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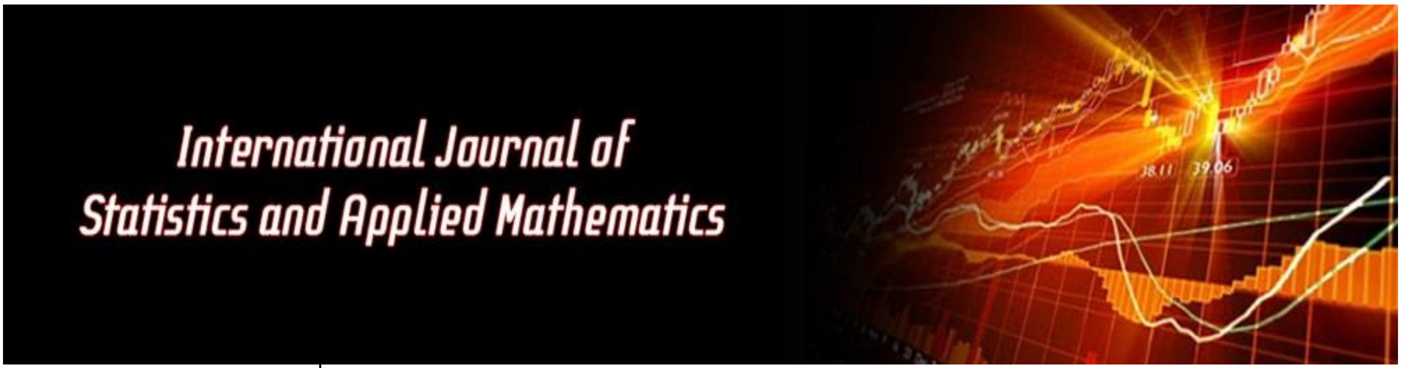

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Dr. Dalia Kamal Fathi Alnagar
Assistant Professor, Department of Statistics, Faculty of Science, Tabuk University Tabuk Sudia Arabia

Dr. Arbab Ismail Babbiker Faris
Associate Professor Department of Econometrics and Social Statistics, Faculty of Economics and Social Studies Khartoum University, Sudan, North Africa

Correspondence
Dr. Dalia Kamal Fathi Alnagar
Assistant Professor, Department of Statistics, Faculty of Science, Tabuk University Tabuk Sudia Arabia

Successive sampling from a changing population proportion over two successive occasions

Dr. Dalia Kamal Fathi Alnagar and Dr. Arbab Ismail Babbiker Faris

Abstract
An efficient estimators procedure to estimate the current population proportion over two occasions successive sampling has been developed.
Suggested estimators have been studied and their respective optimum allocation of the second sample are discussed. The behavior of the optimum subsampling proportions and the gain in precision were tested empirically using different values of the design parameters, namely the correlation and the subgroup weights.

Keywords: Optimum overlap, gain in precision, correlation, unbiased, minimum variance, successive sampling

1. Introduction
When population is subject to change a single survey will only yields information about the properties of that population on a given occasion. If the changes in population values are to examined the survey will have to be repeated on several occasions.
Repeated sampling processes already in use are extended to estimate the current of the population proportion in the second occasions Successive sampling is devised to deal with such situation.
Survey populations are liable to change over time owing to the entry of new units and /or loss of the previous ones.
Suppose that of a population comprising units on the fist and the second occasions:

1. Continuing Members (N_{1c}): members of population on the first occasion which continue to exist to second occasion.
2. New Members (N_{2b}): new members of population which join the population in the time lapse between two occasions.
3. Dropout Members (N_{1d}): members of population on the first occasion that dropout before the second occasion

2. Sample Design
Let a character under study be denoted by x and y on the first and second occasions respectively.

X_{hi}, (Y_{thi}) = values of survey binary variable associated with i^{th} (i=1, 2…, N_h) unit of the h^{th} (h=1c, 1d), (h=2c, 2b) subgroup on the first (second) occasion respectively.

Notation:
= N1d = Number of members of the population on the first occasion that drop out before the second.
N1c = Number of members of the population on the first occasion which continue to exist to the second.
N2b = Number of new units which join the population in the time lapse between the two occasions.

N1 = N1d + N1c = Number of units in the population on the first occasion.
N2 = N1c+ N2b = Number of units in the population on the second occasions.

$N_1 \neq N_2$, and consequently n_1 is expected to be different from n_2

$$W_{1d} = \frac{N_{1d}}{N_1}, \quad W_{1c} = \frac{N_{1c}}{N_1}, \quad W_{2b} = \frac{N_{2b}}{N_2}, \quad W_{2c} = \frac{N_{2c}}{N_2}$$

2.1 To estimate proportion of population $\hat{\theta}_t$:

$\theta_{th} = \frac{A_{th}}{N_{th}}$ The h^{th} sub proportion of population in the whole population on the occasions.

h : For subgroup of the population, $th = h^{th}$ sup group on occasion t.

$A_h = \sum_{i=}^{N} X_{hi}$ Numbers of population who possess the Attribute. $\hat{\theta}_t = \sum_{h}^{H} W_{th}\hat{\theta}_{th}$ The of population on the t occasion.

$\theta_1 = \frac{A_1}{N_1}$, $\theta_2 = \frac{A_2}{N_2}$ The proportion of population who possess the attribute on first and second occasions respectively.

$$S_1^2 = \frac{N_1\theta_1(1-\theta_1)}{N_1 - 1}, \quad S_2^2 = \frac{N_2\theta_2(1-\theta_2)}{N2 - 1}$$

The sub group variance of the sample proportion on the first and second occasion respectively.

$\theta_1 = \sum_{h'} W_1\theta_1$, $\theta_2 = \sum_{h'} W_2\theta_2$ The proportion of population on the first and second occasion respectively.

The correlation coefficient that describes the association of X_{1c} and Y_{2c} is

$$\rho = \frac{(\theta_{1c}\theta_{2c})/n_{1c}}{\sqrt{\{\theta_{1c}(1-\theta_{1c})/n_{1c}\}\{\theta_{2c}(1-\theta_{2c})/n_{2c}))}}$$

ρ The association between the responses of the continuing members of the first stratum on the two occasions.

Throughout this article, we make the customary simplifying assumption that the variances of the subgroups of continuing members remain unchanged over time, i.e. $S_{1c}^2 = S_{2c}^2$. Furthermore, we assume that the size of the population on the tth occasion (t=1, 2, 3...) and the size of the component subgroup is known. For simplicity we assume that the sizes are large so that the subsampling fractions are negligibly zero.

We assume that stratified random sample of units is drawn without replacement from the N_1 members of the population on the first occasion, and their X values are observed. Let n_{1c} denote the number of the first sample members which continue to exist to the second occasion, then the selection procedure may be given for drawing a sample of n_1 (n_1 is usually different from n_2 i.e. ($n_t \neq n_{t+1}$ $for\ t = 1,2,3,\ldots$) units from the N_2 members of the population on the second occasion:

- Select a simple random sample without replacement (SRSWOR) of n_{1cm} from the n_{1c} continuing members.
- Select a SRSWOR of n_{2cu} units from the N_{1c}-n_{1c} units which were not included in to the first sample to replace (either completely or partially) the n_{1cu} units not reselected.
- Select a SRSWOR of n_{2b} from the N_{2b} news members to supplement.

$n_{2c} = n_{1cm} + n_{2cu}$ continuing members in the sample.

Based on the responses of the sample members on both occasions the following sub sample indices education poverty may be formed:

$\hat{\theta}_{1cm}=\dfrac{a_{1cm}}{n_{1cm}}$	The proportion of population possess the attribute of the matched units on first occasion. (1)
$\hat{\theta}_{1cu}=\dfrac{a_{1cu}}{n_{1cu}}$	The proportion of the members of the first sample possess the attribute which a replaced on the second occasion. (2)
$\hat{\theta}_{1c}=\dfrac{n_{1cm}\hat{\theta}_{1cm}+n_{1cu}\hat{\theta}_{1cu}}{n_{1c}}$	The proportion of the first occasion on the continuing members of the first sample. (3)
$\hat{\theta}_{2cm}=\dfrac{a_{2cm}}{n_{2cm}}$	The proportion of the matched units on the second occasion. (4)
$\hat{\theta}_{2cu}=\dfrac{a_{2cu}}{n_{2cu}}$	The proportion of the unmatched units on the second occasion. (5)
$\hat{\theta}_{2c}=\dfrac{n_{2cm}\hat{\theta}_{2cm}+n_{2cu}\hat{\theta}_{2cu}}{n_{2c}}$	The proportion of the continuing members of the sample on the second occasions. (6)
$\hat{\theta}_{2b}=\dfrac{a_{2b}}{n_{2b}}$	The proportion of population of the news of the second sample. (7)

3. Estimation of Population proportion on the Second Occasions θ_2

Suppose we need to estimate the current population proportion θ_2 to achieve this we drive the estimators of the proportions θ_{2c} and θ_{2b} of the two subgroups in the current population. The resulting separate estimators will then be combined to provide an estimator of θ_2 . Consider, then, the estimation of θ_{2c} in theorem we give the minimum variance unbiased estimator (mvue) of θ_{2c} and its variance.

Definition (1-1)

An estimator γ is defined as a minimum variance unbiased estimator (mvue) of the parameter Γ if and only if:

1. $E(\gamma)=\Gamma$ for all values Γ , and
2. Among all estimators γ^* of Γ satisfying (i), $V(\gamma)=V(\gamma^*)$ (Babiker, 1984)

Consider a generalized estimator $\hat{\theta}_{2c}$ of the population proportion of the sensitive characteristic on the second occasion or current occasion as

$$\hat{\theta}_{2c}=a\hat{\theta}_{1cu}+b\hat{\theta}_{1cm}+c\hat{\theta}_{2cu}+d\hat{\theta}_{2cm} \tag{8}$$

Where a, b, c and d are suitable constants its expressed is:

$$E(\hat{\theta}_{2c})=(a+b)\hat{\theta}_{1c}+(c+d)\hat{\theta}_{2c} \tag{9}$$

Then the condition of unbiased ness required that

$$a+b=0 \qquad ,b=-a$$
$$c+d=1 \qquad ,d=1-c$$

Hence:

$$\hat{\theta}_{2c} = a\hat{\theta}_{1cu} - a\hat{\theta}_{1cm} + c\hat{\theta}_{2cu} + (1-c)\hat{\theta}_{2cm} \tag{10}$$

Is unbiased estimator of $\hat{\theta}_{2c}$, now:

$$V(\hat{\theta})_c = a^2 V(\hat{\theta}_{1cu}) + a^2 V(\hat{\theta}_{1cm}) + c^2 V(\hat{\theta}_{2cu}) + (1-c)^2 V(\hat{\theta}_{2cm})$$
$$-2a(1-c)COV(\hat{\theta}_{1cm} - \hat{\theta}_{2cm})$$

$$= a^2 \frac{S_{1c}^2}{n_{1cu}} + a^2 \frac{S_{1c}^2}{n_{1cm}} + c^2 \frac{S_{1c}^2}{n_{2cu}} + (1-c)^2 \frac{S_{1c}^2}{n_{2cm}} - 2a(1-c)\rho_{\phi} \frac{S_{1c}^2}{n_{1cm}} \tag{11}$$

To obtain values of aand c make $V(\hat{\theta}_{2c})$ a minimum we differentiate with respect to both a and c and set the partial derivatives equal to zero.
The optimum values obtained are:

(12)

$$\frac{\partial V(\hat{\theta})_c}{\partial a} = \frac{2a}{n_{1cm}} + \frac{2a}{n_{1cu}} - 2(1-c)\rho \frac{1}{n_{1cm}} = 0$$

$$\frac{\partial V(\hat{\theta})_c}{\partial c} = \frac{2c}{n_{2cu}} - \frac{2(1-c)}{n_{2cm}} + 2a\rho \frac{1}{n_{1cm}} = 0$$

$$a\left[\frac{1}{n_{1cm}} + \frac{1}{n_{1cu}}\right] = (1-c)\rho \frac{1}{n_{1cm}}$$

(13)

$$a\rho \frac{1}{n_{1cm}} = \frac{c}{n_{2cu}} - \frac{(1-c)}{n_{2cm}}$$

We derive and then obtain:

$$c = \frac{n_{2cu} n_{1c} - n_{1cu} n_{2cu} \rho^2}{n_{1c} n_{2c} - n_{1cu} n_{2cu} \rho^2} \tag{14}$$

$$a = \frac{n_{1c} n_{1cm} \rho}{n_{1c} n_{2c} - n_{1cu} n_{2cu} \rho^2} \tag{15}$$

Theorem (1-1)

The mvue of $\hat{\theta}_{2C}$ and its variance are respectively:

$$\hat{\theta}_{2C} = \frac{n_{1c} n_{1cm} \left\{ \hat{\theta}_{2cm} + \rho(\hat{\theta}_{1c} - \hat{\theta}_{1cm} \right\} + n_{2cu}(n_{1c} - n_{1cu}\rho^2)\hat{\theta}_{2cu}}{n_{1c} n_{2c} - n_{1cu} n_{2cu} \rho^2} \tag{1}$$

And: $$V(\hat{\theta}_{2c}) = \frac{(n_{1c} - n_{1cu}\rho^2)}{n_{1c} n_{2c} - n_{1cu} n_{2cu} \rho^2} \quad S_{1c}^2 \tag{2}$$

We now proceed to derive the estimator of $\hat{\theta}_{2b}$ the subsample of new units provides a mvue of $\hat{\theta}_{2b}$ with variance:

$$V(\hat{\theta}_{2b}) = \frac{S_{2b}^2}{n_{2b}} \tag{4}$$

Having obtained the mvue's of $\hat{\theta}_{2c}$ and $\hat{\theta}_{2b}$.

Corollary (1-1)

If γ_i is a mvue of Γ_i and a_i is constant $(i = 1,2,\ldots,k)$ then $\sum_{i=1}^{k} a_i \gamma_i$ is a mvue of $\sum_{i=1}^{k} a_i \Gamma_i$

We apply corollary (3-1) to the estimation of $\hat{\theta}$.

Theorem (1-2)
Under the sample design of selection identifiable a mvue of population proportion on the second occasions is given by:

$$\hat{\theta}_w = W_{2c}\hat{\theta}_c + W_{2b}\hat{\theta}_{2b} \tag{5}$$

$$= \frac{W_{2c} \left\{ n_{1c} n_{1cm} (\hat{\theta}_{2cm} + \rho\ (\hat{\theta}_{1c} - \hat{\theta}_{1cm}) + n_{2cu}(n_{1c} + n_{1cu}\rho^2)\hat{\theta}_{2cu} \right\}}{n_{1c} n_{2c} - n_{1cu} n_{2cu} \rho^2} + W_{2b}\hat{\theta}_{2b} \tag{6}$$

And: $$V(\hat{\theta}_w) = \frac{(n_{1c} - n_{1cu}\rho^2) W_{2c}^2\, S_{1c}^2}{n_{1c} n_{2c} - n_{1cu} n_{2cu} \rho^2} + \frac{W_{2c}^2 S_{2b}^2}{n_{2b}} \tag{7}$$

It's worth remarking here that results pertaining to the situations where the population is fixed (i.e. $W_{1c} = W_{2c} = 1$), or when it contains no new entrants but some of the previous units case to exist (i.e. $0 < W_{1c} < 1$) and $W_{2c} = 1$ follow as special cases.

4. Optimum Allocation of the second sample

The usual problem of choosing sample for more precise, we consider the allocation of the second sample optimum for measuring population proportion $\hat{\theta}$, under this case, the sampling units are identifiable prior to the selection of the sample, then, the number of the first sample members which dropout in the time span between the two occasions, n_{1d}, and consequently n_{1c}

($n_1 = n_{1c} + n_{1d}$), will be known in advance thus given the variance expression (7) we are in a position to determine the optimum allocation of the second sample by obtaining an explicit mathematical statement of the best values of n_{1cm}, n_{2cm} and n_{2b} to be under taken since the rest of the parameters involved are assumed known.

Theorem (1-3)

If under the sample identifiable the required estimator is $\hat{\theta}_{2w}$, then, the minimum sampling variance is attained when:

$$\left.\begin{aligned} n_{1cm.opt} &= \frac{n_{1c}\sqrt{1-\rho^2}}{\alpha_1} \\ n_{2b.opt} &= \frac{W_{2b}\delta_{2b}(n_1\alpha_1 + n_{1c}\alpha_2)}{\alpha_1(W_{2c}+W_{2b}\delta_{2b})} \end{aligned}\right\} if\ n_{1c} \le \frac{n\alpha_1 W_{2c}}{W_{2c}\sqrt{1-\rho^2}+W_{2b}\delta_{2b}}$$

$$\left.\begin{aligned} n_{1cm.opt} &= \frac{nW_{2c}\sqrt{1-\rho^2}}{W_{2b}\delta_{2b}+W_{2c}\sqrt{1-\rho^2}} \\ n_{2b.opt} &= \frac{nW_{2b}\delta_{2b}}{W_{2b}+W_{2c}\sqrt{1-\rho^2}} \end{aligned}\right\} otherwise \qquad (8)$$

Where:

$$\alpha_1 = 1+\sqrt{1-\rho^2}$$
$$\alpha_2 = 1-\sqrt{1-\rho^2}$$

$\delta_{2b} = \dfrac{S_{2b}}{S_{2c}}$ And opt-short for optimum.

The unconstrained problem may now be solved by following the usual minimization procedures. Equating the partial derivatives of $v(\hat{\theta}_w)$ with respect to n_{1cm} and n_{2b} to zero and solving gives the optimum solution:

$$n_{1cm} = \frac{n_{1c}\sqrt{1-\rho^2}}{1+\sqrt{1-\rho^2}}$$

$$n_{2b} = \frac{W_{2b}\delta_{2b}[n(1+\sqrt{1-\rho^2}) + n_{1c}(1-\sqrt{1-\rho^2}]}{(W_{2c}+W_{2b}\delta_{2b})(1+\sqrt{1-\rho^2})}$$

Attains its minimum value when:

$$n_{2b} = \frac{nW_{2b}\delta_{2b}}{W_{2c}\sqrt{1-\rho^2} + W_{2b}\delta_{2b}}$$

Hence:

$$n_{1cm} = \frac{nW_{2c}\sqrt{1-\rho^2}}{W_{2c}\sqrt{1-\rho^2} + W_{2b}\delta_{2b}}$$

As evident from 3-6, $n_{1cm.opt}$ does not depend on the continuing subgroup size or variance except at the boundary of the solution space these parameters (i.e.W_{2c} and S_{1c}) play predominant role in determining the optimum number of units allocated to subgroup, whereas the number of members of the sample on the preceding occasion with continue to exist and the correlation between their responses on the two occasions give the number of matched units in the current sample.

Corollary (1-2)

$$V(\hat{\theta}_w)_{0pt} = \begin{cases} \dfrac{\alpha_1}{n\alpha_1 + n_{1c}\alpha_2}[\sum_{h'} W_{h'}S_{h'}]^2, if \ \ n_{1c} \le \dfrac{nW_{2c}\alpha_1}{W_{2c}\sqrt{1-\rho^2} + W_{2b}\delta_{2b}} \\ \dfrac{\rho^2 W_{2c}^2 S_{1c}^2}{n_{1c}} + (W_{2c}\sqrt{1-\rho^2} + W_{2b}\delta_{2b}\dfrac{S_{1c}^2}{n} \qquad , other\ wise \end{cases} \quad (9)$$

The an optimally allocated cross sectional sample of size n in is drawn from the two subgroups comprising the population on the second occasion then the resulting estimator $\hat{\theta}_{2st}$ (St, stands for stratified) of $\hat{\theta}_2$ will have an optimum variance of form:

$$V(\hat{\theta}_{st})_{opt} = [\sum_{h'} W_{h'}S_{h'}]^2$$

(10)

Its fellows from 3-7 and 3-8 that when $n_{1c} \le \dfrac{n\alpha_1 W_{2c}}{W_{2c}\sqrt{1-\rho^{\ 2}} + W_{2b}\theta_{2b}}$

$$V(\hat{\theta}_w)_{opt} = \psi V(\hat{\theta}_{st})_{opt}$$

Here: $\psi = \dfrac{\alpha_1}{\alpha_1 + \lambda_{1c}\alpha_2}$

Where $\lambda_{1c} = \dfrac{n_{1c}}{n}$

Now since:

$$\alpha_2 = 1 - \sqrt{1-\rho^2} \ge 0$$
$$\lambda_{1c} > 0$$
$$then \ \ \lambda_{1c}\alpha_2 \ge 0$$

i.e. $\alpha_1 \leq \alpha_1 + \lambda_{1c} + \alpha_2$

$$\frac{\alpha_1}{\alpha_1 + \lambda_{1c}\alpha_2} \leq 1$$

Hence $\psi \leq 1$

Accordingly, in this case $V(\hat{\theta}_w)_{opt} \leq V(\hat{\theta}_{st})_{opt}$ gain in precision:

$V(\hat{\theta}_w)_{opt} \leq V(\hat{\theta}_{st})_{opt}$ It may therefore be concluded that for obtaining a more precise estimator of the current proportion of a changing population successive sampling with some degree of overlap continues to be favorable approach.

Definition (1-2)

Let T_1 and T_2 be two estimators of population parameter T based on two different samples of the same size: denote their respective variances by $V(T_1)$ and $V(T_2)$, then, the precision of T_1 relative to that of T_2 is defined as reciprocal of the ratio $\frac{V(T_1)}{V(T_2)}$ i.e. $RP(T_1, T_2) = \frac{V(T_2)}{V(T_1)}$

Note that if T_1 is more precise i.e. has smaller variance than T_2 then $RP(T_1, T_2) > 1$.

This encourages us to assess the superiority of $\hat{\theta}_w$ over $\hat{\theta}_{st}$ consider, then, the gain in the precision of $\hat{\theta}_w$ relative to $\hat{\theta}_{st}$ from definition 1-1 we have:

$$gp(\hat{\theta}_w, \hat{\theta}_{st}) = \begin{cases} \lambda_{1c}\dfrac{\alpha_2}{\alpha_1} & , if\ \lambda_{1c} \leq \dfrac{\alpha_1 W_{2c}}{W_{2c}\sqrt{1-\rho^2} + W_{2b}\delta_{2b}} \\ \dfrac{2\lambda_{1c}W_{2c}W_{2b}\delta_{2b}\alpha_2 - \rho^2 W_{2c}^2(1-\lambda_{1c})}{\rho^2 W_{2c}^2 + \lambda_{1c}(W_{2c}\sqrt{1-\rho^2} + W_{2b}\delta_{2b}} & , otherwise \end{cases} \quad (11)$$

Where $\lambda_{1c} = \frac{n_{1c}}{n}$

We obtain the analogs of the optimum allocation and the corresponding $gp(\hat{\theta}_w, \hat{\theta}_{st})$ as:

$$\left.\begin{array}{l} \lambda_{1cm.opt} = \dfrac{(1-p_d)\sqrt{1-\rho^2}}{\alpha_1} \\ \lambda_{2b.opt} = \dfrac{W_{2b}\delta_{2b}[\alpha_2 + (1-p_d)\alpha_2]}{\alpha_1(W_{2b}\delta_{2b} + (1-W_{2b})} \\ gp(\hat{\theta}_w, \hat{\theta}_{st}) = (1-p_d)\dfrac{\alpha_2}{\alpha_1} \end{array}\right\}, if\,(1-p_d) \leq \frac{\alpha_1(1-W_{2b})}{[W_{2b}\delta_{2b} + (1-W_{2b})\sqrt{1-\rho^2}} \quad (12)$$

Where:

$$\alpha_1 = 1 + \sqrt{1-\rho^2}$$

$$\alpha_2 = 1 - \sqrt{1-\rho^2}$$

$$\lambda_{1cm.opt} = \frac{n_{1cm.opt}}{n} \text{ And: } \lambda_{2b.opt} = \frac{n_{2b.opt}}{n}$$

Are the optimum subsampling proportion, and $p_d = W_{1d} = \frac{N_{1d}}{N}$

5. The allocation of the second sample for measuring the current population proportion (θ_2)

The problem of choosing estimator for measuring the current population proportion, an essential solution for more precise sampling results comprises a best choice of the sample allocation. We now consider the allocation of the second sample for measuring the current population proportion (θ_2), thus we are in a position to determine the optimum allocation of the second by computed numerically the behavior of the optimum subsampling proportions and gain in precision, $\lambda_{1cm.opt}, \lambda_{2b.opt}$ and $gp(\hat{\theta}_w, \hat{\theta}_{st})$. To examine numerically the behavior of the optimum subsampling proportions and the gain in precision were computed under some combinations of values of design parameters $\rho, p_d, W_{2b} and \delta_{2b}$. The limited ranges values chosen for some design parameters, namely δ_{2b} *and* W_{2b} are intended to provide a close resemblance to the true situation for suggest the choice $0 \le \delta_{2b} \le 1$ and $0 \le W_{2b} \le 1$.

6. Results and Discussions

Values of $\lambda_{1cm.opt}, \lambda_{2b.opt}$ and $gp(\hat{\theta}_w, \hat{\theta}_{st})$ obtained from equation (11) are reported in table (4-1) for varying combination of the design parameters.

Table (4-1): optimum subsampling proportions and $gp = gp(\hat{\theta}_w, \hat{\theta}_{st})$ $\delta_{2b} = 0.5$ $W_{2b} = 0.2$

p_d		**Correlation (ρ)**					
		0.00	0.20	0.40	0.60	0.80	0.99
0.00	λ_{1cmopt}	0.50	0.49	0.48	0.44	0.38	0.12
	λ_{2bopt}	0.00	0.00	0.01	0.02	0.06	0.17
	gp	0.00	0.01	0.04	0.11	0.25	0.75
0.40	λ_{1cmopt}	0.30	0.30	0.29	0.27	0.23	0.07
	λ_{2bopt}	0.00	0.00	0.01	0.02	0.04	0.13
	gp	0.00	0.01	0.03	0.07	0.15	0.45
0.80	λ_{1cmopt}	0.10	0.10	0.10	0.09	0.08	0.02
	λ_{2bopt}	0.00	0.00	0.01	0.01	0.03	0.10
	gp	0.00	0.00	0.01	0.02	0.05	0.15
0.95	λ_{1cmopt}	0.03	0.00	0.00	0.00	0.00	0.00
	λ_{2bopt}	0.00	0.00	0.01	0.01	0.03	0.09
	gp	0.00	0.00	0.00	0.01	0.03	0.08

Source: own calculations

Table (4-1): contd $\delta_{2b} = 0.5 \quad W_{2b} = 0.5$

p_d		Correlation (ρ)					
	p	0.00	0.20	0.40	0.60	0.80	0.99
0.00	λ_{1cmopt}	0.50	0.49	0.48	0.44	0.38	0.12
	λ_{2bopt}	0.00	0.01	0.04	0.09	0.21	0.63
	gp	0.00	0.01	0.04	0.11	0.25	0.75
0.40	λ_{1cmopt}	0.30	0.30	0.29	0.27	0.23	0.07
	λ_{2bopt}	0.00	0.01	0.03	0.07	0.17	0.50
	gp	0.00	0.01	0.03	0.07	0.15	0.45
0.80	λ_{1cmopt}	0.10	0.10	0.10	0.09	0.08	0.02
	λ_{2bopt}	0.00	0.00	0.02	0.04	0.10	0.30
	gp	0.00	0.00	0.01	0.02	0.05	0.15
0.95	λ_{1cmopt}	0.03	0.00	0.00	0.00	0.00	0.00
	λ_{2bopt}	0.00	0.00	0.02	0.04	0.09	0.26
	gp	0.00	0.00	0.00	0.01	0.03	0.08

Source: own calculations

Table (4-1): contd $\delta_{2b} = 1.0 \, W_{2b} = 0.2$

P_d		Correlation (ρ)					
	p	0.00	0.20	0.40	0.60	0.80	0.99
0.00	λ_{1cmopt}	0.50	0.49	0.48	0.44	0.38	0.12
	λ_{2bopt}	0.00	0.00	0.02	0.04	0.10	0.30
	gp	0.00	0.01	0.04	0.11	0.25	0.75
0.40	λ_{1cmopt}	0.30	0.30	0.29	0.27	0.23	0.07
	λ_{2bopt}	0.00	0.00	0.01	0.04	0.08	0.24
	gp	0.00	0.01	0.03	0.07	0.15	0.45
0.80	λ_{1cmopt}	0.10	0.10	0.10	0.09	0.08	0.02
	λ_{2bopt}	0.00	0.00	0.01	0.03	0.06	0.18
	gp	0.00	0.00	0.01	0.02	0.05	0.15
0.95	λ_{1cmopt}	0.03	0.00	0.00	0.00	0.00	0.00
	λ_{2bopt}	0.00	0.00	0.01	0.02	0.05	0.16
	gp	0.00	0.00	0.00	0.01	0.03	0.08

Source: own calculations

Table (4-1): contd $\delta_{2b} \quad W_{2b}$ $1.0 \quad 0.5$

p_d		Correlation (ρ)					
		0.00	0.20	0.40	0.60	0.80	0.99
0.00	λ_{1cmopt}	0.50	0.49	0.48	0.44	0.38	0.12
	λ_{2bopt}	0.00	0.01	0.04	0.11	0.25	0.75
	gp	0.00	0.01	0.04	0.11	0.25	0.75
0.40	λ_{1cmopt}	0.30	0.30	0.29	0.27	0.23	0.07
	λ_{2bopt}	0.00	0.01	0.03	0.09	0.20	0.60
	gp	0.00	0.01	0.03	0.07	0.15	0.45
0.80	λ_{1cmopt}	0.10	0.10	0.10	0.09	0.08	0.02
	λ_{2bopt}	0.00	0.01	0.03	0.07	0.15	0.45
	gp	0.00	0.00	0.01	0.02	0.05	0.15
0.95	λ_{1cmopt}	0.03	0.00	0.00	0.00	0.00	0.00
	λ_{2bopt}	0.00	0.01	0.02	0.06	0.13	0.40
	gp	0.00	0.00	0.00	0.01	0.03	0.08

Source: own calculations

Table (4-1): contd $\delta_{2b} = 0.5 \quad W_{2b} = 0.0$

p_d		Correlation (ρ)					
		0.00	0.20	0.40	0.60	0.80	0.99
0.00	λ_{1cmopt}	0.50	0.49	0.48	0.44	0.38	0.12
	λ_{2bopt}	0.00	0.00	0.00	0.00	0.00	0.00
	gp	0.00	0.01	0.04	0.11	0.25	0.75
0.40	λ_{1cmopt}	0.30	0.30	0.29	0.27	0.23	0.07
	λ_{2bopt}	0.00	0.00	0.00	0.00	0.00	0.00
	gp	0.00	0.01	0.03	0.07	0.15	0.45
0.80	λ_{1cmopt}	0.10	0.10	0.10	0.09	0.08	0.02
	λ_{2bopt}	0.00	0.00	0.00	0.00	0.00	0.00
	gp	0.00	0.00	0.01	0.02	0.05	0.15
0.95	λ_{1cmopt}	0.03	0.00	0.00	0.00	0.00	0.00
	λ_{2bopt}	0.00	0.00	0.00	0.00	0.00	0.00
	gp	0.00	0.00	0.00	0.01	0.03	0.08

Source: own calculations

Where:

$$\lambda_{1cm.opt} = \frac{n_{1cm.opt}}{n}$$ And: $$\lambda_{2b.opt} = \frac{n_{2b.opt}}{n}$$

Are the optimum subsampling proportion, and $p_d = W_{1d} = \frac{N_{1d}}{N}$

From the table (4-1) above it may be seen

- optimum overlap proportion for estimating current education poverty level amounted to 0.5
- λ_{1cmopt} is decreasing function of (ρ), it never exceeds $0.5\lambda_{1c}$ and decreases steadily as ρ increases. When we increase λ_{1c} (i.e. decrease p_d) leads to increase λ_{1cmopt}.Contrary wise, λ_{2bopt} tends to increase very slowly as ρ increases. This is not unexpected result, since as the degree of association between values of the continuing members of the population on successive occasions tends to perfection, the overall numbers of units quired to represent the subgroup of those continuing members declines, thereby calling for more new units to be included into the current sample
- λ_{2bopt} Affected by changes in δ_{2b} *and* W_{2b} the large values of δ_{2b} *and or* W_{2b} yields large value of λ_{2bopt}.
- When ρ and p_d are fixed, λ_{1cmopt} remains the same for various values of δ_{2b} *and* W_{2b} .this indicates that the condition:

$$n_{1c} \le \frac{\alpha_1 W_{2c}}{W_{2c}\sqrt{1-\rho_{\phi}^{\;2}} + W_{2b}\theta_{2b}}$$

Is satisfied by all combinations of the design parameters values under taken. In this case, only λ_{2bopt} is affected changes in δ_{2b} *and* W_{2b}

- The behavior of $gp(\hat{\theta}_w \;, \hat{\theta}_{st})$, it is encouraging to note that as λ_{1c} and or ρ increase. The gain in precision due to an optimally allocated rotating sample increase.

- When ρ tends to zero, the gain in precision compared with no overlap. Also become negligible however, large the values of λ_{1c} may be. Thus impels that under such situations the two estimators $\hat{\theta}_w$, $\hat{\theta}_{st}$ equally precise. Moreover, δ_{2b} *and* W_{2b} are not crucial for evaluating $gp(\hat{\theta}_w , \hat{\theta}_{st})$

7. Conclusions

To estimate the current population proportion over population change repeated sampling survey processes already in use are extended to cater for the two dimensional nonstationarity inherent in the population.

The numbers of members of the sample on the preceding occasion which continue to exist and the correlation between their responses on the two occasions gives the number of overlap in the current sample; the correlation between values of the two occasions was the most influential on deciding the optimum allocation to be undertaken and achievable level of precision.

The sampling scheme developed and the estimators drawn therefrom were tested empirically using different values of the design parameters, namely the correlation and the subgroup weights, this led to the main finding that optimum overlap proportion for estimating current population proportion 0.5.

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