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B Amudhambigai
Assistant Professor of Mathematics, Department of Mathematics, Sri Sarada College for Women, Salem, Tamil Nadu, India

G Vasuki
Associate Professor of Mathematics, Department of Mathematics, Sri Sarada College for Women, Salem, Salem, Tamil Nadu, India

J Sasirekha
Research Scholar, Department of Mathematics Sri Sarada College for Women Salem, Tamil Nadu, India

Correspondence
B Amudhambigai
Assistant Professor of Mathematics, Department of Mathematics, Sri Sarada Collegefor Women Salem

A view on Bb-Hausdorff M-spaces in Multiset topological spaces

B Amudhambigai, G Vasuki and J Sasirekha

Abstract
This paper deals with the concepts of B-open M-sets, Bb-closed M-sets and the Bb-continuity M-set function in M-topological spaces are discussed. Also, the concepts of Bb-Compact M-spaces, Bb-Hausdorff M-spaces, Bb-homeomorphism M-set functions and some applications of Bb-Hausdorff M-spaces on product topology are studied and some of their interesting relationships with respect to Bb-open M-sets are established.

Keywords: B-open M-sets, Bb-closed M-sets, Bb-compact M-spaces, Bb-Hausdorff M-spaces, Bb-homeomorphism M-set functions

1. Introduction
The notion of M-topological space and the concept of open M-sets are introduced by Girish and sunil Jacob John [3]. Hausdorff spaces are named after Felix Hausdorff, one of the founders of topology. Hausdorff's original definition of a topological space (in 1914) included the Hausdorff condition as an axiom. The term compact was introduced into mathematics by Maurice Fréchet in 1904.
The main purpose of this paper is to study the concepts of B-open M-sets, Bb-closed M-sets and the Bb-continuity M-set function in M-topological spaces. Also, the concepts of Bb-Compact M-spaces, Bb-Hausdorff M-spaces, Bb-homeomorphism M-set functions and some applications of Bb-Hausdorff M-spaces on product topology are studied
Throughout this paper X denote a non-empty set, $M \in [X]^W$ and $C_M: X \longrightarrow W$ where W is the set of all whole numbers.

1.1 Preliminaries
Definition 1.1.1 [3]: Let M ∈ [X]w and τ ⊆ P∗(M). Then τ is called a Multiset topology of M if τ satisfies the following properties.
1) The M-set M and the empty M-set ϕ are in τ.
2) The M-set union of the elements of any sub collection of τ is in τ.
3) The M-set intersection of the elements of any finite sub collection of τ is in τ.

Definition 1.1.2 [3]: Given a subM-set A of an M-topological space M in [X]w, the interior of A is defined as the M-set union of all open M-sets contained in A and is denoted by Int(A). i.e., Int(A) = $\bigcup$ {G ⊆ M: G is an open M-set and G ⊆ A} and

$$C_{Int(A)}(x) = \max\{C_G(x) : G \subseteq A, G \in \tau\}.$$

Definition 1.1.3 [3]: Given a subM-set A of an M-topological space M in [X]w, the closure of A is defined as the M-set intersection of all closed M-sets containing A and is denoted by Cl(A). i.e., Cl(A) = $\bigcap$ {K ⊆ M: K is a closed M-set and A ⊆ K} and

$$C_{Cl(A)}(x) = \min\{C_K(x) : A \subseteq K, K \in \tau^c\}.$$

Definition 1.1.4 [5]: Let (M, τ) be an M-topological space. If for every two M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$, then there exist G, H ∈ τ such that $\{k_1/x_1\} \subseteq G$, $\{k_2/x_2\} \subseteq H$ and G⋂H = ϕ . Hence (M, τ) is a Hausdorff M-space.

2. ON Bb-Closed M-Set

Definition 2.1: Let (M, T) be an M-topological space in $[X]^W$. A subM-set A of M is said to be a b-open M-set, if A ⊆ int(cl(A)) ∪ cl(int(A)) with $C_A(x) \leq \max\{C_{int(cl(A))}(x), C_{cl(int(A))}(x)\}$, for all x ∈ X.

Definition 2.2: Let (M, T) be an M-topological space in $[X]^W$. A subM-set A of M is said to be a t-open M-set, if int(A) = int(cl(A)) with $C_{int(A)}(x) = C_{int(cl(A))}(x)$, for all x ∈ X.

Definition 2.3: Let (M, T) be an M-topological space and any subM-set P of M is called a B-open M-set if P = Q ⋂ R with $C_p(x) = \min\{C_Q(x), C_R(x)\}$, for all x ∈ X where Q is an open M-set and R is a t-open M-set.

Definition 2.4: Let (M, T) be an M-topological space in $[X]^W$ and A be a subM-set of M. The M-set intersection of all B-closed M-sets of (M, T) containing A is called B-closure of A and is denoted by Bcl(A).
i.e.,Bcl(A) = ⋂ {K ⊆ M: each K is a B-closed M-set and A ⊆ K} with $C_{Bcl(A)}(x) = \min\{C_K(x)$: A ⊆ K, each k ⊆ M is a B-closed M-set}.

Definition 2.5: Let (M, T) be an M-topological space in $[X]^W$ and A be a subM-set of M. The M-set union of all B-open M-sets of (M, T) contained in A is called B-interior of A and is denoted by Bint(A).
i.e., Bint(A) = ⋃ {G ⊆ M: each G is a B-open M-set and G ⊆ A} with $C_{Bint(A)}(x) = \max\{C_G(x)$: G ⊆ M, G is a B-open M-set}.

Definition 2.6: Let (M, T) be an M-topological space in $[X]^W$. A subM-set A of M is said to be a Bb-closed M-set, if Bcl(A) ⊆ B with $C_{Bcl(A)}(x) \leq C_B(x)$, whenever A ⊆ B with $C_A(x) \leq C_B(x)$ and B is a b-open M-set, for all x ∈ X. The complement of a Bb-closed M-set is said to be a Bb-open M-set.

Example: Let X = {a, b, c}, W = 2 and M = {2/a, 1/b, 1/c}. Then P(M) = {M, ϕ, {2/a}, {1/a}, {1/b}, {1/c}, {2/a, 1/b}, {2/a, 1/c}, {1/a, 1/b}, {1/a, 1/c}, {1/b, 1/c}, {1/a, 1/b, 1/c}} which is the power M-set of M. Let T = {M, ϕ, {1/a}, {1/b}, {1/a, 1/b}} then T^c = {ϕ, M, {1/a, 1/b, 1/c}, {2/a, 1/c}, {1/a, 1/c}}. Clearly, T is an M-topology and the ordered pair (M, T) is an M-topological space.
Now the collection of b-open M-sets is {M, ϕ, {1/a}, {1/b}, {1/a, 1/b}, {2/a, 1/b}, {1/a, 1/c}, {1/b, 1/c}, {1/a, 1/b, 1/c}} and the collection of B-closed M-sets is {M, ϕ, {1/a}, {1/b}, {1/a, 1/b}, {1/a, 1/c}, {1/b, 1/c}, {2/a, 1/c}, {1/a, 1/b, 1/c}}.
Let A = {2/a, 1/c} be a subM-set of M and A ⊂ B = M is a b-open M-set in (M, T). Then Bcl(A) = {2/a, 1/c} ⊂ B with $C_{Bcl(A)}(x) < C_B(x)$, whenever A ⊂ B with $C_A(x) < C_B(x)$, for all x ∈ X. Therefore, A is a Bb-closed M-set.

Proposition: Let (M, T) be an M-topological space. Then every B-closed subM-set in (M, T) is a Bb-closed M-set.

Remark: The converse of the Proposition 2.1 need not be true as shown in the Example 2.7

Example: Let X = {a, b, c}, W = 2 and M = {1/a, 2/b, 1/c}. Let T = {ϕ, M, {1/a}, {2/b}, {1/a, 2/b}}. Now T is an M-topology and (M, T) is an M-topological space. Now the collections of B-closed M-set is {M, ϕ, {1/a}, {2/b}, {1/c}, {1/a, 2/b}, {2/b, 1/c}, {1/a, 1/c}} and the collections of b-open M-set is {M, ϕ, {1/a}, {2/b}, {1/b}, {1/a, 2/b}, {2/b, 1/c}, {1/a, 1/c}, {1/a, 1/b}, {1/a, 1/b, 1/c}}.
Let A = {1/a, 1/b} be a subM-set of M. Hence A is a Bb-closed M-set but A is not a B-closed M-set.

Definition 2.7: Let (M, T) and (N, S) be any two M-topological spaces. Any M-set function f: (M, T) → (N, S) is called a closed M-set function if f(V) is a closed M-set in (N, S) for every closed M-set V in (M, T).

Definition 2.8: Let (M, T) and (N, S) be any two M-topological spaces. Any M-set function f: (M, T) → (N, S) is called an open M-set function if f(V) is an open M-set in (N, S) for every open M-set V in (M, T).

Definition 2.9: Let (M, T) and (N, S) be any two M-topological spaces. Any M-set function f: (M, T) ⟶ (N, S) is called a Bb-continuous M-set function if $f^{-1}(V)$ is a Bb-open M-set in (M, T) for every open M-set V of (N, S).

Proposition: Let (M, T) and (N, S) be any two M-topological spaces. For any M-set function f: (M, T) ⟶ (N, S), the following statements are equivalent:
(1) f is a Bb-continuous M-set function.
(2) f(Bb-Cl(A)) ⊆ Cl (f(A)), for each A ⊆ M with $C_{f(Bb-cl(A))}(x) \leq C_{Clf(A)}(x)$, for all x ∈ X.
(3) Bb-Cl(f^{-1}(B)) ⊆ f^{-1}(Cl(B)), for each B ⊆ N with $C_{Bb-Cl(f^{-1}(B))}(x) \leq C_{f^{-1}(Cl(B))}(x)$, for all x ∈ X.
(4) f^{-1}(Int(B)) ⊆ Bb-Int(f^{-1}(B)), for each B ⊆ N with $C_{(f^{-1}(Int(B))}(x) \leq C_{Bb-Int(f^{-1}(B))}(x)$, for all x ∈ X.

Proposition: Let (M, T), (N, S) and (P, R) be any three M-topological spaces. If a function f: (M, T) → (N, S) is a surjective Bb-continuous M-set function and g: (N, S) → (P, R) is a continuous M-set function then g∘f: (M, T) → (P, R) is a Bb-continuous M-set function.

3. On Bb-Hausdorff M-Spaces

In this section, the concepts of Bb-Compact M-spaces, Bb-Hausdorff M-spaces and Bb-Homeomorphism M-set functions are studied and some applications of Bb-Hausdorff M-spaces on product topology are also discussed.

Definition 3.1: An M-topological space (M, T) is said to be a Bb-Hausdorff M-space, if for each pair of distinct points m/x and m/y in M, there exist Bb-open M-sets A and B containing m/x and m/y respectively such that A ∩ B = ϕ with $C_{A\cap B}(x)$ = min $\{C_A(x), C_B(x)\}$ = 0, for all x ∈ X.

Definition 3.2: Let (M, T) be an M-topological space. A collection $\mathcal{B}$ = {B_i ⊆ M, i ∈ I is an indexed set} of subM-set of M is said to be a Bb-cover M or to be a Bb-covering of M, if $\bigcup_{i\in I} B_i$ = M, with max {$C_{B_i}(x)$, where B_i ⊆ M and i ∈ I is an indexed set} = $C_M(x)$, for all x ∈ X. Such a Bb-cover of M is said to be a Bb-open covering of M if each $B_i \in \mathcal{B}$ is a Bb-open M-set of (M, T).

Definition 3.3: An M-topological space (M, T) is said to be Bb-compact M-space if every Bb-open cover of M has a finite subcover. i.e., for any collectio n {B_i: each B_i ⊆ M is an Bb-open M-set of M} with $\bigcup_{i\in I} B_i$ = M, where I is an indexed set and max { $C_{B_i}(x)$, i ∈ I is an indexed set} = $C_M(x)$, for all x ∈ X, there exists a finite subset J of I such that $\bigcup_{i\in J} B_i$ = M with max { $C_{B_i}(x)$, i ∈ J is an indexed set} = $C_M(x)$, for all x ∈ X.

Definition 3.4: Let (M, T) be an M-topological spaces. A Bb-subM-space A of M is a Bb-compact M-space if every Bb-open cover of A by Bb-open M-sets in M has a finite subcover. i.e., for any collection {B_i: each B_i ⊆ A, is an Bb-open M-set of M} with $\bigcup_{i\in I} B_i \supseteq A$, where I is an indexed set and max { $C_{B_i}(x)$, i ∈ I is an indexed set} ≥ $C_A(x)$, for all x ∈ X, there exists a finite subset J of I such that $\bigcup_{i\in J} B_i \supseteq A$ with max { $C_{B_i}(x)$, i ∈ J is an indexed set} ≥ $C_A(x)$, for all x ∈ X.

Proposition: Let (M, T) be an M-topological space. Each Bb-closed subM-set of a Bb-compact M-space is Bb-compact.

Proof Let A be a Bb-closed subM-set of the Bb-compact M-space M and let $\mathcal{B}$ = {B_i: each B_i ⊆ M is a Bb-open M-set of M i ∈ I is an indexed set} Bb-open cover of A by Bb-open M-sets in M. Since A is Bb-closed, Then M − A is Bb-open and $\mathcal{B}^* = \mathcal{B} \cup$ {M – A} with $C'_{\mathcal{B}^*}$ (x) = max$\left\{ C_{\mathcal{B}}(x), C_{\{M-A\}}(x)\right\}$, for all x ∈ X is a Bb-open cover of M. Since M is Bb-compact, it has a finite subcover, containing only finite many members $B_1, B_2, B_3, \ldots, B_n$ of $\mathcal{B}$ and may contain M − A. Since M = (M – A) ∪ $\bigcup_{i=1}^{n} B_i$ with

$C_M(x) = \max\{C_{(M-A)}, \max\{C_{B_i}(x), i = 1,2, \ldots . n \text{ and } B_i \in \mathcal{B}\}\}$

for all x ∈ X. Hence it follows that, A ⊆ $\bigcup_{i=1}^{n} B_i$ with $C_A(x) \leq \max\left\{ C_{B_i}(x)\ i = 1, 2, \ldots \ldots \ldots n \text{ and } B_i \in \mathcal{B} \right\}$ for all x ∈ X and A has a finite subcover. Therefore $\bigcup_{i\in I} B_i$ = A with max { $C_{B_i}(x)$, i ∈ I is an indexed set}. Hence A is Bb-compact.

Proposition: Let (M, T) be an M-topological space. Each Bb-compact subM-set of a Bb-Hausdorff M-space is Bb-closed.

Proof Let A be a Bb-compact subM-set of the Bb-Hausdorff M-space. To show that A is Bb-closed, we will show that M − A is Bb-open.

Let m/x ∈ M − A with $C_{m/x}(x) \leq C_{M-A}(x)$, for all x ∈ X. Then for each m/x ∈ A there exist a disjoint M-sets $U_{\{m/x\}}$ and $V_{\{m/x\}}$ with m/x ∈ $V_{\{m/x\}}$ and m/x ∈ $U_{\{m/x\}}$. The collection of Bb-open M-sets $\left\{U_{\{m/x\}}: \{m/x\} \in A\right\}$ forms a Bb-open cover of A. Since A is a Bb-compact, this Bb-open cover has a finite subcover $\left\{U_{\{m/x\}} : i = 1, 2, \ldots n\right\}$. Let U = $\bigcup_{i=1}^{n} U_{y_i}$ with

$C_U(x) = \max\{ C_{U_{\{m/x\}}}(x): i = 1, \ldots n\}$ and V = $\bigcap_{i=1}^{n} V_{y_i}$ with $C_V(x) = \max\{ C_{V_{\{m/x\}}}(x): i = 1, \ldots n\}$.

Since each $U_{\{m/x\}}$ and $V_{\{m/x\}}$ are disjoint, we have U and V are disjoint. Also, A ⊆ U with $C_A(x) \leq C_U(x)$ and {m/x} ∈ M − A, for all x in X. We have found a Bb-open M-set V containing {m/x} which is disjoint from A. Thus, M − A is Bb-open and A is Bb-closed. Hence, each Bb-compact subM-set of a Bb-Haussdorff M-space is Bb-closed.

Proposition: Let (M, T) and (N, S) two M-topological spaces and let (M, T) be a Bb-compact M-space and f: (M, T) → (N, S). Then (N, S) is a Bb-compact.

Proof Let f: (M, T) → (N, S) be a Bb-continuous M-set function and M be a Bb-compact M-space. Let $\mathcal{B}$ be a covering of the M-set f of M by M sets open in (N, S) and f is Bb-continuous then the M-set $f^{-1}(A)$ is Bb-open in (M, T). Therefore the collection {f^{-1} (A)/ A ∈ $\mathcal{B}$} is a collection of M-sets covering M. Since M is Bb-compact, the finite subcollection $f^{-1}(A_1)$, ….., $f^{-1}(A_n)$ cover M.

Then the M-sets A_1 ,….., A_n cover f(M). Therefore f(M) = N is Bb-compact.

Definition 3.5: Let (M, T) be an M-topological spaces and A be a subM-set of M. The collection T_A = {N ∩ U: U is a open M-set} is an M-topology on A, called the subM-space M-topology on A. With the M-topology T_A, the ordered pair (A, T_A) is called a sub M-space of M.

Definition 3.6: Let (M, T) be an M-topological space and let m/x_∞ denote an ideal M-point, called the M-point at infinity, not included in M. Let $M_\infty = M \cup m/x_\infty$ with $C_{M_\infty} = \max\{C_M(x), C_{m/x_\infty}(x)\}$ for all x ∈ X and define an M-topology T_∞ on M_∞ by specifying the following Bb-open M-sets:

a) The Bb-open M-sets of (M, T), considered as subM-sets of M_∞.
b) The subM-sets of M_∞ whose complements are Bb-closed, Bb-compact subM-sets of M, and
c) The M-set M_∞. The space (X_∞, M_∞) is called the one-point Bb-compactification of M.

Proposition: Let (M, T) be an M-topological space and M_∞ its one-M-point Bb-compactification. Then

a) (M_∞, T_∞) is Bb-compact.
b) (M_∞, T_∞) is Bb-Hausdorff if and only if M is Bb-Hausdorff and locally Bb-compact.
c) M is a dense subM-set of M_∞ if and only if (M, T) is not Bb-compact.

Proof a) Any Bb-open cover $\mathcal{B}$ of M_∞ must have a member U containing m/x_∞. Since the complement $M_\infty - U$ is Bb-compact, it has a finite subcover $\{B_i\}_{i=1}^{n}$ derived from $\mathcal{B}$. Thus, finite subcover U, $B_1, B_2, \ldots, B_n$ is a finite subcover of M_∞. Hence, M_∞ is Bb-compact.

b) Suppose that M_∞ is Bb-Hausdorff. Then (M, T) is Bb-Hausdorff since the property is hereditary. Now, let {m/x} ∈ M. Since M_∞ is Bb-Hausdorff, there exists Bb-open disjoint M-sets U and V in M_∞ so that $m/x_\infty \in U$ and {m/x} ∈ V.

Thus, $V \subseteq M_\infty - U$ with $C_V(x) \le C_{(M_\infty - U)}(x)$, for all x ∈ X and this M-set is Bb-closed and Bb-compact in (M, T).

Hence $M - V \subseteq M_\infty - U$ with $C_{M-V}(x) \le C_{(M_\infty - U)}(x)$, for all x ∈ X, So M − V is Bb-compact, since it is a Bb-closed subM-set of a Bb-compact M-set (By proposition 3.1.1). Thus, M is locally Bb-compact at {m/x}.

Now, suppose that M is Bb-Hausdorff and locally Bb-compact. To show that M_∞ is Bb-Hausdorff, we only need to be able to separate m/x_∞ from any M-point in {m/x} ∈ M. Since, (M, T) is locally Bb-compact, there exists an Bb-open M-set U so that {m/x} ∈ U and M − U is Bb-compact. Then U and $M_\infty - \overline{U}$ are two disjoint Bb-open M-sets in M_∞, containing m/x and m/x_∞ respectively.

c) If (M, T) is Bb-compact, then $\{m/x_\infty\}$ is a Bb-open M-set in M_∞, since $\{m/x_\infty\} = M_\infty - M$. Thus, m/x_∞ is not a limit point of M, and $\overline{M} \ne M_\infty$. Hence, M is not dense. If M is not dense in M_∞, then $\overline{M}$ =M, since $m/x_\infty \notin \overline{M}$. Hence, $\{m/x_\infty\}$ is Bb-open in M_∞. Thus, (M, T) is Bb-compact.

Proposition: Let (M, T) be an M-topological space. If (M, T) is a Bb-Hausdorff M-space, then the one point {m/x} are Bb-closed.

Proof To show that A = M \ {m / x} is Bb-open M-set. But, if {m/y} ∈ A, then {m/x} ≠ {m/y} with $C_{\{m/x\}}(x) \ne C_{\{m/y\}}(x)$, for all x ∈ X. So by the definition of Bb-Hausdorff, there exist a Bb-open M-sets U and V containing {m/x} and {m/y} respectively such that U ∩ V = ϕ with $\min\{C_U(x), C_V(x)\} = 0$, for all x ∈ X. Thus, {m/y} ∈ V ⊆ A, so every point of A has an Bb-open neighbourhood lying entirely within A. Thus A is Bb-open M-set. Hence {m/x} is Bb-closed M-set.

Remark: The converse of the Proposition 3.5 need not be true as shown in the following example 3.1

Example: Let X = {a, b, c}, W = 2 and M = {2/a, 1/b, 1/c}. Let T = {M, ϕ, {1/a}, {1/b}, {1/a, 1/b}}. Clearly, T is an M-topology and the ordered pair (M, T) is an M-topological space. The collection of Bb-closed M-sets is {M, ϕ, {1/a}, {1/b}, {1/c}, {1/a, 1/b}, {1/a, 1/c}, {1/b, 1/c}, {2/a, 1/c}, {1/a, 1/b, 1/c}}.

Let A = {1/a} and B = {1/a, 1/b} be a Bb-closed M-set. But A ∩ B = {1/a} with $\min\{C_A(x), C_B(x)\} = 1$, for all x ∈ X which is not a Bb-Hausdorff M-space.

Definition 3.7 Let (M, T) be an M-topological spaces and N be a subM-set of M. The collection T_N = {N ∩ U: U is a Bb-open M-set} is an M-topology on N, called the Bb-subM-space M-topology on N. With the M-topology T_N, the ordered pair (N, T_N) is called a Bb-subM-space of M.

Proposition 3.6 Let (M, T) be a Bb-Hausdorff M-Space and if N ⊆ M, where N is Bb-subM-space M-topology. Then N is also a Bb-Hausdorff M-Space.

Definition 3.8 Let (M, T) and (N, S) be any two M-topological Spaces. Any M-set function f: (M, T) → (N, S) is called Bb-homeomorphism M-set function if it has the following properties:

1) f is a bijection (one-to-one and onto),
2) f is Bb-continuous,
3) The inverse function f^{-1} is Bb-continuous M-set.

If such an M-set function exists, we say (M, T) and (N, S) are Bb-homeomorphic.

Proposition: Let (M, T) and (N, S) be any two M-topological Spaces with (M, T) is Bb-homeomorphic M-set to (N, S) and (N, S) is a Hausdorff M-Space. Then (M, T) is a Bb-Hausdorff M-Space.

Proof Let f: (M, T) → (N, S) be a homeomorphism M-set function. Let $\{m/x_1\}$, $\{m/x_2\}$ ∈ M with $\{m/x_1\} \ne \{m/x_2\}$. Since f is 1-1 $f(m/x_1)$, $f(m/x_2)$ ∈ N and $f(m/x_1) \ne f(m/x_2)$ with $C_{f\{m/x_1\}}(x) \ne C_{f\{m/x_2\}}(x)$, for all x ∈ X. Since (N, S) is a Hausdorff M-space, there exist open M-sets V_1, V_2 of N such that $f(m/x_1) \in V_1$, $f(m/x_2) \in V_2$ and $V_1 \cap V_2 = \phi$ with $C_{V_1 \cap V_2}(x) = C_\phi(x) = 0$, for all x

$\in$ X. But now {m/x_1} $\in f^{-1}(V_1)$, {m/x_2} $\in f^{-1}(V_2)$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = f^{-1}(\phi) = \phi$ with $C_{f^{-1}(V1) \cap f^{-1}(V2)}$ (x) = C_ϕ(x) = 0, for all x $\in$ X. Hence (M, T) is a Bb-Hausdorff M-spaces.

Proposition 3.8: Let (M, T) and (N, S) be any two M-topological spaces. Let f, g: (M, T) → (N, S) be any two M-set functions and (N, S) be a Bb-Hausdorff M-space. Then {{m/x} ∈ M: f (m/x) = g (m/x)} is a Bb-closed M-set.

Proof Let A = {{m/x} ∈ M: f (m/x) ≠ g (m/x)} and {m/x} ∈ A. Since f (m/x) ≠ g (m/x) and (N, S) is a Bb-Hausdorff M-space there are Bb-open M-sets U, V ⊂ N such that f(m/x) ∈ U, g(m/x) ∈ V and U ∩ V = ϕ with $C_{U\cap V}$ (x) = C_ϕ (x) = 0, for all x ∈ X. Let W = $f^{-1}(U) \cap g^{-1}(V)$ with W = min {$C_{f^{-1}(U)}$ (x), $C_{g^{-1}(V)}$ (x)}, for all x ∈ X. Finite intersection of Bb-open M-sets is Bb-open M-sets and so W is Bb-open and {m/x} ∈ W. Moreover, W ⊂ A = {{m/x} ∈ M: f (m/x) ≠ g (m/x)}. Thus it follows that A is Bb-open and so {{m/x} ∈ M: f (m/x) = g (m/x)} is a Bb-closed M-set.

Definition 3.9: Let (M, T) and (N, S) be any two M-topological spaces. The product Bb-M-topology on M × N is an M-topology having as basis the collection B of all M-sets of the form U × V, Where U is an Bb-open sub M-set of M and V is a Bb-open sub M-set of N.

Definition 3.10: Let (M, T) be an M-topological space in $[X]^W$. Let the diagonal △ = {(m/x, m/x) ; m/x ∈ M} be a sub M-set of M × M. Then the complement of the diagonal is defined as D = $\mathcal{C}_{MxM}$ (△) = {(m/x_1, m/x_2); m/x_1 ≠ m/x_2 with $C_{\{m/x_1\}}$ (x) ≠ $C_{\{m/x_2\}}$ (x), for all x ∈ X}.

Proposition: Let (M, T) be an M-topological Spaces. Let the diagonal △ = {(m/x, m/x) ; m/x ∈ M}. Then (M, T) is a Bb-Hausdorff M-space if and only if △ is a Bb-Closed M-set in M× M.

Proof Assume (M, T) is a Bb-Hausdorff M-space. We have to prove that △ is Bb-closed M-Set. It is enough to show that D = $\mathcal{C}_{MxM}$ (△) = {(m/x_1, m/x_2) ; m/x_1 ≠ m/x_2 with $C_{\{m/x_1\}}$(x) ≠ $C_{\{m/x_2\}}$ (x) for all x ∈ X, is a Bb-open M-set. So let w/x = (m/x_1, m/x_2) ∈ D. Then m/x_1 ≠ m/x_2 so there exist Bb-open M-Sets U, V in (M, T) such that m/x_1 ∈ U, m/x_2 ∈ V and U ∩ V = ϕ with $C_{U\cap V}$(x) = C_ϕ(x) = 0, for all x ∈ X.

We claim that U × V ⊂ D. If U × V ⊄ D then there exists some element (m/t, m/t) ∈ U X V (with m/t ∈ M) such that (m/t, m/t) ∉ D. But then m/t ∈ U ∩ V, and since U ∩ V = ϕ with $C_{U\cap V}$(x) = C_ϕ(x) = 0 for all x ∈ X, this is impossible. So U × V ⊂ D.

We have shown that for each m/x ∈ D there exist Bb-open M-sets U, V such that m/x ∈ U × V ⊂ D.

Hence D is Bb-open (by the definition of the product Bb-M-topology on M × M) and so △ is Bb-closed.

Conversely, we assume △ is Bb-closed. Thus D = $\mathcal{C}_{MxM}$ (△) is a Bb-open M-set. Let m/x_1, m/x_2 ∈ M with m/x_1 ≠ m/x_2. Then (m/x_1, m/x_2) ∈ D and D is Bb-open so there exist Bb-open M-sets U, V such that (m/x_1, m/x_2) ∈ U × V ⊂ D.

Now m/x_1 ∈ U, m/x_2 ∈ V and U ∩ V = ϕ, for if m/t ∈ U ∩ V then (m/t, m/t) ∈ U × V ⊂ D and D is the M-set of all elements (m/a, m/b) ∈ M × M such that m/a ≠ m/b with $C_{\{m/a\}}$ (x) ≠ $C_{\{m/b\}}$ (x) for all x ∈ X, so this is impossible. Hence (M, T) is a Bb-Hausdorff M-space.

Definition 3.11: Let (M, T) and (N, S) be any two M-topological space. Then the spaces is said to be a Bb-Hausdorff M-Space on M × N, if there exist disjoint Bb-open M-sets P, Q in M × N such that (m/x, m/y) ∈ P, (m/x', m/y') ∈ Q and P ∩ Q = ϕ with $C_{P\cap Q}$(x) = C_ϕ(x) = 0, for all x ∈ X.

Proposition: Let (M, T) and (N, S) be any two M-topological spaces. Then M × N is a Bb-Hausdorff M-Space if and only if both (M, T) and (N, S) are Bb-Hausdorff M-Spaces.

Proof Assume that M × N be a Bb-Hausdorff M-space. We have to prove that (M, T) and (N, S) are Bb-Hausdorff M-spaces.

Suppose that m/x, m/x' ∈ M with m/x ≠ m/x'. Choose m/y ∈ N. Then there are Bb-open M-sets P, Q in M × N such that (m/x, m/y) ∈ P, (m/x', m/y) ∈ Q and P ∩ Q = ϕ with $C_{P\cap Q}$(x) = C_ϕ(x) = 0, for all x ∈ X. Since P is a Bb-open M-set by the definition Bb-product M-space on M × N, there exist Bb-open M-set U, V in (M, T) and (N, S) respectively such that (m/x, m/y) ∈ U × V ⊂ P, and similarly there exist Bb-open M-sets U', V' in (M, T) and (N, S) respectively such that (m/x', m/y) ∈ U'× V' ⊆ Q. We have (U × V) ∩ (U' × V') ⊂ P ∩ Q = ϕ with $C_{P\cap Q}$(x) = C_ϕ(x) = 0, for all x ∈ X. So that, (U ∩ U') × (V ∩ V')= ϕ with $C_{(U\cap U') x (V\cap V')}$(x) = C_ϕ(x) = 0.

Now if m/t ∈ U ∩ U' then (m/t, m/y) ∈ (U ∩ U') × (V ∩ V') so there is no such element m/t, i.e. U ∩ U'= ϕ with $C_{U\cap U'}$ (x) = C_ϕ(x) = 0. Thus for m/x, m/x' ∈ M with m/x ≠ m/x' we have produced Bb-open M-sets U, U' such that m/x ∈ U, m/x' ∈ U' and U ∩ U' = ϕ with $C_{U\cap U'}$ (x) = C_ϕ(x) = 0. Hence (M, T) is a Bb-Hausdorff M-space. Similarly, (N, S) is a Bb-Hausdorff M-space.

Conversely, Suppose (M, T) and (N, S) are Bb-Hausdorff M-spaces. Let w/x = (m/x, m/y), w/x' = (m/x', m/y') ∈ M × N with w/x = w/x'. Then m/x ≠ m/x' or m/y ≠ m/y'. We assume m/x ≠ m/x'. (The other case is similar). Then there exist Bb-open M-sets U, U' in M with m/x ∈ U, m/x' ∈ U' and U ∩ U'= ϕ with $C_{U\cap U'}$ (x) = C_ϕ(x) = 0. Put P = U × M with C_p(x) = $C_{U x M}$(x), Q = U' ×

N with $C_Q(x) = C_{U' \times N}(x)$, Then $w/x \in P$, $w/x' \in Q$ and $P \cap Q = (U \times N) \cap (U' \times N) = (U \cap U') \times N = \phi \times N = \phi$ with $C_{P \cap Q}(x) = C_{\phi}(x) = 0$, for all $x \in X$. Hence $M \times N$ is a Bb-Hausdorff M-Space.

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5. References

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