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Akintunde Oyetunde A
Department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti State, Nigeria

Correspondence
Akintunde Oyetunde A
Department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti State, Nigeria

On the introduction of location parameter to Nakagami-m distribution

Akintunde Oyetunde A

Abstract
This research paper examined the Nakagami-m distribution with its properties and then investigated in the new proposed three-parameter extended Nakagami by the introduction of the location parameter. The new proposed distribution, also named as Nakagami-Akintunde distribution, was examined with some basic properties including moments, skewness, kurtosis, asymptotic behaviors and estimation of parameters. The proposed model is much more flexibility and has better representation of data than other existing three-parameter probability distributions like Wei bull distribution, Gamma distribution, Frechet distribution and others. A simulated data set is used to illustrate the importance the potentiality of the new model.

Keywords: Nakagami distribution, location parameter, moments, skewness, kurtosis

1. Introduction

Of the probability distributions, Nakagami-m distribution can be considered as a flexible lifetime distribution. The distribution has been used to model attenuation of wireless signals traversing multiple paths, fading of radio signals, data regarding communicational engineering, and so forth since the time of its derivation and formulation by Nakagami, M. (and after whom the distribution was named) in 1960 [1]. The distribution has also been employed to model failure times of a variety of products (and electrical components) such as ball bearing, vacuum tubes, and electrical insulation. It is also widely considered in biomedical fields, such as to model the time to the occurrence of tumors and appearance of lung cancer. It has the applications in medical imaging studies to model the ultrasounds especially in Echo (heart efficiency test). Shanker *et al.* [2] And Tsui *et al.* [3] used the Nakagami distribution to model ultrasound data in medical imaging studies. Nakagami distribution is extensively used in reliability theory and reliability engineering.

Yang and Lin [4] during their investigations to derive the statistical model of spatial-chromatic distribution of images through extensive evaluation of large image database discovered that a two-parameter Nakagami distribution well suits the purpose. Kim and Latchman [5] also used the Nakagami distribution in their analysis of multimedia. However, several distributions have been proposed to describe the composite fast fading and shadowing. Initially, the Rayleigh-lognormal distribution was used by Suzuki [6] to model this compound fading. Also, the Nakagami-lognormal distribution has been employed to model the composite fading by Abu-Dayya and Beaulieu [7], Ho and Stuber [8], Tjhung and Chai [9], and many others. Recently, some other distributions such as the α-μ by Yacoub [10], the generalized-K by Shankar [11], or the mixture Gamma (MG) distribution by Atapattu, Tellambura and Jiang [12] have been proposed in the literature. Nevertheless, the results derived from a measurement campaign carried out in a macro-cellular urban environment by Reig and Rubio [13] show that the Nakagami distribution is the best fit distribution when compared with the Rayleigh-lognormal, α-μ and generalized-K distributions.

2. Estimation of Two Parameters of Nakagami-m Distribution

There are two main philosophical approaches to estimation of parameters and properties in probability distributions in Statistics. The first is called the classical approach which was

founded by Professor R.A. Fisher in a series of fundamental papers around 1930. In classical approach, we use the same method as obtained by Ahmad *et al*. [14]. The second approach is the Bayesian approach, which was first discovered by Reverend Thomas Bayes. In this approach, parameters are treated as random variables and data is treated as fixed. Recently Bayesian estimation approach has received great attention by most researchers among them are Al-Aboud [15] who studied Bayesian estimation for the extreme value distribution using progressive censored data and asymmetric loss. Ahmed *et al*. [16] considered Bayesian Survival Estimator for Wei bull distribution with censored data. An important pre-requisite in this approach is the appropriate choice of prior(s) for the parameters. Very often, priors are chosen according to one's subjective knowledge and beliefs. The other integral part of Bayesian inference is the choice of loss function. A number of symmetric and asymmetric loss functions have been shown to be functional; see Pandey *et al*. [17], Al-Athari [18], S.P. Ahmad and K. Ahmad [19], Ahmad *et al*. [20, 21], and so forth.
In this research paper, the classical approach is used in estimating the parameters and properties by methods of raw-moments and method of maximum likelihood.

2.1 Estimation of-The k-th Raw Moment of Two Parameters Nakagami-m Distribution
The probability density function (p.d.f) of the Nakagami-m distribution [1] with two parameters $\{m, \Omega\}$ is given as:

$$f(x/m,\Omega) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{m}{\Omega}x^2\right) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1} \exp\left(-\frac{m}{\Omega}x^2\right) \quad \begin{matrix} 0 \le x \le \infty \\ 0.5 < m \le \infty \\ 0 \le \Omega \le \infty \end{matrix} \tag{1}$$

Where m is the shape parameter and Ω is the speed parameter.
Then by the raw moment method,

$$E(x^k) = \int_{-\infty}^{\infty} x^k f(x)\, dx$$

$$= \int_0^{\infty} x^k \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1} \exp\left(-\frac{m}{\Omega}x^2\right) dx$$

$$= \int_0^{\infty} \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m+k-1} \exp\left(-\frac{m}{\Omega}x^2\right) dx$$

If $t = \frac{m}{\Omega}x^2 \Rightarrow x = \left(\frac{\Omega}{m}t\right)^{1/2}$ *and* $\frac{dx}{dt} = \frac{1}{2}\left(\frac{\Omega}{m}\right)^{1/2} t^{-1/2} \Rightarrow dx = \frac{1}{2}\left(\frac{\Omega}{m}\right)^{1/2} t^{-1/2}dt$

Also recall:

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1}e^{-y}dy = (\alpha-1)!$$

Thus,

$$E(x^k) = \int_0^{\infty} \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m \left[\left(\frac{\Omega}{m}t\right)^{1/2}\right]^{2m+k-1} e^{-t}\frac{1}{2}\left(\frac{\Omega}{m}\right)^{1/2} t^{-1/2}dt$$

$$= \frac{1}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{k/2} \int_0^{\infty} t^{m+k/2-1} e^{-t}\, dt$$

$$= \frac{\Gamma(m+k/2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{k/2} \tag{2}$$

Then when k=1, $E(x) = \frac{\Gamma(m+1/2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{1/2}$. This is the mean of the distribution. (3)

When k=2, $E(x^2) = \frac{\Gamma(m+1)}{\Gamma(m)}\left(\frac{\Omega}{m}\right) = \Omega$ (4)

When k=3, $E(x^3) = \frac{\Gamma(m+3/2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{3/2}$ (5)

When k=4, $E(x^4) = \frac{\Gamma(m+2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^2 = \frac{(m+1)}{m}\Omega^2$ (6)

2.2.1 Estimation of-The Variance of Two Parameters Nakagami-m Distribution

$$Var\,(x) = E(x^2) - [E(x)]^2$$

$$= \Omega - \left[\frac{\Gamma(m+1/2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{1/2}\right]^2$$

$$= \Omega\left[1 - \frac{1}{m}\left(\frac{\Gamma(m+1/2)}{\Gamma(m)}\right)^2\right] \quad (7)$$

2.2.2 Estimation of-The Skewness of Two Parameters Nakagami-m Distribution

$$skewness = \frac{E(x^3)}{[\sqrt{E(x^2)}]^3}$$

$$= \frac{\Gamma(m+3/2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{3/2}.\frac{1}{\Omega^{3/2}}$$

$$= \frac{\Gamma(m+3/2)}{\mathrm{m}^{3/2}\Gamma(m)} \quad (8)$$

2.2.3 Estimation of-The Kurtosis of Two Parameters Nakagami-m Distribution

$$kurtosis = \frac{E(x^4)}{[E(x^2)]^2}$$

$$= \frac{(m+1)}{m}\Omega^2.\frac{1}{\Omega^2} = \frac{(m+1)}{m}$$

$$= \frac{(m+1)}{m} \quad (9)$$

2.3 Estimation of-The Speed Parameter of Two Parameters Nakagami-m Distribution Using Maximum Likelihood Function method

Let $(x_1, x_2, \dots, x_n)$ be a random sample of size n having pdf (1); then the maximum likelihood estimator of speed parameter Ω, when the shape parameter m is known, is obtained by:

$$L(x_1, x_2, \dots, x_n \,/\, m, \Omega) = f(x_1/m,\Omega).f(x_2/m,\Omega) \;\dots\; f(x_n/m,\Omega)$$

$$= \frac{2m^m}{\Gamma(m)\Omega^m}x_1^{2m-1}\,exp\left(-\frac{m}{\Omega}x_1^2\right).\frac{2m^m}{\Gamma(m)\Omega^m}x_2^{2m-1}\,exp\left(-\frac{m}{\Omega}x_2^2\right)\dots\frac{2m^m}{\Gamma(m)\Omega^m}x_n^{2m-1}\,exp\left(-\frac{m}{\Omega}x_n^2\right)$$

$$= \frac{2^{\mathrm{n}}\mathrm{m}^{\mathrm{nm}}}{[\Gamma(m)]^{\mathrm{n}}\Omega^{nm}}\prod_{i=1}^{n}x_i^{2m-1}\exp\left(-\frac{m}{\Omega}\sum_{i=1}^{n}x_i^2\right) \quad (10)$$

The logarithm of the likelihood function is obtained as:

$$\log L(x_1, x_2, \dots, x_n \,/\, m, \Omega) = n\log 2 + nm\log m - n\log\Gamma(m) - nm\log\Omega + (2m-1)\log\sum_{i=1}^{n}x_i - \frac{m}{\Omega}\sum_{i=1}^{n}x_i^2$$

Taking the derivatives of $\log L(x_1, x_2, \dots, x_n \,/\, m, \Omega)$ with respect to Ω and setting it to zero, we have:

$$\frac{\delta\log L(x_1, x_2, \dots, x_n \,/\, m, \Omega)}{\delta\Omega} = -\frac{nm}{\Omega} + \frac{m}{\Omega^2}\sum_{i=1}^{n}x_i^2 = 0$$

$$\Rightarrow \Omega = \frac{\sum_{i=1}^{n}x_i^2}{n} \quad (11)$$

Equation (13) affirms that equation (4) is true, thus,

$$\Omega = E(x^2) = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

The graph below shows the curve of the pdf of the Nakagami-m distribution at several values of the two parameters $\{m, \Omega\}$ as given by [22]

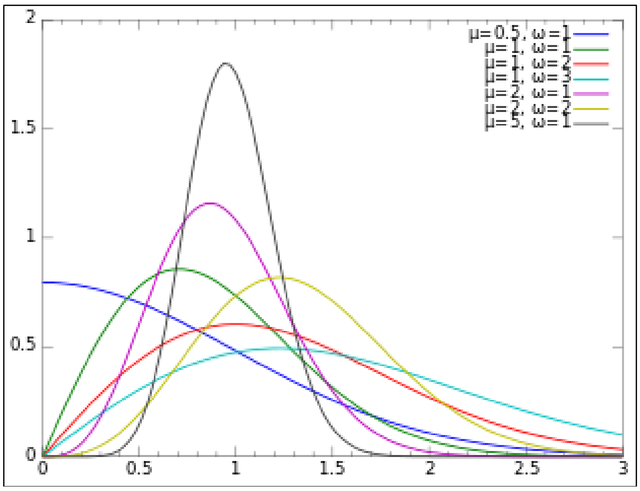

Fig 1: Nakagami-m probability density function curve under different values of m *and* Ω

3. Extending the Two-Parameters Nakagami Distribution to Three Parameters Nakagami Distribution

Like other distribution having three parameters such as Gamma distribution, Wei bull distribution, Frechet distribution and many others, Nakagami-m distribution is also sought out to be extended to three parameters which are m −the shape parameter, Ω −the speed parameter and s - the location shape parameter (which is just introduced).

This research paper proposes the three parameters Nakagami distribution which the researcher develops and the classical approach is used in estimating the parameters and properties by methods of raw-moments and method of maximum likelihood.

Definition: The probability density function (P.D.F) of the extended Nakagami distribution with three parameters $\{m, \Omega, s\}$ is given as:

$$f(x/m, \Omega, s) = \frac{2m^m}{\Gamma(m)\Omega^m}(x-s)^{2m-1} exp\left(-\frac{m}{\Omega}(x-s)^2\right)$$

$$= \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m (x-s)^{2m-1} \exp\left(-\frac{m}{\Omega}(x-s)^2\right) \quad \begin{matrix} 0 \le x \le \infty \\ 0.5 < m \le \infty \\ 0 \le \Omega \le \infty \\ x \le s \le \infty \end{matrix} \qquad (12)$$

Where m is the shape parameter, Ω is the speed parameter and s is the location parameter.

3.1 Estimation of-The k-th Raw Moment of Three Parameters extended Nakagami Distribution

Using the raw moment method,

$$E(x^k) = \int_{-\infty}^{\infty} x^k f(x)\, dx$$

$$= \int_{0}^{\infty} x^k \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m (x-s)^{2m-1} \exp\left(-\frac{m}{\Omega}(x-s)^2\right) dx$$

If $t = \frac{m}{\Omega}(x-s)^2 \Rightarrow (x-s) = \left(\frac{\Omega}{m}t\right)^{1/2} \Rightarrow x = s + \left(\frac{\Omega}{m}t\right)^{1/2}$ *and* $\frac{dx}{dt} = \frac{1}{2}\left(\frac{\Omega}{m}\right)^{1/2} t^{-1/2}$

$$\Rightarrow dx = \frac{1}{2}\left(\frac{\Omega}{m}\right)^{1/2} t^{-1/2} dt$$

Thus,

$$E(x^k) = \int_0^\infty \left(s + \left(\frac{\Omega}{m}t\right)^{1/2}\right)^k \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m \left(\left(\frac{\Omega}{m}t\right)^{1/2}\right)^{2m-1} e^{-t}\frac{1}{2}\left(\frac{\Omega}{m}\right)^{1/2} t^{-1/2}dt$$

$$= \frac{1}{\Gamma(m)}\int_0^\infty \left(s + \left(\frac{\Omega}{m}t\right)^{1/2}\right)^k t^{m-1}e^{-t}dt \qquad (13)$$

Then when k=1.

$$E(x) = \frac{1}{\Gamma(m)}\int_0^\infty \left(s + \left(\frac{\Omega}{m}t\right)^{1/2}\right) t^{m-1}e^{-t}dt$$

$$= s + \left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} \qquad (14)$$

This is the mean of the distribution.
When k=2,

$$E(x^2) = \frac{1}{\Gamma(m)}\int_0^\infty \left(s + \left(\frac{\Omega}{m}t\right)^{1/2}\right)^2 t^{m-1}e^{-t}dt$$

$$= s^2 + 2s\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega \qquad (15)$$

When k=3,

$$E(x^3) = \frac{1}{\Gamma(m)}\int_0^\infty \left(s + \left(\frac{\Omega}{m}t\right)^{1/2}\right)^3 t^{m-1}e^{-t}dt$$

$$= s^3 + 3s^2\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + 3s\Omega + \left(\frac{\Omega}{m}\right)^{3/2}\frac{\Gamma(m+3/2)}{\Gamma(m)} \qquad (16)$$

When k=4,

$$E(x^4) = \frac{1}{\Gamma(m)}\int_0^\infty \left(s + \left(\frac{\Omega}{m}t\right)^{1/2}\right)^4 t^{m-1}e^{-t}dt$$

$$= s^4 + 4s^2\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + 6s^2\Omega + 4s\left(\frac{\Omega}{m}\right)^{3/2}\frac{\Gamma(m+3/2)}{\Gamma(m)} + \frac{\Omega^2(m+1)}{m} \qquad (17)$$

3.2.1 Estimation of-The Variance of Three Parameters extended Nakagami Distribution

$$Var\ (x) = E(x^2) - [E(x)]^2$$

$$= \left[s^2 + 2s\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega\right] - \left[s + \left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)}\right]^2 = \Omega\left[1 - \frac{1}{m}\left(\frac{\Gamma(m+1/2)}{\Gamma(m)}\right)^2\right] \qquad (18)$$

3.2.2 Estimation of-The Skewness of Three Parameters extended Nakagami Distribution

$$skewness = \frac{E(x^3)}{[\sqrt{E(x^2)}]^3}$$

$$= \frac{\left[s^3 + 3s^2\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + 3s\Omega + \left(\frac{\Omega}{m}\right)^{3/2}\frac{\Gamma(m+3/2)}{\Gamma(m)}\right]}{\left[s^2 + 2s\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega\right]^{3/2}}$$

$$= \left[s^3 + 3s^2\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + 3s\Omega + \left(\frac{\Omega}{m}\right)^{3/2}\frac{\Gamma(m+3/2)}{\Gamma(m)}\right]\left[s^2 + 2s\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega\right]^{-3/2} \quad (19)$$

3.2.3 Estimation of-The Kurtosis of Three Parameters extended Nakagami Distribution

$$kurtosis = \frac{E(x^4)}{[E(x^2)]^2}$$

$$= \frac{\left[s^4 + 4s^2\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + 6s^2\Omega + 4s\left(\frac{\Omega}{m}\right)^{3/2}\frac{\Gamma(m+3/2)}{\Gamma(m)} + \frac{\Omega^2(m+1)}{m}\right]}{\left[s^2 + 2s\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega\right]^2}$$

$$= \left[s^4 + 4s^2\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + 6s^2\Omega + 4s\left(\frac{\Omega}{m}\right)^{3/2}\frac{\Gamma(m+3/2)}{\Gamma(m)} + \frac{\Omega^2(m+1)}{m}\right]\left[s^2 + 2s\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega\right]^{-2} \quad (20)$$

3.3 Estimation of-The Speed Parameter of Three Parameters extended Nakagami Distribution Using Maximum Likelihood Function method

Let $(x_1, x_{2,} \dots, x_n)$ be a random sample of size n having pdf (12); then the maximum likelihood estimator of speed parameterΩ, when the shape parameter m is known, is obtained by:

$$L(x_1, x_2, \dots, x_n \,/\, m, \Omega, s) = f(x_1/m, \Omega, s).f(x_2/m, \Omega, s) \;\dots\; f(x_n/m, \Omega, s)$$

$$= \frac{2m^m}{\Gamma(m)\Omega^m}(x_1 - s)^{2m-1}\exp\left(-\frac{m}{\Omega}(x_1 - s)^2\right).\frac{2m^m}{\Gamma(m)\Omega^m}(x_2 - s)^{2m-1}\exp\left(-\frac{m}{\Omega}(x_2 - s)^2\right)\dots.\frac{2m^m}{\Gamma(m)\Omega^m}(x_n - s)^{2m-1}\exp\left(-\frac{m}{\Omega}(x_n - s)^2\right)$$

$$= \frac{2^n m^{nm}}{[\Gamma(m)]^n \Omega^{nm}}\prod_{i=1}^{n}(x_i - s)^{2m-1}\exp\left(-\frac{m}{\Omega}\sum_{i=1}^{n}(x_i - s)^2\right) \quad (21)$$

Taking logarithms of the Likelihood function,

$$\log L(x_1, x_2, \dots, x_n \,/\, m, \Omega, s) = n\log 2 + nm\log m - n\log\Gamma(m) - nm\log\Omega + (2m-1)\sum_{i=1}^{n}(x_i - s) - \frac{m}{\Omega}\sum_{i=1}^{n}(x_i - s)^2$$

Taking the derivatives of $\log L(x_1, x_2, \dots, x_n \,/\, m, \Omega, s)$ with respect to Ω and setting it to zero, we have:

$$\frac{\delta \log L(x_1, x_2, \dots, x_n \,/\, m, \Omega, s)}{\delta\Omega} = -\frac{nm}{\Omega} + \frac{m}{\Omega^2}\sum_{i=1}^{n}(x_i - s)^2 = 0$$

$$\Rightarrow \Omega = \frac{\sum_{i=1}^{n}(x_i - s)^2}{n} \quad (22)$$

The graph below shows the curve of the pdf (12) of the three parameters Nakagami distribution at several values of the three parameters $\{m, \Omega\}$ as shown below:

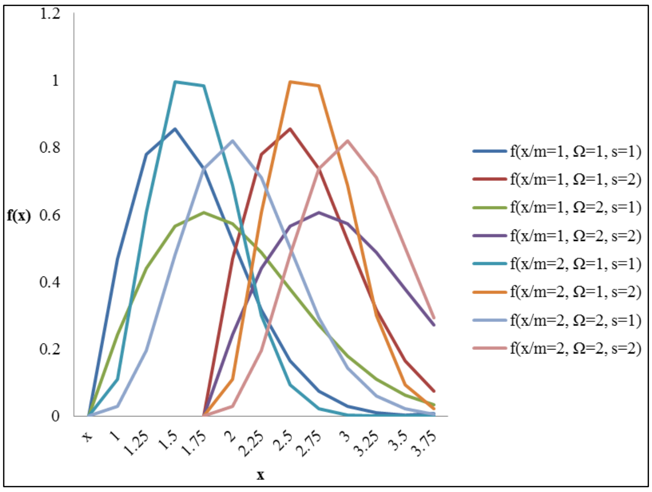

Fig 2: Three Parameters Extended Nakagami probability density function curve under different values of m, Ω and s

It is noted from the graph the distribution is rightly skewed.

4. Relationship of Three Parameter Extended Nakagami-m Distribution to Other Distribution

The three-parameter extended Nakagami distribution is closely related to three-parameter Weibull distribution. By setting$2 = \alpha, m = 1\ \gamma = s\ and\ \Omega = \beta^{\alpha}$, a three-parameter Weibull distribution is generated with probability density function (pdf) [23]:

$$f(x/\alpha,\beta,\gamma) = \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right] \quad (23)$$

Furthermore, normal distribution can be generated from the three-parameter extended Nakagami distribution by setting$m = \frac{1}{2}, s = \mu\ and\ \Omega = \sigma^2$. This resulted in the normal distribution with probability density function (pdf):

$$f(x/\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right] \quad (24)$$

Thus, $X{\sim}Nakagami(\alpha = 2, m = 1\ s = \gamma\ \Omega = \beta^{\alpha}) = Y{\sim}\mathrm{Weibull}(\alpha,\beta,\gamma)$ and $X{\sim}Nakagami\left(m = \frac{1}{2}, s = \mu, \Omega = \sigma^2\right) = Y{\sim}Normal(\mu,\sigma^2)$. It can then be easily deduced that the location parameter s can serve as any of the measures of central tendency/average/location. It also examines how much the random variable deviates from the measures of central tendency/average/location.

In addition, by setting the location parameter$s = 0$, the three-parameter extended Nakagami distribution will become the (two-parameter) Nakagami-m distribution. Thus as Nakagami-m distribution can be used to generate Gamma distribution and Chi-squared distribution, so also the three-parameter extended Nakagami distribution by methods of mathematical logic/reasoning and induction [22].

The new three-parameter Nakagami distribution was then studied alongside with other three-parameter distributions, most specially Wei bull distribution, Gamma distribution and Frechet distribution by using the same values for the scale, speed and location parameters. As shown in the Figure 3 and 4, it was observed that the new three-parameter Nakagami distribution is well spread and skewed than all others.

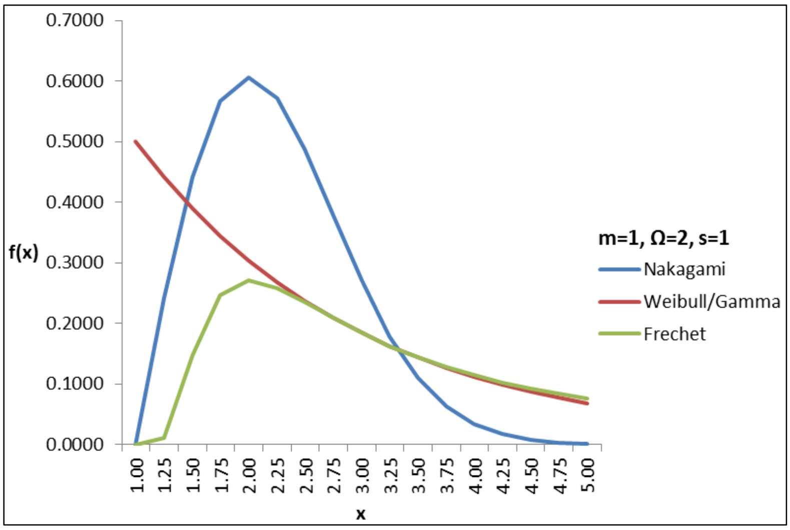

Fig 3: Comparison of pdf's of Nakagami distribution and other distributions at m=1, Ω=2, s=1

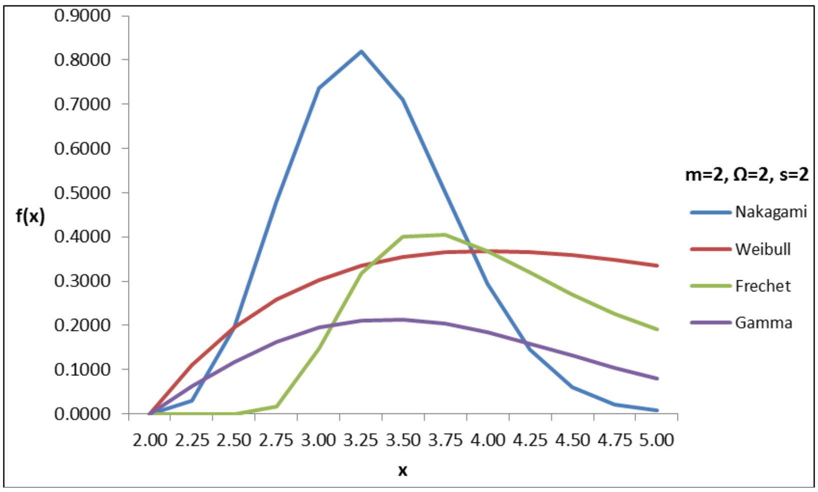

Fig 3: Comparison of pdf's of Nakagami distribution and other distributions at m=2, Ω=2, s=2

5. Conclusion

In this research paper, the probability density function (pdf) of the new extended three-parameter Nakagami distribution developed has been presented with all its parameters estimation and properties. The new proposed distribution can be known as Nakagami-Akintunde distribution. Furthermore, the new model may be applicable to many areas such as survival analysis, economics, pollution, engineering (including communication engineering), environmental, and many others.

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