possible, because x $\in$ {2n-1,2n} for exactly one n $\in \mathbb{N}$. Let e be the other element of {2n-1,2n}. Define a fuzzy set h such that h (x) = h(e) = 1 and h(y) = 0 for all y $\neq$ x, e. Clearly h $\in \tau_2$ and { y$\in \mathbb{N}$ / $\bar{\alpha}_2$(y) + B(y) > 1} is a finite subset of $\mathbb{N}$. Hence $(\bar{\alpha}_2)^*$(x) $= 0$ for allx$\in \mathbb{N}$.Therefore $(\bar{\alpha}_{1,2})^* \le (\bar{\alpha}_1)^* \vee (\bar{\alpha}_2)^* = \bar{\alpha} \vee \bar{0} = \bar{\alpha}$.That is $(\bar{\alpha}_{1,2})^* \le \bar{\alpha}$.

4. Induced pairwise fuzzy topological space

4.1. Definition *Let (Y, τ_1, τ_2) be a fbts with fuzzy ideal $\mathcal{g}$ on Y. We define a pairwise fuzzy topology $\tau^*_{1,2}(\mathcal{g})$ induced by $\tau_{1,2}$ and $\mathcal{g}$ as $\tau^*_{1,2}(\mathcal{g}) = \{W \in I^Y : \psi(1-W) = 1-W\}$ Where $\psi(A) = A \vee A^*_{1,2}$: $\psi(A)$ is denoted by $cl^*(A)_{1,2}$.*

4.2 Theorem *The pairwise fuzzy topology $\tau^*_{1,2}(\mathcal{g})$ is finer than the pairwise fuzzy topology $\tau_{1,2}$.*

Proof. Let (Y, τ_1, τ_2) be a fbts and $\mathcal{g}$ be a fuzzy ideal . We claim that $\tau_{1,2} \subseteq \tau^*_{1,2}(\mathcal{g})$. Let A be a pairwise fuzzy closed set in $\tau_{1,2}$. Then $A^*_{1,2} \leq cl(A)_{1,2} = A$ (by theorem 3.6). Therfore $A^*_{1,2} \leq A$ and A = A $\vee A^*_{1,2}$. Therefore A is pairwise fuzzy closed in $\tau^*_{1,2}(\mathcal{g})$. Thus every $\tau_{1,2}$ - closed set A is $\tau^*_{1,2}$ - closed set. Hence $\tau_{1,2} \subseteq \tau^*_{1,2}(\mathcal{g})$.

4.3. Example Let $\mathbb{N}$ be the set of all natural numbers. Let $(\mathbb{N}, \tau_1, \tau_2)$ be a fuzzy bitopolgical space with $\tau_1 = \{\bar{\alpha} / 0 \leq \alpha \leq 1\}$ and $\tau_2 = \{$f : $\mathbb{N} \to [0,1]$ / for each n$\in \mathbb{N}$, f(2n) = 0 iff f(2n-1) = 0$\}$. Consider the fuzzy ideal $\mathcal{I}_f = \{ A \in I^{\mathbb{N}}$ / S (A) is a finite subset of $\mathbb{N}\}$. Note that $\tau^*_{1,2}(\mathcal{I}_f) = \{ (1-f) \in I^{\mathbb{N}} / f = cl^*(f)_{1,2}\}$. Now $f^*_{1,2} \leq f \Rightarrow$ for all n $\in \mathbb{N}$, $f^*_{1,2}$ (n) $\leq$ f(n). If f $\neq \bar{1}$, fix $n_0 \in \mathbb{N}$, such that f$(n_0) \neq 1$. Then $f^*_{1,2}(n_0) \leq$ f$(n_0) < 1$. Take λ such that f$(n_0) < \lambda < 1$. As $\lambda > f^*_{1,2}(n_0)$, there exists $B \in \tau_{1,2}$ with $B(n_0) > 1 - \lambda$ and B $\cap$ f $\in \mathcal{I}_f$. As $B \in \tau_{1,2}$, B = $\bar{\gamma}$ where $B(n_0) = \gamma$ and $B(n_0) = B_1(n_0) \vee B_2(n_0)$ with $B_1 \in \tau_1$ and $B_2 \in \tau_2$.
= $\mu \vee \eta$ where $\mu, \eta \in [0,1]$.
= θ (say).
Now B $\cap$ f $\in \mathcal{I}_f \Rightarrow$ S (B $\cap$ f) is finite. That is S (B $\cap$ f) = $\{$ n $\in \mathbb{N}$ / B(n) + f (n) $> 1\}$
= $\{$ n $\in \mathbb{N}$ / f(n) $>$ 1- $B(n_0)\}$ is a finite subset of $\mathbb{N}$. As $\lambda >$1- $B(n_0)$, the set $\{$n $\in \mathbb{N}$ / f(x) $> \lambda\}$ is a finite set . Therefore $\tau^*_{1,2} = \{$(1-f) $\in I^{\mathbb{N}}$ / f = $\bar{1}$ (or) to each λ such that f $(n_0) < \lambda < 1$ for some $n_0 \in \mathbb{N}$, the set $\{$n $\in \mathbb{N}$ / f(n) $> \lambda\}$ is a finite set$\}$.

4. Conclusion

In this paper we have investigated some properties of fuzzy bitopological spaces in Lowen's sense. We have also obtained new pairwise fuzzy topology from a given one via fuzzy ideals.

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