International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(4): 184-189

www.mathsjournal.com
Received: 20-05-2018
Accepted: 21-06-2018

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Forecasting of mobile phone services subscription in a saturating market

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Abstract
The aim of this paper is to make a short-term forecast for the growth of mobile phone subscribers in India. Using data of mobile phone subscribers from the year 1995 to 2016, a forecast was performed for four years (2020). Gompertz distribution function was used in this forecasting. The Gompertz distribution function is a type of mathematical model for a time series, where growth is slowest at the start and end of a time period. In case of mobile phone subscription, the assumption is made that the growth is slow at the beginning and it becomes saturated towards the end when most of the population has already subscribed to the services. The result of this study suggests that by the year 2020, India will reach a density of 94 subscribers per 100 population.

Keywords: Forecasting, mobile phone subscribers, gompertz density function, saturating market

Introduction

Today, the mobile phones are no more a luxury as they used to be way back in mid 1990s when wireless communications system such as mobile phones started as services in India. In the beginning of wireless communications services on offer, the mobile phones were not common, the devices (mobile phones) were expensive and cost of services were very high. But over the period of time, the users of mobile services increased and this factor helped organization to overcome operational expenses leading to further decrease in cost of services, passing benefits to consumers and hence cheaper services available for everyone in the country. Today, mobile phones are smart, feature rich, inexpensive, easy to use and cost of maintaining these services are one of the cheapest in the world.

The growth of mobile phone service subscribers and its usage is expanding rapidly in the country. In a 2016 report by TRAI, India's telecom regulator, it stated that mobile phone subscriber base crossed the 1 billion users mark (Trak.in & TRAI, 2017) [13].

According to Diffusion of Innovation Theory (Rogers, 2003) [10], technological innovations are not adopted by all individuals in a social system at the same time. In mobile service case, the initial services being expensive, were not adopted by everyone, but as the users grow, these services become affordable due to price reduction and growth picks up until it reached to almost everyone leading to a saturation level in the business. Keeping this in mind, the objectives of this research are to explore the density of mobile subscribers in the country and to forecast the growth of mobile phone subscribers. For forecasting we will use Gompertz Density Function (Gompertz, 1825) [5] which also takes care of diffusion theory.

Literature Review

The sigmoid models are very often used in technological change and in general, economic growth and diffusion studies. An early use of such models to analyse the economic growth can be observed in the research work of French sociologist Tarde (Tarde, 1903) [11]. Following Tarde's idea, Mansfield (1961) [7] explained the observed patterns of diffusion in terms of the expected profitability of the innovation, and the dissemination of information about its technical and economic characteristics.

Christian (2012) [3] in his research explored necessity and penetration rate of 3G service in India, Mexico and Thailand by using the Gompertz curve model. He examined the drivers behind 3G

diffusion using linear probability models and qualitative choice (logit) analysis and concluded that time series diffusion patterns follow an s-curve pattern, favouring the Gompertz curve.
Ahmad *et al*. (2014) [1] in their study examined different models to estimate the drivers for the growth of mobile phone subscribers in Oman. The growth rates were estimated by fitting four statistical models namely, Logistic, Gompertz, Exponential and Autoregressive model. They assessed the forecasting performance of these models using Mean Absolute Deviation (MAD) and Mean Square Error. Finally, they concluded that based on the criterion for assessment of the model using the minimum forecast error, the Gompertz model outperformed the other models.
In another research, Das (2013) [4] forecasted the maximum possible GSM subscriber base to 760 million but beyond the year 2020, which in fact crossed 1 billion-mark in 2017 (Trak. in, 2017) [13]. This conflict in forecasting naturally leads us to investigate the use of statistical tests that require simple estimation and easy computation. We will model the growth of mobile services in India using diffusion framework, mode specifically Gompertz Diffusion Function, while incorporating other variables that might impact growth and make a forecast in short term.
In a manner conforming to diffusion theory and plethora of research findings, it is hypothesized that the number of mobile phone subscribers in India would grow slowly initially and saturates later when density of subscribers reaches towards 100 percent hence forming a S-shaped curve. This phenomenon can be described using Gompertz Density Function (sometimes also known as Gompertz Curve or Gompertz Function, named after Benjamin Gompertz, is a sigmoid function). The right-hand (also known as future value) asymptote of the function is approached much more gradually by the curve than the left-hand (or lower valued) asymptote. This is typically in contrast to the simple logistic function in which both asymptotes are approached by the curve symmetrically.
The univariate analysis like ARIMA (Kumar & Sharma, 2016) [6] could also be exploited to make a similar forecast but that would conflict the basic scenario where growth during initial years are slow (one reason behind it poor infrastructure and high cost of operations), then rises exponentially as infrastructure improves and cost of operations falls but later sustainability (of that business growth) becomes a challenge as saturation phase arrives (where all the consumers are brought into its services with diminishing potentials of adding more customers to the base).

Gompertz Density Function

Benjamin Gompertz (1825) [5], in his research and published a paper in the Philosophical Transactions of the Royal Society, "On the Nature of the Function Expressive of the Law of Human Mortality," showed that "if the average exhaustions of a man's power to avoid death were such that at the end of equal infinitely small intervals of time, he lost equal portions of his remaining power to oppose destruction," then the number of survivors at any age ‘x’ would be given by the equation 1 below:

$$L_x = \alpha\, g^{cx} \qquad \dots (1)$$

For any historical time series period’t’ [t > 0] and given Y_t as variable of interest, the relative increase in Y_t is obtained by the equation 2 below:

$$y_t = ((Y_t - Y_{t-1})/Y_{t-1}) \qquad \dots (2)$$

And if Y_t is a time series taking nonnegative values, the Gompertz diffusion curve for Y_t is given by:

$$Y_t = \alpha e^{-\beta e^{-\gamma t}} \qquad \dots (3)$$

Here α, β, γ > 0 i.e. the parameters α, β and γ are all positive and upper limit of the growth curve is controlled by parameter ‘α’ [0% < α ≤ 100%]. The ‘β’ [0 < β ≤ 1] sets the displacement along the x-axis, ‘γ’ [0 < γ ≤ 1] sets the growth rate (i.e. y scaling) and ‘e’ is Euler's number (i.e. ‘e’ = 2.71828….), as presented by Charles P. Winsor [13].

Data and Sources

Below shown in table 1 is the data of yearly total Mobile Subscribers sourced from Telecom Regulatory Authority of India (TRAI), Cellular Operators Association of India (COAI, 2017) [2], and online from Track. In. The yearly total population data is sourced from World Bank (2017) [16] and Worldometers.info (2017) [17]. The density of mobile subscribers is calculated for per 100 people in India.

Table 1: Mobile Subscribers, Population and Density per 100 People in India

Year	Mobile Cellular Subscribers#	Population*	Mobile Subscribers Density&
1995	76680	955804355	0.008
1996	327967	973147577	0.034
1997	881839	990460131	0.089
1998	1195400	1007746556	0.119
1999	1884311	1025014711	0.184
2000	3577095	1042261758	0.343
2001	6540000	1059500888	0.617
2002	13000000	1076705723	1.207
2003	33690000	1093786762	3.080
2004	52220000	1110626108	4.702

2005	90140000	1127143548	7.997
2006	166050000	1143289350	14.524
2007	233620000	1159095250	20.155
2008	346890000	1174662334	29.531
2009	525090000	1190138069	44.120
2010	752190000	1205624648	62.390
2011	893862478	1221156319	73.198
2012	864720917	1236686732	69.922
2013	886304245	1252139596	70.783
2014	960579472	1267401849	75.791
2015	1010890000	1311050527	77.105
2016	1127370000	1326801576	84.969
Sources:			
# TRAI, COAI and TRAK.in			
** World Bank and http://www.worldometers.info/*			
& Per 100 People			

From raw data above, below in Figure 1, we have a plot for the yearly population growth in India between 1995 and 2016. Clearly, we can see that the population is growing at almost a constant rate (about 1.5-2% Year on Year) with some small spike near year 2014-15 and later returning to same average in rest of the recent years.

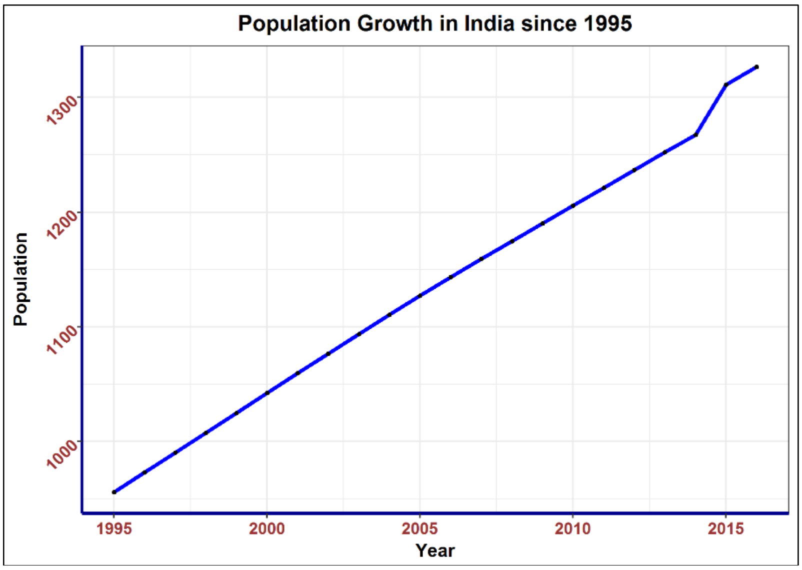

Fig 1: India's Total Yearly Population in Millions

Next, we would explore the data for Yearly Mobile Subscribers for the same period. The mobile services were launched in India in 1995 and below Figure 2 represents the growth in subscription since inception until most recent year for which complete data is available at the sources (Track.in, World Bank, Worldometers):

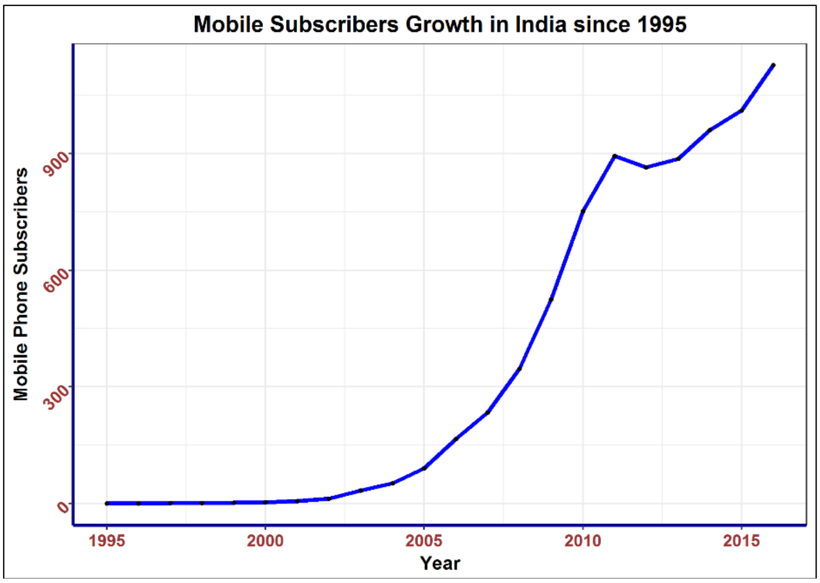

Fig 2: Total Subscribers for Mobile Services in India (in Millions)

Clearly, we see that there has been a slow growth in the numbers during initial phase of service offerings and then it followed sharp rise towards the mid years and slowing growth in most recent years. These most recent years could be seen as the beginning towards saturated growth. This shows up that the trend is taking a turn towards slowing down growth.

Let us further explore the trend using density of subscribers between the periods. Below Figure 3 represents the same:

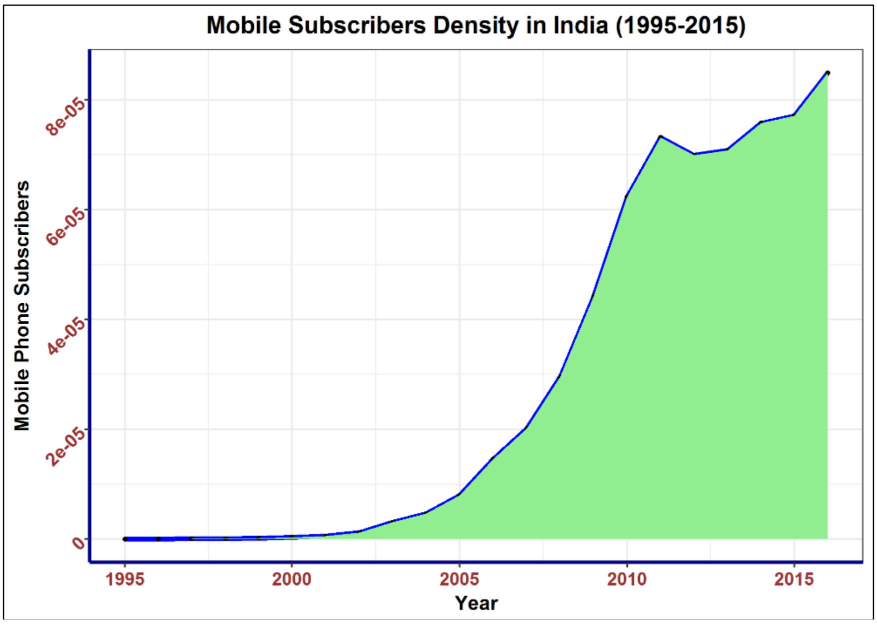

Fig 3: Total Mobile Subscribers Density (in Millions) Per 100 People in India

And the Figure 4 below is a comparative Figure for total subscribers' growth and growth in density of subscribers (scaled to per 1000 people):

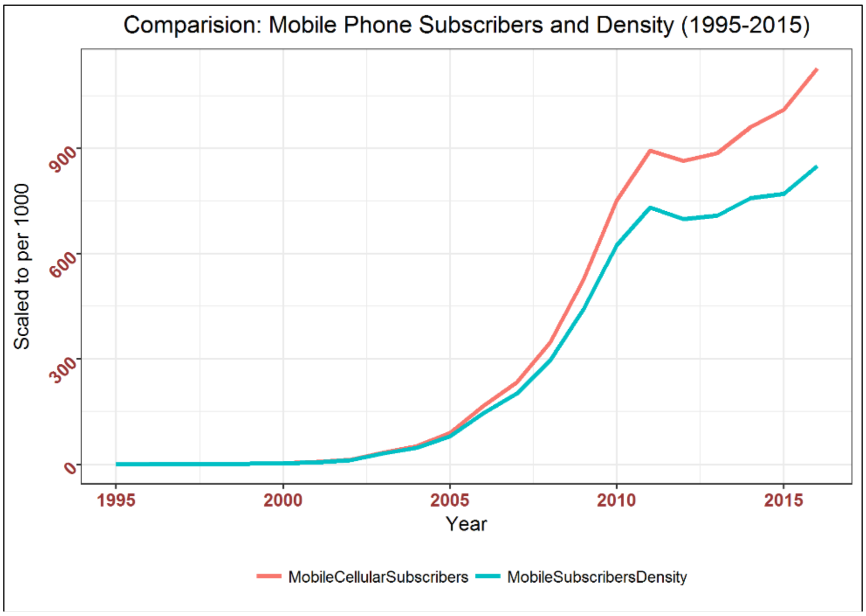

Fig 4: Comparative Figure

This is quite evident that there has been slow growth in the initial decade of mobile phone technology (which could be because of infrastructure development phase and cost to customers) and then sudden rise in users in mid years and slow growth in later years. Although trend still not tending towards saturation, the plot suggests that it will move towards saturation in coming years. Also, because population growth, as we saw earlier, in all these years were almost constant, the density almost follows the subscribers growth trend.

Modelling the Data

Y_t is the density of mobile phone subscribers per 100 persons at time't' (year). Here't' is taken as 0 for 1995 (the year when mobile services started in India), 1 for 1996, 2 for 1997, and so forth. In our case since density varies between 0 and 100, the parameter 'α' takes a max value of 100 (at saturation).

The Gompertz density function, as in equation (3) above, reaches its maximum penetration rate at $Y_t = \alpha/e$ which is the point of inflection of the curve that occurs at $t = \ln(\beta)/\gamma$.

When t = 0, $Y_t = \alpha e^{-\beta}$ is the starting point of mobile phone subscribers density and when't' is large (end of the time series), $Y_t = \alpha$, i.e. Y_t reaches towards saturation. Using saturation parameter 'α', time variable 't' and ordinary least square method, the parameters 'β' and 'γ' are estimated. The open source statistical computational tool 'R' (version 3.4.0), Packages 'drc' (Ritz *et al.* 2015, 2016) [8, 9] and gg Figure 2 (Wickham, 2009) [14] were used for statistical analysis.

Below table 2 shows the result obtained from data modeling:

Table 2: Parameters

α	β	γ	R^2	MSE
84.4033	-0.4023	13.6762	0.88	4.102

These parameters, as obtained in the table 2 above, are used to fit the model on original data. Below Figure 5 shows the original data series of subscribers' density, the fitted regression model and the fitted Gompertz Model:

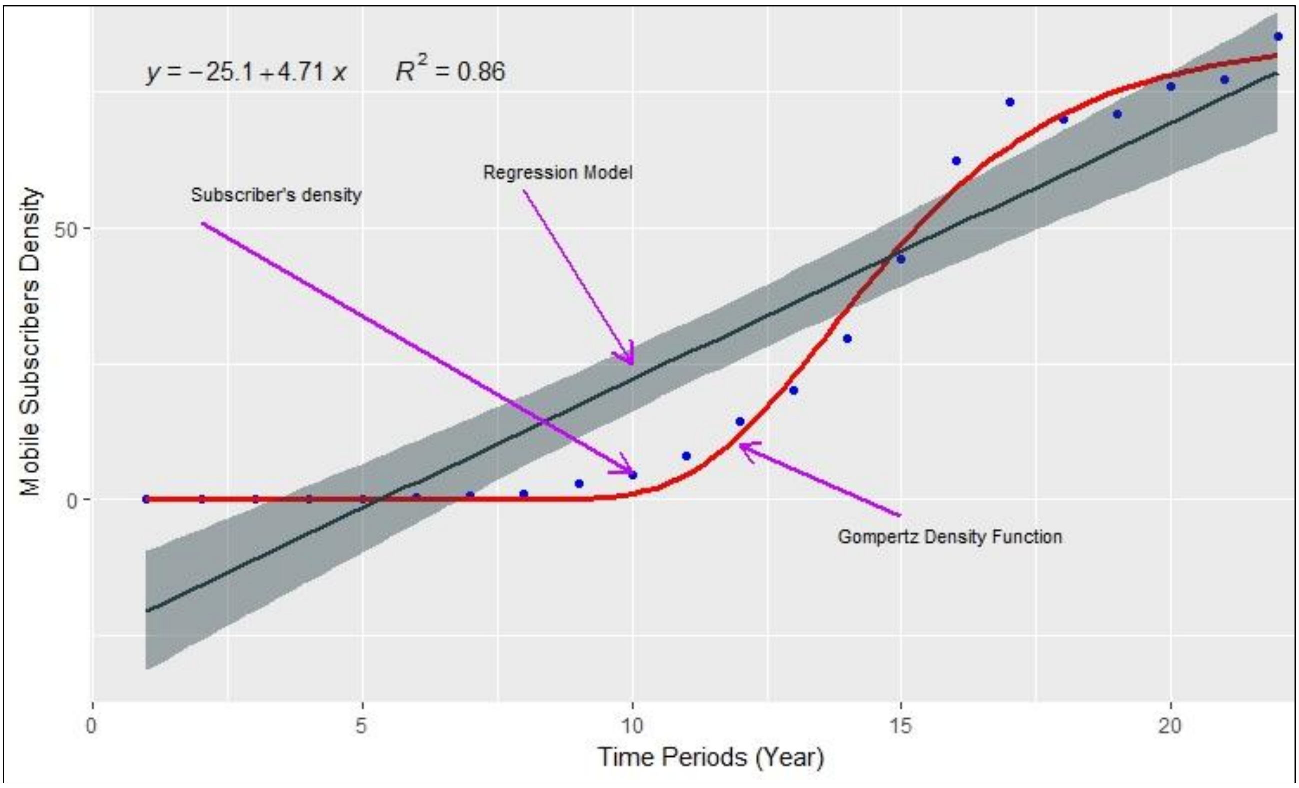

Fig 5: Fitted Regression and Gompertz Models

We can observe that the Gompertz model fits well on the data as compare to regression model. Using the above parameters, we will make a prediction for future growth using Gompertz function. This prediction (and estimation) of future growth of mobile phone subscriptions in India might help researchers and policy makers in estimating the remaining untapped market of mobile services in India. Table 3 below shows the forecasting of growth in population as well as for subscribers in India for the four years in future (i.e. 2017 through 2020) along with 95% confidence interval bounds:

Table 3: Forecast of Subscribers' Growth

Year	Population	Density	Mobile Phone Subscribers		
			Lower Bound (Pessimistic)	Actual Forecast	Upper Bound (Optimistic)
2017	1344468110	88.56089	1140151711	1190672924	1163215121
2018	1362134645	90.12635	1187630895	1227642238	1278550658
2019	1379801179	92.60495	1248738418	1277764192	1304032019
2020	1397467713	93.75544	1287520683	1310202003	1331763253

Conclusion

In this research, we studied the growth of mobile phone subscriptions in India and we observed that the growth followed an S-curve (precisely, Gompertz function). The growth during the first decade of inception of services (i.e. from year 1995 to 2003) was almost constant but during mid-years (i.e. 2004-2011), it was exponentially high and during later years (i.e. 2011 to 2016), the growth approximately started approaching the saturation (or slowing growth) phase.

Using annual data of mobile phone subscribers since inception of services in India (year 1995) to 2016, the parameters of the Gompertz function were estimated using ordinary least squares methodology. These parameters were found statistically significant under the assumed saturation levels. The R^2 value and lower mean squared error (MSE) suggest that model can be assumed to be a good fit for forecasting of mobile phone subscribers in near future.

Using the obtained parameter, we then made four periods ahead forecast of population, subscribers' density and phone subscribers. We observed that almost 84 subscriptions of mobile services per 100 persons in 2015 further rises to 93 subscriptions per 100 persons by 2020 (as forecasted) and forecast suggests that the subscribers' density (or total phone subscribers) will be flatten out further (i.e. move towards more saturation) in four years.

This study and forecast for density growth would serve as one of the supporting references to help the telecom services providing companies, mobile handset manufacturers and other related accessories suppliers to re-design their operational and growth strategies in India. However, this research can further be extended to divide the growth between various segments such as rural and urban populations, low, medium and high-income populations etc. where there could be some significant differences in the growth.

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