International Journal of Statistics and Applied Mathematics 2018; 3(4): 01-11

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(4): 01-11

www.mathsjournal.com
Received: 01-05-2018
Accepted: 02-06-2018

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Two component mixture of Laplace and Laplace type distributions with applications to manpower planning models

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Abstract
This paper is on the mixtures of Laplace and Laplace type bimodal distribution with the probability p and 1-p respectively. In addition to the distribution properties and inferential aspects studied the same distribution was applied to the real-life data on the manpower planning of an organization under the two different promotional policies namely promotion by seniority and promotion by random.

Keywords: Laplace, two parameter Laplace, bimodal, mixtures, man power planning models, promotion by seniority, promotion by random

1. Introduction

In two parameters Laplace type distributions and three parameter Laplace type distributions. Distributions it was observed that the point at $X = \mu$ is zero, which is seemingly not useful in some practical situations. So to lift the drift at the mean point μ one way of consideration is assuming that the random variable follows Laplace distribution with probability p and two parameter Laplace type distribution with probability $1-p(0 \leq p \leq 1)$. This idea is imbedded in generating the mixture distributions. Hence in this paper it was developed the probability density function of two component mixture of Laplace and Laplace type distributions. The distributional properties of this distribution are also studied. This was applied to a practical manpower situation.

2. Mixtures of Laplace and Laplace Type Distributions

Let us consider a random variable X, which is such that, with probability p, it comes from Laplace distribution with probability density function

$$f(x,\mu,\beta) = \frac{1}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|}, -\infty < x < \infty, -\infty < \mu < \infty, \beta > 0$$

And with probability $1-p$ comes from a two parameter Laplace type distribution with probability density function

$$f(x,\mu,\beta) = \frac{1}{4\beta}\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}, -\infty < x < \infty, -\infty < \mu < \infty, \beta > 0$$

The probability density function of X is

$$f(x,\mu,\beta,p) = \frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta}\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}$$

$$= \left[\frac{2p+(1-p)\left(\frac{x-\mu}{\beta}\right)^2}{4\beta}\right] e^{-\left|\frac{x-\mu}{\beta}\right|} \quad -\infty < x < \infty, -\infty < \mu < \infty, \beta > 0 \qquad \rightarrow 1$$

This contains two parameters μ *and* β**.** To maintain the parsimonious property of the modal in two components of the mixture population, the same parameters are considered. If $p = 1$, this distribution becomes Laplace distribution with parameters μ *and* β**.** And if $p = 0$, the distribution becomes two parameter Laplace type distributions.

3. Properties of the Distribution

The distribution function of the random variable X is

$$F_X(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta}\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

$$= p\left[1 - \frac{e^{-\left|\frac{x-\mu}{\beta}\right|}}{2}\right] + (1-p)\left[1 - \left\{\frac{1}{2} + \left(\frac{x-\mu}{2\beta}\right) + \left(\frac{x-\mu}{2\beta}\right)^2\right\}\right] e^{-\left(\frac{x-\mu}{\beta}\right)}, for\ x \geq \mu$$

$$= p\left[\frac{e^{-\left|\frac{x-\mu}{\beta}\right|}}{2}\right] + (1-p)\left[\frac{1}{2} + \left(\frac{x-\mu}{2\beta}\right) + \left(\frac{x-\mu}{2\beta}\right)^2\right] e^{-\left(\frac{x-\mu}{\beta}\right)}, for\ x < \mu \qquad \rightarrow 2$$

For different values of $\mu, \beta and\ p$**,** the frequency curves of the distribution are plotted and shown in figure: 1

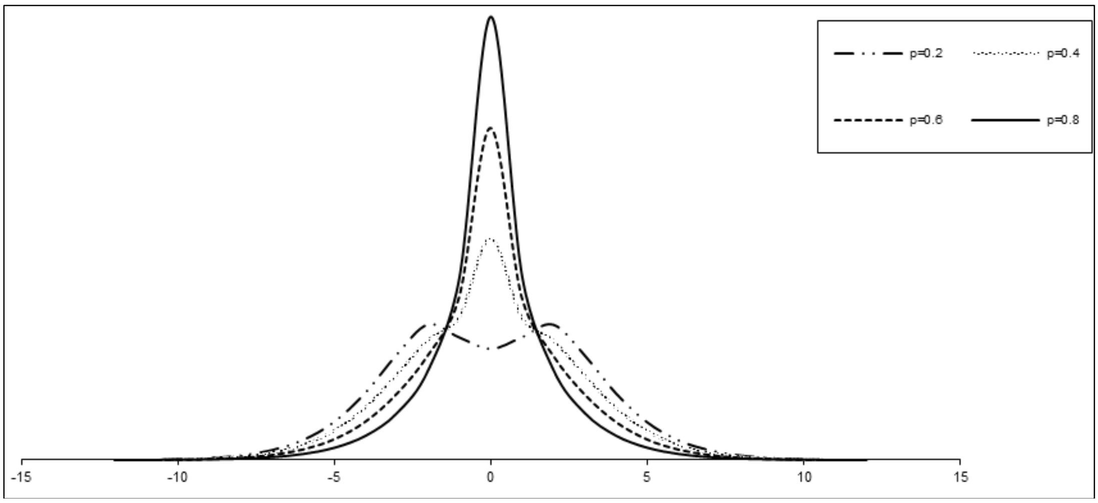

Fig 1: The frequency curves of the distribution

Mean, Mode and median of the distribution

The mean of this random variable X is

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} x\left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta}\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

$$= \frac{p}{2\beta}\int_{-\infty}^{\infty} xe^{-\left|\frac{x-\mu}{\beta}\right|}dx + + \frac{1-p}{4\beta}\int_{-\infty}^{\infty} x\left[\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

$$= p\mu + (1-p)\mu = \mu \qquad \rightarrow 3$$

This distribution is a bimodal distribution for the value of $0 \leq p \leq \frac{1}{3}$. For $p = \frac{1}{3}$, this is unimodal. For $0 < p < \frac{1}{3}$, the modes are

$\mu + \beta\left(1 + \sqrt{1 - \frac{2p}{1-p}}\right)$ And $\mu - \beta\left(1 + \sqrt{1 - \frac{2p}{1-p}}\right)$

The distance between two modes is $2\left[\beta\left(1 + \sqrt{1 - \frac{2p}{1-p}}\right)\right]$ $\rightarrow 4$

The median of this distribution can be obtained by solving the equation

$$\int_{-\infty}^{M} f(x)\,dx + \int_{M}^{\infty} f(x)\,dx = 1$$

That is, $F(M) = 1 - F(M)$ and $\Rightarrow F(M) = \frac{1}{2}$

Therefore $\int_{-\infty}^{M} \left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta}\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx = \frac{1}{2}$

$\Rightarrow M = \mu$ →5

That is for this distribution the Median = Mean and hence the distribution is said to be symmetric at $x = \mu$

Moment generating function

The moment generating function of the two-component mixture of Laplace and Laplace type variate is

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tX}\left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta}\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

$$= \frac{p}{2\beta}\int_{-\infty}^{\infty} e^{tX} e^{-\left|\frac{x-\mu}{\beta}\right|} dx + + \frac{1-p}{4\beta}\int_{-\infty}^{\infty} e^{tX}\left[\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

Using the transformations and on simplification we get

$$M_X(t) = p\left[\frac{\frac{e^{t\mu}}{2}}{1+\beta^2 t^2}\right] + (1-p)\left[\frac{e^{t\mu}}{2}\left\{\frac{2+6\beta^2 t^2}{(1-\beta^2 t^2)^3}\right\}\right]$$

$$= \frac{e^{t\mu}}{2}\left[\left(\frac{p}{1+\beta^2 t^2}\right) + (1-p)\left\{\frac{2+6\beta^2 t^2}{(1-\beta^2 t^2)^3}\right\}\right] \qquad \to 6$$

Characteristic function

The characteristic function of the variate X is

$$\emptyset_X(t) = \int_{-\infty}^{\infty} e^{itx} f(x)\, dx = \int_{-\infty}^{\infty} e^{itX}\left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta}\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

$$= \frac{p}{2\beta}\int_{-\infty}^{\infty} e^{itX} e^{-\left|\frac{x-\mu}{\beta}\right|} dx + + \frac{1-p}{4\beta}\int_{-\infty}^{\infty} e^{itX}\left[\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

Using the transformation, y= $x - \mu$ and on simplification one can get

$$\emptyset_X(t) = p\left[\frac{\frac{e^{it\mu}}{2}}{1+\beta^2 t^2}\right] + (1-p)\left[\frac{e^{it\mu}}{2}\left\{\frac{2+6\beta^2 t^2}{(1-\beta^2 t^2)^3}\right\}\right] \qquad \to 7$$

Central Moments of the Distribution

The central moments of this distribution can be obtained as

$$\mu_{2n} = \int_{-\infty}^{\infty} (x-\mu)^{2n} f(x)\, dx = \int_{-\infty}^{\infty} (x-\mu)^{2n}\left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta}\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

$$= \frac{p}{2\beta}\int_{-\infty}^{\infty} (x-\mu)^{2n} e^{-\left|\frac{x-\mu}{\beta}\right|} dx + + \frac{1-p}{4\beta}\int_{-\infty}^{\infty} (x-\mu)^{2n}\left[\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}\right] dx$$

Using the transformation and on simplification, one can get

$\mu_{2n} = p(n)! + (1-p)\frac{(2n+2)!\beta^{2n}}{2}$ If n is even

= 0 if n is odd

In particular the first four central moments are

$\mu_1 = 0, \mu_2 = 2\beta^2, \mu_3 = 0, \mu_4 = \beta^4(360 - 366p)$ →8

Hence the variance of the distribution is $2\beta^2(6 - 5p)$. Since the distribution is symmetric, its skewness is zero and the kurtosis of the distribution is

$$\beta_2 = \frac{(360-366p)}{4(6-5p)^2} \quad \rightarrow 9$$

Hazard rate of the distribution

The hazard rate of the distribution is $h(x) = \frac{f(x)}{1-F(x)}$

Where $f(x)$ and $F(x)$ are as given in equations (1) and (2) respectively. →10

Additive property of the distribution

For verify the additive property of the distribution, consider two independent variables X_1 and X_2 with density function

$$f(x_1, \mu_1, \beta_1, p) = \frac{p}{2\beta_1} e^{-\left|\frac{x_1-\mu_1}{\beta_1}\right|} + \frac{1-p}{4\beta_1}\left(\frac{x_1-\mu_1}{\beta_1}\right)^2 e^{-\left|\frac{x_1-\mu_1}{\beta_1}\right|}$$

$$f(x_2, \mu_2, \beta_2, p) = \frac{p}{2\beta_2} e^{-\left|\frac{x_2-\mu_2}{\beta_2}\right|} + \frac{1-p}{4\beta_2}\left(\frac{x_2-\mu_2}{\beta_2}\right)^2 e^{-\left|\frac{x_2-\mu_2}{\beta_2}\right|}$$

The characteristic functions are

$$\emptyset_{X_1}(t) = p\left[\frac{\frac{e^{it\mu_1}}{2}}{1+{\beta_1}^2t^2}\right] + (1-p)\left[\frac{e^{it\mu_1}}{2}\left\{\frac{2+6{\beta_1}^2t^2}{\left(1-{\beta_1}^2t^2\right)^3}\right\}\right] \text{ And}$$

$$\emptyset_{X_2}(t) = p\left[\frac{\frac{e^{it\mu_2}}{2}}{1+{\beta_2}^2t^2}\right] + (1-p)\left[\frac{e^{it\mu_2}}{2}\left\{\frac{2+6{\beta_2}^2t^2}{\left(1-{\beta_2}^2t^2\right)^3}\right\}\right]$$

Then the characteristic function of the sum of the random variables, $X_1 + X_2$ is

$$\emptyset_{X_1+X_2}(t) = \emptyset_{X_1}(t)\emptyset_{X_2}(t)$$

$$= \left\{\frac{e^{it\mu_1}}{2}\left[\frac{p}{1+{\beta_1}^2t^2} + (1-p)\left\{\frac{2+6{\beta_1}^2t^2}{\left(1-{\beta_1}^2t^2\right)^3}\right\}\right]\right\}\left\{\frac{e^{it\mu_2}}{2}\left[\frac{p}{1+{\beta_2}^2t^2} + (1-p)\left\{\frac{2+6{\beta_2}^2t^2}{\left(1-{\beta_2}^2t^2\right)^3}\right\}\right]\right\}$$

The lack of closed form of the characteristic function does not permit to have a single way of identifying the distribution of the sum of the random variables. That is additive property was not possessing by this distribution. However, from Cramer 1955 one can invent the characteristic function by using the inversion formula

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx}\emptyset_X(t)\, dt$$

4. Estimation of the Parameters

This section is on the the estimation of the parameters arising from the mixture of Laplace and Laplace type distribution. The drift parameter p (mixing parameter) can be estimated by using the method of moments along with the location and scale parameters. Since the estimation of the location and scale parameter involves the drift parameter, estimate p can be obtained by equating the sample kurtosis with the population kurtosis.

We have $\mu_4 = \beta^4(360 - 366p)$ and $\mu_2 = 2\beta^2(6 - 5p)$

Therefore p can be estimated by solving the following quadratic equation and picking up the value which lie between 0 and 1 as an estimate of the parameter

$$p = \frac{\mu_4}{\mu_2^2} = \frac{n\sum_{i=1}^{n}(x_i-\mu)^4}{\left[\sum_{i=1}^{n}(x_i-\mu)^2\right]^2} = \frac{(360-366p)}{4(6-5p)^2} \quad \rightarrow 11$$

Where the RHS is the sample Kurtosis. The moment estimators μ and β^2 can be obtained as conditional to p as $\hat{\mu} = \bar{x}$ and

$$2\beta^2(6-5p) = s^2$$

$$2\beta^2(6-5p) = s^2 \Rightarrow \hat{\beta}^2 = \frac{s^2}{2(6-5p)} \quad \rightarrow 12$$

Where, s^2 is the sample variance.

The variance of these estimators is given by $V(\bar{x}) = \frac{2\beta^2(6-5p)}{n}$

$$V(s^2) = \frac{\mu_4-\mu_2^2}{n} - \frac{2(\mu_4-2\mu_2^2)}{n^2} + \frac{(\mu_4-3\mu_2^2)}{n^3} \quad \rightarrow 13$$

Where μ_i is the ith central moment

Therefore,

$$V(s^2) = \left[\frac{(360-366p)\beta^4-4\beta^4(6-5p)^2}{n}\right] - 2\left[\frac{(360-366p)\beta^4-8\beta^4(6-5p)^2}{n^2}\right] + \left[\frac{(360-366p)\beta^4-12\beta^4(6-5p)^2}{n^3}\right]$$

As these variances tends to zero as$n \to \infty$, these estimators $\bar{x}$ and s^2 are consistent for μ and σ^2.

The maximum likelihood estimation

The maximum likelihood estimation of p and β^2 can be obtained after estimating the value of μ through median similar to the location parameter estimated in two parameter Laplace type distributions. After substituting the estimate of μ the likely hood equation of the sample is

$$L = \prod_{i=1}^{n} f(x_i;\mu,\beta,p) = \left(\frac{p}{2\beta}\right)^n + \left(\frac{1-p}{4\beta}\right)^n \prod_{i=1}^{n}\left[\left(\frac{x_i-\mu}{\beta}\right)^2\right] e^{-\sum_{i=1}^{n}\left|\frac{x_i-\mu}{\beta}\right|}$$

$$logL = -nlogp - nlog2\beta + nlog(1-p) - nlog4\beta + 2\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\beta}\right) - \sum_{i=1}^{n}\left|\frac{x_i-\mu}{\beta}\right|$$

Differentiating with respect to p and β^2 the likelihood equations are

$$\frac{\partial LogL}{\partial p} = \frac{n}{p} - \frac{n}{(1-p)} = 0$$

$$\frac{\partial LogL}{\partial \beta} = \frac{-n}{2\beta} - \frac{n}{4\beta} - \frac{n}{2\beta^2}\sum(x_i-\mu) + \frac{1}{\beta^2}\sum|x_i-\mu|$$

Let $\beta^2 = \theta \Rightarrow \beta = \sqrt{\theta}$

$$\frac{\partial LogL}{\partial \beta^2} = \frac{n}{\beta^2} - \sum(x_i-\mu)\beta^{-3} + \frac{1}{2}\sum|x_i-\mu|\beta^{-3} \quad \rightarrow 14$$

Respectively.

Solving these equations iteratively for p and β^2 using the Numerical techniques like Newton Raphson's Method, we can obtain the maximum likely hood estimators.

5. Application of mixture of Laplace and two Parameters Laplace type bimodal distribution to Manpower Planning

In this section we consider the application of the developed distribution to the manpower planning models. The complete length of service of an employee in an organization is the total duration of time that employee spends in an organization. In general, the complete length of service can be viewed as a random variable as it is determined by a number of factors in nature. This is one of the most important features characterizing the manpower situation and is important in manpower models, which describe the manpower structure of the organization and useful in obtaining optimal man power policies.

Several researchers have approximated the complete length of service distributions by different probability distributions. Silcock (1954) [1] have considered that the complete length of service of an employee fallows and exponential distribution. However exponential distribution is unimodal having long right tail with constant hazard rate. But in many private and public-sector organizations the frequency curve of the complete length of service of an employee in an organization gradually increase up to a point and decline up to some other point and again increase and decrease, that is the curve is bimodal. This phenomenon may be due to the imposition of probation period in many organizations. After an employee joining the organization, because of social and behavioral factors he is leaving and his complete length of service forms bimodal. Since the complete length of service of an employee cannot be negative, and cannot be greater than a specified value A, we can truncate the theoretical distribution given in section -2 at '0' in left and at A in right and impose the condition that the location parameter $\mu > 0$. Then we can approximate the complete length of service of an employee fallows left truncated mixture of Laplace and Laplace type distribution.

Then the probability density function of the complete length of service is

$$g(x) = \frac{f(x)}{F(A)-F(0)} \quad \rightarrow 15$$

Where $f(x)$ is the probability density function of given in equation (1)

From equation (2), $F(A) = p\left[1-\frac{e^{-\left(\frac{A-\mu}{\beta}\right)}}{2}\right] + (1-p)\left[\frac{1}{2}+\left(\frac{A-\mu}{2\beta}\right)+\left(\frac{A-\mu}{2\beta}\right)^2\right]e^{-\left(\frac{A-\mu}{\beta}\right)}$

And $F(0) = \left[\frac{1}{2}+(1-p)\left(\frac{\mu^2}{4\beta^2}-\frac{\mu}{2}\right)\right]e^{-\frac{\mu}{\beta}}$

Let $= F(A) - F(0)$, then

$$M = \left\{p\left[1-\frac{e^{-\left(\frac{A-\mu}{\beta}\right)}}{2}\right] + (1-p)\left[\frac{1}{2}+\left(\frac{A-\mu}{2\beta}\right)+\left(\frac{A-\mu}{2\beta}\right)^2\right]e^{-\left(\frac{A-\mu}{\beta}\right)}\right\} - \left[\frac{1}{2}+(1-p)\left(\frac{\mu^2}{4\beta^2}-\frac{\mu}{2}\right)\right]e^{-\frac{\mu}{\beta}}$$

Then $g(x) = \frac{1}{M}\left[\frac{2p+(1-p)\left(\frac{x-\mu}{\beta}\right)^2}{4\beta}\right]e^{-\left|\frac{x-\mu}{\beta}\right|}$

For different values of p, μ *and* β the frequency curves of their distribution shown in figure - 2

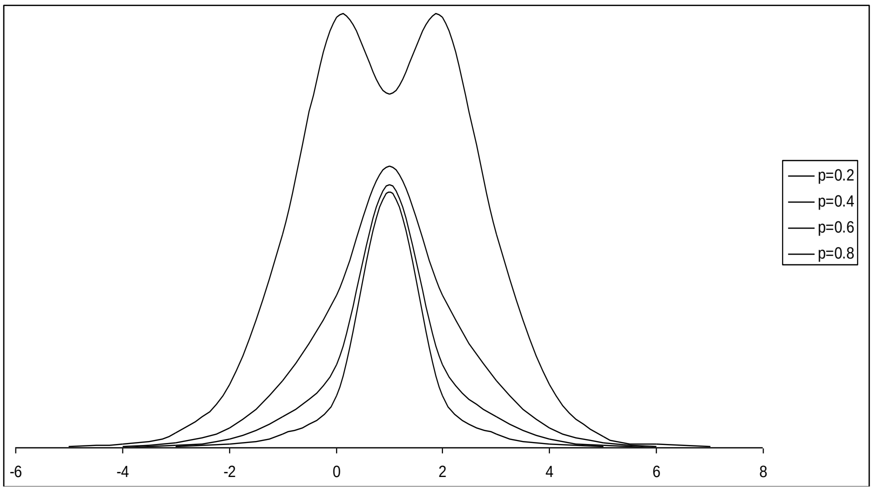

Fig 2

The probability that an employee served in an organization for the length of time ‘t’ is $G(t)$, which is survival function of the employee in the organization is defined as the compliment of distribution function. We have distribution function of complete length of service as

$$F(t) = \int_0^t g(t)dt$$

$$= \frac{1}{M}\left[\frac{p}{2\beta}\int_0^t e^{-\left|\frac{t-\mu}{\beta}\right|}dt + \frac{(1-p)}{4\beta}\int_0^t\left(\frac{t-\mu}{\beta}\right)^2 e^{-\left|\frac{t-\mu}{\beta}\right|}dt\right]$$

$$= \frac{1}{M}\left[\frac{p}{2}\left(1-e^{-\frac{\mu}{\beta}}\right)+\frac{(1-p)}{2}\left\{1-e^{-\frac{\mu}{\beta}}\left(1+\frac{\mu}{\beta}+\frac{\mu^2}{\beta^2}\right)\right\}\right] \; for\ t < \mu \quad \rightarrow 16$$

$$= \frac{1}{M}\left[\frac{p}{2}\left(1-e^{-\left(\frac{t-\mu}{\beta}\right)}\right)+\frac{(1-p)}{2}\left\{1-e^{-\left(\frac{t-\mu}{\beta}\right)}\left(1-\left(\frac{t-\mu}{\beta}\right)+\frac{1}{2}\left(\frac{t-\mu}{\beta}\right)^2\right)\right\}\right] \; for\ t \geq \mu \quad \rightarrow 17$$

Since$G(t) = 1 - F(t)$,

$$G(t) = \frac{1}{M}\left[\frac{p}{2}\left(1-e^{-\frac{\mu}{\beta}}\right)+\frac{(1-p)}{2}\left\{1-e^{-\frac{\mu}{\beta}}\left(1+\frac{\mu}{\beta}+\frac{\mu^2}{\beta^2}\right)\right\}\right] \; for\ t < \mu$$

$$= \frac{1}{M}\left[\frac{p}{2}\left(1 - e^{-\left(\frac{t-\mu}{\beta}\right)}\right) + \frac{(1-p)}{2}\left\{1 - e^{-\left(\frac{t-\mu}{\beta}\right)}\left(1 - \left(\frac{t-\mu}{\beta}\right) + \frac{1}{2}\left(\frac{t-\mu}{\beta}\right)^2\right)\right\}\right] \; for\; t \geq \mu \quad \rightarrow 18$$

The force of separation, which is known as rate of labour wastage can be obtained as

$$L(t) = \frac{g(t)}{G(t)}$$

Therefore, for this manpower model the rate labour wastage is

$$= \frac{\left[\frac{2p+(1-p)\left(\frac{x-\mu}{\beta}\right)^2}{4\beta}\right]e^{-\left|\frac{x-\mu}{\beta}\right|}}{4\beta\left[M - \left\{\frac{p}{2}\left(1-e^{-\frac{\mu}{\beta}}\right) + \frac{(1-p)}{2}\left\{1-e^{-\frac{\mu}{\beta}}\left(1+\frac{\mu}{\beta}+\frac{\mu^2}{\beta^2}\right)\right\}\right\}\right]} \; for\; t < \mu$$

$$= \frac{2p + (1-p)\left(\frac{x-\mu}{\beta}\right)^2}{4\beta\left[M - \left\{\frac{p}{2}\left(1 - e^{-\left(\frac{t-\mu}{\beta}\right)}\right) + \frac{(1-p)}{2}\left\{1 - e^{-\left(\frac{t-\mu}{\beta}\right)}\left(1 - \left(\frac{t-\mu}{\beta}\right) + \frac{1}{2}\left(\frac{t-\mu}{\beta}\right)^2\right)\right\}\right\}\right]} \; for\; t \geq \mu$$

Where

$$M = \left\{p\left[1 - \frac{e^{-\left(\frac{A-\mu}{\beta}\right)}}{2}\right] + (1-p)\left[\frac{1}{2} + \left(\frac{A-\mu}{2\beta}\right) + \left(\frac{A-\mu}{2\beta}\right)^2\right]e^{-\left(\frac{A-\mu}{\beta}\right)}\right\} - \left[\frac{1}{2} + (1-p)\left(\frac{\mu^2}{4\beta^2} - \frac{\mu}{2}\right)\right]e^{-\frac{\mu}{\beta}} \quad \rightarrow 19$$

The average time that an employee spends in the organization is

$$E(X) = \int_0^A t g(t)\, dt = \int_0^A t \frac{1}{M}\left[\frac{2p + (1-p)\left(\frac{x-\mu}{\beta}\right)^2}{4\beta}\right] e^{-\left|\frac{t-\mu}{\beta}\right|} dt$$

$$= \frac{1}{M}\left[\frac{p}{2\beta}\int_0^A t e^{-\left|\frac{x-\mu}{\beta}\right|} dt + \frac{(1-p)}{4\beta}\int_0^A t\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} dt\right]$$

Using the transformation and on simplification, one can get

$$\frac{1}{M}\left\{\frac{p}{2}\left[\left[e^{\frac{\mu}{\beta}}(\beta-\mu) - e^{-\left(\frac{A-\mu}{\beta}\right)}(\beta + A - \mu)\right] + \mu\left(e^{\frac{\mu}{\beta}} - e^{-\left(\frac{A-\mu}{\beta}\right)}\right)\right]\right\}$$

$$+ \frac{1}{M}\left\{\frac{1-p}{4}\left[\left[e^{\frac{\mu}{\beta}}\left(6\beta - 6\mu + \frac{3\mu^2}{\beta} - \frac{\mu^3}{\beta^2}\right) - e^{-\left(\frac{A-\mu}{\beta}\right)}\left(6\beta + 6(A-\mu) + \frac{3(A-\mu)^2}{\beta} - \frac{(A-\mu)^3}{\beta^2}\right)\right] + \mu e^{\frac{\mu}{\beta}}\left(\frac{\mu^2}{\beta^2} + \frac{2\mu}{\beta} + 1\right) - e^{-\left(\frac{A-\mu}{\beta}\right)}\left(\left(\frac{A-\mu}{\beta}\right)^2 + \left(\frac{A-\mu}{\beta}\right) + 1\right)\right]\right\} \quad \rightarrow 20$$

$$E(t^2) = \int_0^A t^2 g(t)\, dt = \int_0^A t^2 \frac{1}{M}\left[\frac{2p + (1-p)\left(\frac{x-\mu}{\beta}\right)^2}{4\beta}\right] e^{-\left|\frac{x-\mu}{\beta}\right|} dt$$

$$= \frac{1}{M}\left[\frac{p}{2\beta}\int_0^A t^2 e^{-\left|\frac{x-\mu}{\beta}\right|} dt + \frac{(1-p)}{4\beta}\int_0^A t^2\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} dt\right]$$

Using the transformation and on simplification, one can get

$$E(t^2) = \frac{1}{M}\left[\frac{p}{2}S + \frac{(1-p)}{4}K\right]$$

Where

$$S = \beta^2\left[e^{\frac{\mu}{\beta}}\left(\frac{\mu^2}{\beta^2} + \frac{2\mu}{\beta} + 1\right) - e^{-\left(\frac{A-\mu}{\beta}\right)}\left(\left(\frac{A-\mu}{\beta}\right)^2 + 2\left(\frac{A-\mu}{\beta}\right) + 1\right)\right] + \mu^2\left[e^{\frac{\mu}{\beta}} - e^{-\left(\frac{A-\mu}{\beta}\right)}\right]$$

$$+2\beta\left[e^{\frac{\mu}{\beta}}\left(1 - e^{\frac{\mu}{\beta}}\right) - e^{-\left(\frac{A-\mu}{\beta}\right)}\left(1 + \left(\frac{A-\mu}{\beta}\right)\right)\right] \quad \rightarrow 21$$

$$K = \beta^2\left[e^{\frac{\mu}{\beta}}\left(\frac{\mu^4}{\beta^4} - \frac{4\mu^3}{\beta^3} + \frac{12\mu^2}{\beta^2} - \frac{\mu}{\beta}11\right) - e^{-\left(\frac{A-\mu}{\beta}\right)}\left[\left(\frac{A-\mu}{\beta}\right)^4 + 4\left(\frac{A-\mu}{\beta}\right)^3 + 12\left(\frac{A-\mu}{\beta}\right)^2 - 24\left(\frac{A-\mu}{\beta}\right) - 1\right]\right]$$

$$+\mu^2\left[e^{\frac{\mu}{\beta}}\left(\frac{\mu^2}{\beta^2} + \frac{2\mu}{\beta} + 1\right) - e^{-\left(\frac{A-\mu}{\beta}\right)}\left(\left(\frac{A-\mu}{\beta}\right)^2 - 2\left(\frac{A-\mu}{\beta}\right) - 1\right)\right]$$

$$+2\beta\mu\left[e^{\frac{\mu}{\beta}}\left(6 - 6\frac{\mu}{\beta} + \frac{3\mu^2}{\beta^2} - \frac{\mu^3}{\beta^3}\right) - e^{-\left(\frac{A-\mu}{\beta}\right)}\left(6 + 6\left(\frac{A-\mu}{\beta}\right) + 3\left(\frac{A-\mu}{\beta}\right)^2 - \left(\frac{A-\mu}{\beta}\right)^3\right)\right] \quad \rightarrow 22$$

That is the variability of the complete length of service of an employee is

$$Var(t) = E(t^2) - [E(t)]^2 = \frac{1}{M}\left[\frac{p}{2}S + \frac{(1-p)}{4}T\right] - [E(X)]^2 \quad \rightarrow 23$$

The renewal density of this model is

$$h(t) = g(t) + \int_0^t G(t-x)g(x)dx \quad \rightarrow 24$$

Where $g(t)$ and $G(t)$are as defined in (18)

Using the values of $g(t)$ and $G(t)$ in the (19) one can get the $h(t)$

6. Recruitment Polices For Manpower Model

In this section, we study the different recruitment policies of an organization, which play a vital role in Manpower Planning strategies. An organization can be viewed as having two grades. The first grade is training grade and the second grade is organization grade. The organization itself in general possess an hierarchical structure. The transfer between training grade (grade I) and organization grade (grade II) is considered as promotion. The whole organization is considered as single unit. Let N_1 and N_2 are the sizes of training grade and organization grade respectively. Then the total number of employees in the organization is $N_1 + N_2 = N$. Here N_2 is assumed to be known as fixed constant.

In any organization the promotion policies are of two types namely (i) promotion by seniority (i.e.) the most senior member of the grade I is promoted to grade II and (ii) promotion by random. This is usually done based on merit. Let w_1 be the individual loss rate in training grade and w_2 is the individual loss rate in organization grade and p is the promotion rate. Since the input and output of each grade must balance, for grade II, it can be written as

$$N_1 p = N_2 w_2 \quad \rightarrow 25$$

In equilibrium, the expected input to the organization per unit time is $\frac{N_1}{\theta}$ when θ is the mean length of completed service. Thus for grade I

$$\frac{N}{\theta} = N_1(p + w_1) = N_1 w_1 + N_2 w_2 \quad \rightarrow 26$$

In general, $w_1, w_2 and\ p$ are function of time. The value of $p, w_1\ and\ w_2$ are independent of time, when the system is equilibrium. Let the θ_1 be the average time spent in grade I. Then under study state the expected number of vacancies arriving per unit time in this grade is

$$\frac{N_1}{\theta_1} = N_1(p + w_1)$$

$$\frac{1}{\theta_1} = (p + w_1) \quad \rightarrow 27$$

From (26) and (27) $\frac{N}{\theta} = \frac{N_1}{\theta_1}$ →28

From this equation the throughput for the whole system is equal to the number of vacancies in the grade I.

Promotion by Seniority: Let a $\left(\frac{t}{T}\right)$ denote the age distribution of the system at time 't' give that the organization is established at time$t = 0$. Thus, the probability that an employee chosen at random is having a length of service as T in $(t, t + \delta t)$ is $\left(\frac{t}{T}\right)\delta t$

Therefore $\alpha\left(\frac{t}{T}\right)\delta t =$ Probability (individual joined in $(T - t, T - t + \delta)$ and remaining for time$t) = h(T-t)\delta t G(t)$ →29

Where $h(t)$ is the renewal density of the whole System.

Hence $\alpha\left(\frac{t}{T}\right)\delta t = h(T-t)\delta t G(t)$ this hold for $t < T$

Since $\lim_{T\to\infty} h(t) = \frac{1}{\theta}$ And $\lim_{T\to\infty} G(t) = 0$, then $\lim_{T\to\infty} \alpha\left(\frac{t}{T}\right) = \frac{G(t)}{\theta} = a(t)$ →30

Which gives the study state age distribution.
In promotion by seniority, the loss form grade II is replaced by the seniority of member of grade I. It follows that at any time every individual in grade II has length of service at least as long as any individual in grade I and hence there is some thresh hold value t_1 such that all individuals with length of service less than t_1 are in grade I. t_1 is a random variable, but for large values of grade sizes the approximate formula for its expected value is given by Bartholomew, D.J. (1973) [2] as

$$\int_t^\infty a(t) = \frac{N_2}{N_1+N_2}$$ →31

Therefore, under study state the expected number of proportion per unit time will be the promotion of new recruits whose service is at least at the threshold length t_1. i.e.

$$N_1 p = a(t_1)\frac{N}{\theta}$$ →32

Let μ_L be the average length of time spent in grade I by those who leave while in grade I. Let μ_p be the average length of time spent in grade I by those who are eventually promoted to grade II. Then consider the promotion problem as determining N_1, such that μ_p is the average value of t_1. Therefore, for equations (31) and (32), we have

$$\int_{\mu_p}^\infty a(t)dt = \frac{N_2}{N}$$ →33

That implies $\frac{1}{\theta}\int_{\mu_p}^\infty G(t)dt = \frac{N_2}{N}$ →34

And $N_1 p = G(\mu_p)\frac{N}{\theta}$
For equation (34), one can have $R = \frac{N_1}{N_2}$ as a function of μ_p and hence, knowing the total size of organization (N), one can obtain N_1 for any given value of μ_p. This implies

$$\frac{1}{\theta}\int_{\mu_p}^A G(t)dt = \frac{1}{1+R}$$ →35

For the model under consideration $G(t)$ is as given in equation (18) and using this in equation n (35), gives $\frac{1}{1+R} = \frac{1}{\theta}\int_{\mu_p}^A G(t)dt$ →36

By solving the (36), the explicit values can be obtained.
For different values of μ_p, β *and* p we complete the value of R and presented in Table-1

Table 1

p	β	$\mu_{p,}$	A	μ	θ	R
0.7	2	2.8	50	3	10	0.477
		2.9				0.466
		3				0.456
		2.8	58	4		0.612
		2.9				0.601
		3				0.591
		3.8	50	4		0.642
		3.9				0.631
		4				0.621
		3.8	58	4		0.809
		3.9				0.799
		4				0.788
		4.8	50	5		0.72
		4.9				0.709
		5				0.699
		4.8	58	5		0.174
		4.9				0.163
		5				0.153
0.6		4.8	50	5		0.103
		4.9				0.092
		5				0.082

7. Promotion by Random

In some of the organization the promotion policy is carried out by virtue of merit considering the efficiency and expertise and not by seniority. Let $F_1(t)$ be the probability that an individual remains in the organization for time t, without setting promotion and let n_t be the promotion during$(0, t)$.

Then $F_1(t)$ = Probability {individuals not promoted in $(0, t)$/ does not leave in $(0, t)$}

= Probability {does not leave in$(0, t)$} $= \left(1 - \frac{1}{N_1}\right)^{n_t} G(t)$

Now the expected value of n_t is $N_1 P_t$

Then as a first approximation, one can take

$$F_1(t) = \left(1 - \frac{1}{N_1}\right)^{N_1 P_t} G(t)$$

As, $N_1 \to \infty$ gives $F_1(t) = G(t)e^{-p_t}$

Then the average time spend in grade I is

$$\theta_1 = \int_0^\infty F_1(t)dt = \int_0^\infty G(t)e^{-p_t}dt$$

Using equation (28), $\frac{1}{\theta}\int_0^\infty G(t)e^{-p_t}dt = \frac{N_1}{N}$ →37

Let $Q(t)$ is the probability density function of the time interval between entry of individual into grade I and his promotion to grade II given that he is promoted before leaving.

Let $Q(t) = \int_t^\infty q(x)dx = \left(1 - \frac{1}{N_1}\right)^{n_t}$ →38

Replacing n_t by its expected value$N_1 P_t$, then an approximate value of $Q(t)$ is

$$Q(t) = \left(1 - \frac{1}{N_1}\right)^{N_1 P_t}$$

Then $\mu_p = \int_0^\infty Q(t)dt = \int_0^\infty e^{-p_t}dt = \frac{1}{P}$ →39

Using equation (37), (38) and equation (39), one can have $\frac{N_1}{N} = \frac{1}{\theta}\int_0^A G(t)e^{\frac{1}{\mu_p}}dt$

This implies that $\frac{1}{\theta}\int_0^A G(t)e^{\frac{1}{\mu_p}}dt = \frac{R}{1+R}$ →40

For our model under consideration the $G(t)$ value is given equation (18)

Substituting $G(t)$ in equation (40) gives $\frac{R}{1+R} = \frac{1}{\theta}\int_0^A G(t)e^{\frac{1}{\mu_p}}dt$ →41

By solving the (41), the explicit expression for various values of μ, β *and* θ the R value are computed and presented in Table 2

Table 2

p	β	μ	k	A	θ	R
0.5	2	3	2.8	45	10	0.521
				50		0.362
				58		0.109
0.6				45		1.404
				50		1.412
				58		1.423
0.7				45		2.006
				50		2.126
				58		2.318
0.5			2.9	45		0.508
				50		0.352
				58		0.102

0.6				45		1.381
				50		1.388
				58		1.4
0.7				45		1.976
				50		2.094
				58		2.284
0.5			3	45		0.497
				50		0.342
				58		0.096
0.6				45		1.36
				50		1.367
				58		1.378
0.7				45		1.947
				50		2.065
				58		2.252
0.5			3.1	45		0.486
				50		0.333
				58		0.089
0.6				45		1.34
				50		1.347
				58		1.358

8. Conclusions

This paper is on the mixture of Laplace and two parameter Laplace type bimodal Distribution and its application on Manpower modelling. Initially the distributional properties were discussed. While applying to the Manpower modelling, two types of promotion policies namely promotion by seniority and promotion by random was considered and discussed.

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