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Legendre wavelet collocation method for the numerical solution of singular initial value problems

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Abstract
Wavelet theory is a recently developed mathematical tool for many problems. In this paper, method based on collocation points with Legendre wavelet, an efficient and new numerical technique is proposed for the numerical solution of singular initial value problems. The present method is developed by using the Legendre wavelet and its operational matrices to obtain higher accuracy. A comparative analysis of Legendre wavelet collocation method (LWCM) is carried out with Haar wavelet collocation method (HWCM), Chebyshev wavelet collocation method (CWCM).

Keywords: Wavelets, Legendre wavelet collocation method, Singular initial value, problems, Haar wavelets collocation method, Chebyshev wavelets collocation method

1. Introduction

Recently, the studies of singular initial value problems in the second order ordinary differential equations (ODEs) have fascinated the attention of many mathematicians and physicists. Many methods including numerical and perturbation methods have been used to solve such type of problems. The approximate solutions for these problems were presented by many researchers for examples Wazwaz [1, 2] were using the adomain decomposition method (ADM) and Yildirim and Ozis [3] were using the variational iteration method (VIM).

In numerical analysis, classical discritization methods, such as finite differences, finite elements, spectral elements, are powerful tools for solving differential equations. However, singularities and step changes often emerge in many phenomena, like stress concentration, elastoplasticity, shock wave and crack. Since small-scale features only exist in a small percentage of the solution domain, if one chooses a uniform numerical grid fine enough to resolve the small-scale characteristics, then the solution to the equations will be over-resolved in the majority of the domain. One would like ideally, to have a dense grid where small-scale structure exists and a sparse grid where the solution is only composed of large-scale features [4, 5, 6]. It demands for the usage of non-uniform grids and adaptive grids or moving elements to dynamically adapt to the changes in the solution [7]. That is where wavelets play a role.

Among the different wavelet families, most simple are the Haar wavelets. Haar wavelets have been used by many researchers because of their simplicity and better convergence. Some of the relevant work regarding Haar and Chebyshev wavelets can be found in [8-18]. The weaker side of using the Haar and Chebyshev basis functions for approximating functions is that they are lower in accuracy. To cover this aspect, Legendre wavelets are considered to get more accurate approximation in the absence of jumps or sharp transitions. Legendre wavelets use Legendre polynomials as their basis functions. Because of their improved smoothness and good interpolating properties, accuracy of Legendre wavelets is better than Haar and Chebyshev wavelets in case of approximating smooth functions even on a coarse grid. Applications of Legendre wavelets for numerical approximations can be found in [11, 19, 20 & 21].

In this paper, an attempt is made to propose the Legendre wavelet collocation method (LWCM) for the numerical solution of singular initial value problems.

The paper is organized in the following way. Section 2 is devoted to the preliminaries of wavelets. Method of solution is presented in section 3. Section 4 deals with the numerical examples. Concluding remarks of the paper is discussed in Section 5.

2. Wavelet Preliminaries

2.1 Wavelets

In the recent years, wavelets have been applied extensively for signal processing in communications and physics research, and have proved to be a wonderful mathematical tool. Wavelets can be used for algebraic manipulations in the system of equations obtained which leads to better resulting system. Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter *a* and the translation parameter *b* vary continuously, we have the following family of continuous wavelets;

$$\psi_{a,b}(t)=\frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right),\ \ a,b\in R,\ a\neq 0 \tag{2.1}$$

The best way to understand wavelets is through a multi-resolution analysis. Given a function $f\in L_2(R)$ a multi-resolution analysis (MRA) of $L_2(R)$ produces a sequence of subspaces $V_j, V_{j+1},\ldots,$ such that the projections of *f* onto these spaces give finer and finer approximations of the function *f* as $j\to\infty$.

A multi-resolution analysis of $L_2(R)$ is defined as a sequence of closed subspaces $V_j\subset L_2(R),\ j\in Z$ with the following properties

1. $\ldots\subset V_{-1}\subset V_0\subset V_1\subset\ldots$.
2. The spaces V_j satisfy $\bigcup_{j\in Z}V_j$ is dense in $L_2(R)$ and $\bigcap_{j\in Z}V_j=0$.
3. If $f(t)\in V_0,\ f(2^j t)\in V_j$, i.e. the spaces V_j are scaled versions of the central space V_0.
4. If $f(t)\in V_0,\ f(2^j t-m)\in V_j$ i.e. all the V_j are invariant under translation.
5. There exists $\phi\in V_0$ such that $\phi(t-m);\ m\in Z$ is a Riesz basis in V_0.

The space V_j is used to approximate general functions by defining appropriate projection of these functions onto these spaces. Since the union of all the V_j is dense in $L_2(R)$, so it guarantees that any function in $L_2(R)$ can be approximated arbitrarily close by such projections. As an example the space $\{V_j,\ j\in Z\}$ can be defined like

$$V_j=W_j\oplus V_{j-1}=W_{j-1}\oplus W_{j-2}\oplus V_{j-2}=\ldots=\bigoplus_{j=1}^{J+1}W_j\oplus V_0$$

For each *j* the space W_j serves as the orthogonal complement of V_j in V_{j+1}. The space W_j include all the functions in V_{j+1} that are orthogonal to all those in V_j under some chosen inner product. The set of functions which form basis for the space W_j are called wavelets.

2.2 Legendre wavelets and operational matrix of integration

For any positive integer k, the Legendre wavelets family is defined [19] as follows;

$$L_{n,m}(t)=\begin{cases}\sqrt{m+1/2}\,2^{k/2}p_m(2^k t-2n+1) & for\ \ \frac{2n-2}{2^k}\le t\le\frac{2n}{2^k}\\ 0, & Otherwise\end{cases} \tag{2.2}$$

where $n=1,2,\ldots,2^{k-1}$ and $m=0,1,\ldots,M-1$, M is the maximum order of the Legendre polynomial. Here $p_m(t)$ are the Legendre polynomials of order m which are defined on the interval [-1, 1]. Legendre polynomials can be calculated recursively with the help of the following equations;

$$p_0(t)=1,\ p_1(t)=t,\ p_{m+1}(t)=\frac{2m+1}{m+1}tp_m(t)-\frac{m}{m+1}p_{m-1}(t),\ m=1,2,3,\ldots$$

Equivalently, for any positive integer k, the Legendre wavelets family is defined [11] as fallows;

$$L(t)=L_i(t)=\begin{cases}\sqrt{m+1/2}\,2^{k/2}p_m(2^k t-2n+1) & for\ \ \frac{2n-2}{2^k}\le t\le\frac{2n}{2^k}\\ 0, & Otherwise\end{cases} \tag{2.3}$$

where $i=n+2^{k-1}m$. By varying the values of i with respect to the collocation points $t_j=\frac{j-0.5}{N},\ j=1,2,\ldots,N$, we get the Legendre matrix of order $N\times N$, where $N=2^{k-1}M$ and Legendre polynomials used in the approximation are of degree less than M.

The integration of Legendre wavelets is given [19] as

$$\int_0^t L(t)dt = PL(t) = P_1 \tag{2.4}$$

$$\int_0^t PL(t)dt = P^2L(t) = P_2 \tag{2.5}$$

and so on,

$$\int_0^t P^{n-1}L(t)dt = P^nL(t) = P_n \tag{2.6}$$

where P is the $N \times N$ operational matrix for integration and is given in [19] as

$$P = \frac{1}{2^k}\begin{pmatrix} D & U & U & \cdots & U \\ 0 & D & U & \cdots & U \\ 0 & 0 & \ddots & \ddots & U \\ \vdots & \vdots & \ddots & D & U \\ 0 & 0 & \cdots & 0 & D \end{pmatrix}$$

Where U and D are $M \times M$ matrices given by

$$U = \begin{pmatrix} 2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

And

$$D = \begin{pmatrix} 1 & \frac{1}{\sqrt{3}} & 0 & 0 & \cdots & 0 & 0 \\ -\frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{3}}{3\sqrt{5}} & 0 & \cdots & 0 & 0 \\ 0 & -\frac{\sqrt{5}}{5\sqrt{3}} & 0 & \frac{\sqrt{5}}{5\sqrt{7}} & \cdots & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{7}}{7\sqrt{5}} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 0 & \frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \\ 0 & 0 & 0 & 0 & \cdots & -\frac{\sqrt{2M-1}}{(2M-1)\sqrt{2M-3}} & 0 \end{pmatrix}$$

3. Method of solution

Consider the singular initial value problem of the form

$$u''(t) + \frac{c}{t}u'(t) + f\left(u(t)\right) = g(t) \tag{3.1}$$

Subject to the initial condition $u(0) = \alpha,\ u'(0) = \beta$ (3.2)

where $c,\ \alpha$ & β are real constants, $f\left(u(t)\right)$ is a real valued function and $g(t)$ is non homogeneous term.

Let us assume that $u''(t)=\sum_{i=1}^{N} a_i L_i(t)$ (3.3)

where a_i's, $i=1, 2,\ldots, N$ are Legendre wavelet coefficients to be determined.
Integrating (3.3) twice with respect to the given condition (3.2) we get,

$$u'(t)=\beta+\sum_{i=1}^{N} a_i P_{1i}(t) \quad (3.4)$$

and $$u(t)=\alpha+\beta t+\sum_{i=1}^{N} a_i P_{2i}(t) \quad (3.5)$$

Substituting the values of (3.3)-(3.5) in (3.1) then we have,

$$\sum_{i=1}^{N} a_i L_i(t)+\frac{c}{t}\left(\beta+\sum_{i=1}^{N} a_i P_{1i}(t)\right)+f\left(\alpha+\beta t+\sum_{i=1}^{N} a_i P_{2i}(t)\right)=g(t) \quad (3.6)$$

Solving (3.6), we obtain a_i, substituting these a_i in (3.5) we get the solution of the problem (3.1). The error will be calculated by using $E=|u_e-u_a|$ and $E_{\max}=\max|u_e-u_a|$, where u_e & u_a are exact and approximate solutions respectively.
The convergence analysis of the Legendre wavelets is given through the following Lemma,

Lemma: Assume that the $u(t)\in L_2(R)$ with the bounded first derivative on (0, 1), then the error norm at k^{th} level satisfies the following inequality $\|e_k(t)\|\le A2^{-(3/2)(N/2)}$, where $A=\sqrt{\frac{K}{7}}C$ is some real constant.

Proof. The error at k^{th} level may be defined as,

$$|e_k(t)|=|u(t)-u_k(t)|=\left|\sum_{i=N+1}^{\infty} a_i L_i(t)\right|$$

where $$u_k(t)=\sum_{i=1}^{N=2^{k+1}} a_i L_i(t)$$

$$\|e_k(t)\|^2=\int_{-\infty}^{\infty}\left\langle \sum_{i=N+1}^{\infty} a_i L_i(t), \sum_{l=N+1}^{\infty} a_l L_l(t)\right\rangle dt=\sum_{i=N+1}^{\infty}\sum_{l=N+1}^{\infty} a_i a_l \int_{-\infty}^{\infty} L_i(t)L_l(t)dt$$

$$\|e_k(t)\|^2\le\sum_{i=N+1}^{\infty}|a_i|^2$$

But $|a_i|\le C2^{-\frac{3i}{2}}\max|u'(t)|$

where $C=\int_0^1 |tC(t)|dt$ and $t\in\left(\frac{n-1}{2^k}, \frac{n}{2^k}\right)$

Then
$$\|e_k(t)\|^2\le\sum_{i=N+1}^{\infty} KC^2 2^{-3i}$$
Where $|u'(t)|\le K \ \forall t\in(0,1)$, where K is positive constant.

$$\|e_k(t)\|^2\le KC^2\frac{1}{7}2^{-3(N/2)}$$

$$\|e_k(t)\|\le\sqrt{\frac{K}{7}}C2^{-(3/2)(N/2)}$$

$$\|e_k(t)\|\le A2^{-(3/2)(N/2)}$$, where $A=\sqrt{\frac{K}{7}}C$ is some real constant.

From the above lemma, the error bound is inversely proportional to the level of the resolution of the Legendre wavelets. This ensures that the convergence of the Legendre wavelets approximation by increasing the level of resolution.

Rate of convergence $R_c(N)$**:**
The rate of convergence is defined as

$$R_c(N) = \frac{\log\left(E_{\max}(N/2)/E_{\max}(N)\right)}{\log 2}.$$

4. Numerical examples

In this section, we consider some of the singular initial value problems to demonstrate the applicability of the proposed Legendre wavelet collocation method.

Problem 1. First consider the equation of the type [3],

$$u''(t) + \frac{2}{t}u'(t) - 2(2t^2+3)u(t) = 0 \tag{4.1}$$

with the initial conditions $u(0)=1,\ u'(0)=0$.

Using the method explained in section 3, we obtained the LWCM solution and is presented in comparison with exact solution $u(t)=e^{t^2}$, Haar wavelet collocation method (HWCM) (as the method explained in [15]), Chebyshev wavelet collocation method (CWCM) [18] solutions and VIM solution in Table 1 for *N=16 (M=8 & k=2)* & Fig. 1 for *N=32 (M=8 & k=3).* The error analysis for higher values of *N* is given in Table 2.

Table 1: Comparison of numerical solutions with exact solution for *N=16* of Problem 1.

$t(=1/32)$	VIM	HWCM	CWCM	LWCM	Exact
1	1.00097703	1.00097815	1.00026748	1.00117891	1.00097703
3	1.00882779	1.00882642	1.00811104	1.00900887	1.00882779
5	1.02471452	1.02470820	1.02398686	1.02487530	1.02471452
7	1.04901493	1.04900127	1.04826932	1.04915474	1.04901493
9	1.08231423	1.08229096	1.08154578	1.08242961	1.08231423
11	1.12542873	1.12539373	1.12462986	1.12551656	1.12542873
13	1.17943916	1.17939066	1.17860099	1.17949601	1.17943918
15	1.24573589	1.24567263	1.24485199	1.24575587	1.24573605
17	1.32607839	1.32600014	1.32211963	1.32673859	1.32607912
19	1.42267242	1.42258103	1.41799363	1.42325240	1.42267522
21	1.53826926	1.53817116	1.53288963	1.53871647	1.53827869
23	1.67629260	1.67620433	1.67019824	1.67656637	1.67632108
25	1.84099996	1.84095995	1.83416112	1.84107835	1.84107853
27	2.03768719	2.03777990	2.03008186	2.03758574	2.03788817
29	2.27294665	2.27334961	2.26460332	2.27275946	2.27342854
31	2.55499169	2.55606378	2.54607106	2.55497375	2.55608441

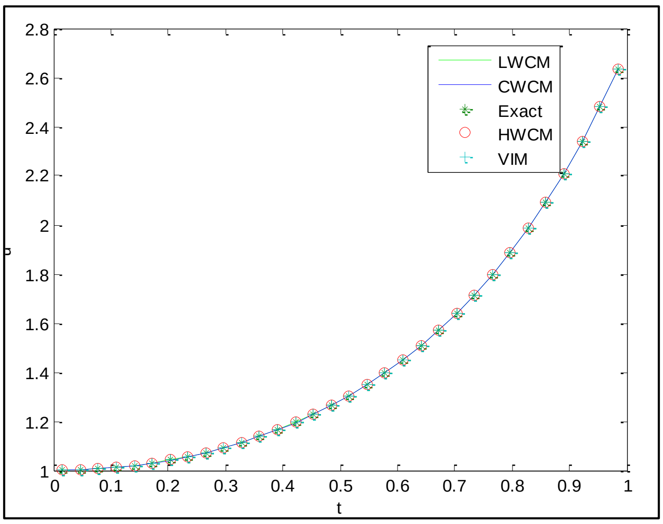

Fig 1: Comparison of numerical solutions with exact solution for *N=32* of Problem 1.

Table 2: Error analysis of Problem 1.

M	k	N	$E_{\max}$				Rate of convergence $R_C(N)$			
			VIM	HWCM	CWCM	LWCM	VIM	HWCM	CWCM	LWCM
8	2	16	1.0927 e-03	1.1858 e-04	1.0013 e-02	1.1106 e-03	-	-	-	-
8	3	32	1.3304 e-03	3.1418 e-05	8.0366 e-04	4.7102 e-05	0.2840	1.9162	3.6391	4.5594
8	4	64	1.4664 e-03	7.9647 e-06	5.6995 e-05	2.7398 e-06	0.1404	1.9799	3.8177	4.1036
8	5	128	1.5391 e-03	1.9977 e-06	3.8107 e-06	1.4987 e-07	0.0698	1.9953	3.9027	4.1923
8	6	256	1.5767 e-03	4.9990 e-07	2.4673 e-07	8.3214 e-09	0.0348	1.9986	3.9491	4.1707
8	7	512	1.5958 e-03	1.2500 e-07	1.5702 e-08	4.7990 e-10	0.0174	1.9997	3.9739	4.1160

Problem 2. Next, consider the equation of the form [2],

$$u''(t)+\frac{2}{t}u'(t)+u(t)=0 \tag{4.2}$$

with the initial conditions $u(0)=1,\ u'(0)=0$.

Using the method explained in section 3, we obtained the LWCM solution and is presented in comparison with ADM, HWCM, CWCM solutions and exact solution $u(t)=\frac{\sin t}{t}$ in Table 3 for *N=16 (M=8 & k=2)* & Fig. 2 for *N=32 (M=8 & k=3)*. The error analysis for higher values of *N* is given in Table 4.

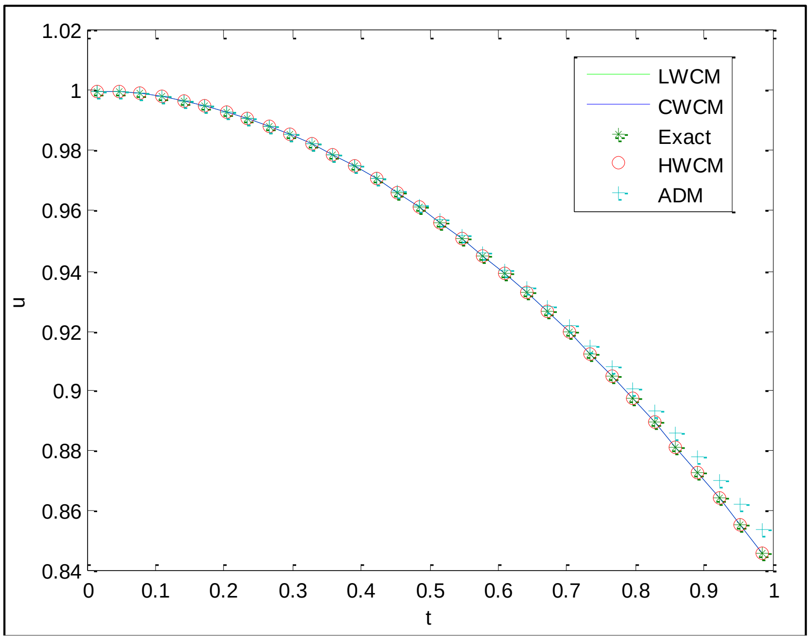

Fig 2: Comparison of numerical solutions with exact solution for *N=32* of Problem 2.

Table 3: Comparison of numerical solutions with exact solution for *N=16* of Problem 2.

$t(=1/32)$	ADM	HWCM	CWCM	LWCM	Exact
1	0.99983725	0.99983726	0.99982934	0.99983999	0.99983724
3	0.99853644	0.99853577	0.99852790	0.99853857	0.99853579
5	0.99594092	0.99593584	0.99592807	0.99593873	0.99593595
7	0.99206288	0.99204356	0.99203595	0.99204659	0.99204379
9	0.98692059	0.98686804	0.98686064	0.98687125	0.98686845
11	0.98053837	0.98042140	0.98041426	0.98042483	0.98042201
13	0.97294656	0.97271871	0.97271189	0.97272241	0.97271958
15	0.96418146	0.96377797	0.96377151	0.96378197	0.96377913
17	0.95428530	0.95362005	0.95361549	0.95362402	0.95362156
19	0.94330619	0.94226865	0.94226471	0.94227301	0.94227053
21	0.93129800	0.92975019	0.92974688	0.92975499	0.92975249
23	0.91832033	0.91609379	0.91609114	0.91609907	0.91609655
25	0.90443844	0.90133115	0.90132918	0.90133695	0.90133441
27	0.88972309	0.88549648	0.88549523	0.88550285	0.88550028
29	0.87425050	0.86862640	0.86862590	0.86863338	0.86863078
31	0.85810221	0.85075984	0.85076013	0.85076748	0.85076484

Table 4: Error analysis of Problem 2.

M	k	N	E_{max}				Rate of convergence $R_C(N)$			
			VIM	**HWCM**	**CWCM**	**LWCM**	**VIM**	**HWCM**	**CWCM**	**LWCM**
8	2	16	7.3373 e-03	5.0012 e-06	7.9071 e-06	2.8381 e-06	-	-	-	-
8	3	32	7.8221 e-03	1.2932 e-06	5.0500 e-07	1.7517 e-07	0.0923	1.9513	3.9688	4.0181
8	4	64	8.0733 e-03	3.2854 e-07	3.1732 e-08	1.0911 e-08	0.0456	1.9768	3.9923	4.0049
8	5	128	8.2012 e-03	8.2783 e-08	1.9859 e-09	6.8139 e-10	0.0227	1.9887	3.9981	4.0012
8	6	256	8.2657 e-03	2.0776 e-08	1.2416 e-10	4.2577 e-11	0.0113	1.9944	3.9995	4.0003
8	7	512	8.2981 e-03	5.2040 e-09	7.7608 e-12	2.6609 e-12	0.0056	1.9972	3.9999	4.0001

Problem 3. Now, consider the non homogeneuos equation of the type [1],

$$u''(t)+\frac{8}{t}u'(t)+tu(t)=t^5-t^4+44t^2-30t \qquad (4.3)$$

with the initial conditions $u(0)=0,\ u'(0)=0$.

As in the previous examples, we obtained the LWCM solution and is presented in comparison with exact solution $u(t)=t^4-t^3$, ADM, HWCM and CWCM solutions in Table 5 for *N=16 (M=8 & k=2)* & Fig. 3 for *N=32 (M=8 & k=3)*. The error analysis for higher values of *N* is given in Table 6.

Table 5: Comparison of numerical solutions with exact solution for *N=16* of Problem 3.

$t(=1/32)$	**ADM**	**HWCM**	**CWCM**	**LWCM**	**Exact**
1	0	-0.00004853	-0.00100612	0.00030525	-0.00002956
3	0	-0.00064069	-0.00172326	-0.00041191	-0.00074672
5	0	-0.00310421	-0.00419508	-0.00288387	-0.00321865
7	0	-0.00803492	-0.00915397	-0.00784305	-0.00817775
9	0	-0.01583280	-0.01696609	-0.01565568	-0.01599025
11	0	-0.02649727	-0.02763139	-0.02632178	-0.02665615
13	0	-0.03966067	-0.04078360	-0.03947515	-0.03980922
15	0	-0.05459072	-0.05569027	-0.05438339	-0.05471706
17	0	-0.07018880	-0.07125271	-0.06994787	-0.07028102
19	0	-0.08499002	-0.08600604	-0.08470378	-0.08503627
21	0	-0.09716330	-0.09811914	-0.09682007	-0.09715175
23	0	-0.10451129	-0.10539472	-0.10409950	-0.10443019
25	0	-0.10447044	-0.10526925	-0.10397859	-0.10430812
27	0	-0.09411095	-0.09481301	-0.09352768	-0.09385585
29	0	-0.07013679	-0.07073004	-0.06945089	-0.06977748
31	-0.02841102	-0.02888567	-0.02935821	-0.02808611	-0.02841091

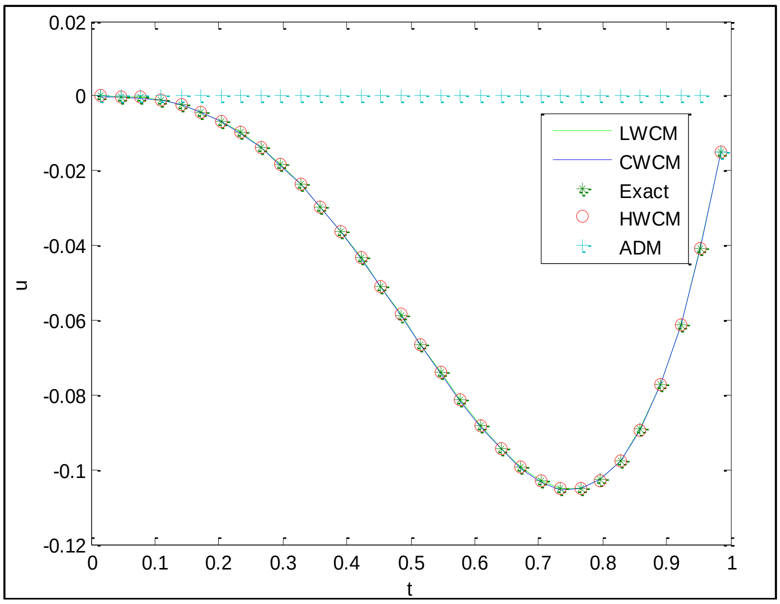

Fig 3: Comparison of numerical solutions with exact solution for *N=32* of Problem 3.

Table 6: Error analysis of Problem 3.

M	k	N	E_{max} VIM	HWCM	CWCM	LWCM	Rate of convergence $R_C(N)$ VIM	HWCM	CWCM	LWCM
8	2	16	1.0443 e-01	4.7475 e-04	9.7656 e-04	3.3482 e-04	-	-	-	-
8	3	32	1.0520 e-01	1.2685 e-04	6.1035 e-05	2.0926 e-05	0.0106	1.9040	4.0000	4.0000
8	4	64	1.0540 e-01	3.2718 e-05	3.8147 e-06	1.3078 e-06	0.0027	1.9550	4.0000	4.0001
8	5	128	1.0545 e-01	8.3041 e-06	2.3842 e-07	8.1734 e-08	0.0006	1.9782	4.0000	4.0001
8	6	256	1.0546 e-01	2.0915 e-06	1.4901 e-08	5.1089 e-09	0.0001	1.9893	4.0000	3.9999
8	7	512	1.0546 e-01	5.2482 e-07	9.3132 e-10	3.1931 e-10	0	1.9946	4.0000	4.0000

Problem 4. Finally consider another non homogeneuos equation of the form [3],

$$u''(t)+\frac{2}{t}u'(t)+u(t)=t^3+t^2+12t+6 \tag{4.4}$$

with the initial conditions $u(0)=0,\ u'(0)=0$.

As in the previous examples, we obtained the LWCM solution and is presented in comparison with VIM, HWCM, CWCM solutions and exact solution $u(t)=t^2+t^3$ in Table 7 for *N=16 (M=8 & k=2)* & Fig. 4 for *N=32 (M=8 & k=3)*. The error analysis for higher values of *N* is given in Table 8.

Table 7: Comparison of numerical solutions with exact solution for *N=16* of Problem 4.

$t(=1/32)$	VIM	HWCM	CWCM	LWCM	Exact
1	0.00100708	0.00103759	0.00100708	0.00100708	0.00100708
3	0.00961303	0.00955808	0.00961303	0.00961303	0.00961303
5	0.02822875	0.02809547	0.02822875	0.02822875	0.02822875
7	0.05831909	0.05811939	0.05831909	0.05831909	0.05831909
9	0.10134887	0.10108593	0.10134887	0.10134887	0.10134887
11	0.15878296	0.15845846	0.15878295	0.15878295	0.15878295
13	0.23208619	0.23170138	0.23208618	0.23208618	0.23208618
15	0.32272343	0.32227940	0.32272338	0.32272338	0.32272338
17	0.43215955	0.43165735	0.43215942	0.43215942	0.43215942
19	0.56185944	0.56130010	0.56185913	0.56185913	0.56185913
21	0.71328807	0.71267258	0.71328735	0.71328735	0.71328735
23	0.88791045	0.88723969	0.88790893	0.88790893	0.88790893
25	1.08719173	1.08646636	1.08718872	1.08718872	1.08718872
27	1.31259723	1.31181752	1.31259155	1.31259155	1.31259155
29	1.56559252	1.56475809	1.56558227	1.56558227	1.56558227
31	1.84764352	1.84675301	1.84762573	1.84762573	1.84762573

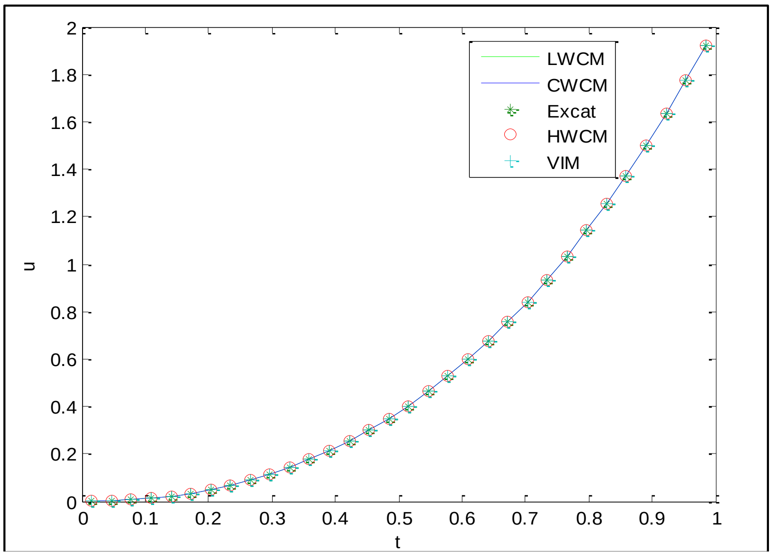

Fig 4: Comparison of numerical solutions with exact solution for *N=32* of Problem 4.

Table 8: Error analysis of Problem 4.

M	k	N	$E_{\max}$			
			VIM	HWCM	CWCM	LWCM
8	2	16	1.7795 e-05	8.7272 e-04	1.7764 e-15	3.3306 e-16
8	3	32	2.0316 e-05	2.2143 e-04	8.8816 e-16	6.6613 e-16
8	4	64	2.1691 e-05	5.5744 e-05	4.4409 e-16	4.4408 e-16
8	5	128	2.2409 e-05	1.3982 e-05	6.6613 e-16	6.6613 e-16
8	6	256	2.2776 e-05	3.5015 e-06	6.6613 e-16	4.4408 e-16
8	7	512	2.2961 e-05	8.7609 e-07	8.8818 e-16	2.2204 e-16

5. Conclusion

A new efficient method that is, Legendre wavelet collocation method has been proposed for the numerical solution of singular initial value problems. The accurate implementation of the classical numerical methods for the numerical solution of the singular initial value problems has been found to involve some difficulty. It has been shown here that the present method can be easily implemented and the results obtained are most accurate. The performance of LWCM is superior to the HWCM, CWCM and other classical methods for example VIM & ADM, which is justified through the illustrative problems. Hence the present method has a clear advantage over the classical methods. A distinct feature of the proposed method is its simple applicability for a variety of singular initial value problems.

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