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More about edge domination in hyper graph

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Abstract
In this paper we further prove more results about edge domination in hypergraphs. In particular we prove necessary & sufficient conditions under which the edge domination number of a hypergraph increases or decreases when a vertex is removed from the hypergraph. We have proved that a subset F of E (G) is an edge dominating set of G iff it is a dominating set in G* (Where G* is dual hypergraph of G) also we have proved that if γ_E (G-v) > γ_E (G) & if F is a minimum edge dominating set of G then there is an edge e containing v ∋ e ∈ F & Prn [e, F] contains two distinct edges.

Keywords: : Hypergraph, dominating set in hypergraph, edge dominating set, edge domination number, minimal edge dominating set, minimum edge dominating set, edge degree**,** edge neighbourhood, subhypergraph, partial subhypergraph, dual hypergraph
AMS Subject Classification (2010): 05C15, 05C69, 05C65

1. Introduction
Edge dominating set & edge domination number have been explored by several authors [5, 6]. The concept of edge domination requires the adjacency relation among the edges of a graph. The same relation is also available in hypergraphs and therefore we have considered edge domination in hypergraphs [7].
The change in the edge domination number when a vertex is removed from the hypergraph has been studied here. For this purpose we have considered the subhypergraph & the partial subhypergraph obtain by removing a vertex from the hypergraph.

2. Preliminaries
Definition 2.1 Hypergraph [4] A *hypergraph* G is an ordered pair $(V(G), E(G))$ where $V(G)$ is a non-empty finite set & $E(G)$ is a family of non-empty subsets of $V(G)$ ∋ their union = $V(G)$. The elements of $V(G)$ are called *vertices* & the members of $E(G)$ are called *edges of the hypergraph* G.
We make the following assumption about the hypergraph.
(1) Any two distinct edges intersect in at most one vertex.
(2) If e_1 and e_2 are distinct edges with $|e_1|, |e_2| > 1$ then $e_1 \not\subseteq e_2$ & $e_2 \not\subseteq e_1$

Definition 2.2 Edge Degree [4]: Let G be a hypergraph & v ∈ V (G) then the *edge degree* of v = $d_E(v)$ = the number of edges containing the vertex v. The minimum edge degree among all the vertices of G is denoted as δ_E (G) and the maximum edge degree is denoted as Δ_E (G).

Definition 2.3 Dual Hypergraph [4]: Let G be a hypergraph. For every v ∈ V (G) define $\bar{v}$ as $\bar{v} = \{e \in E(G) \,/\, v \in e\}$. Let E (G*) = {$\bar{v}$ / v ∈ V (G)} and let V (G*) = E (G). Then the *dual hypergraph* of the given hypergraph G is the hypergraph G* whose vertex set is V (G*) & the edge set is E (G*). We will write G* = (V (G*), E (G*)).

Definition 2.4 Dominating Set in Hypergraph [1]: Let G be a hypergraph & $S \subseteq V(G)$ then S is said to be a *dominating set* of G if for every $v \in V(G) - S$ t here is $u \in S$ ∋u and v are

adjacent vertices.
A dominating set with minimum cardinality is called *minimum dominating set* and cardinality of such a set is called *domination number* of G and it is denoted as $\gamma(G)$.

Definition 2.5 Edge Dominating Set [7]: Let G be a hypergraph & $S \subseteq E(G)$ then S is said to be an *edge dominating set* of G if for every $e \in E(G) - S$ there is some f in S ∍ e and f are adjacent edges.
An edge dominating set with minimum cardinality is called a *minimum edge dominating set* and cardinality of such a set is called *edge domination number* of G and it is denoted as γ_E (G).

Definition 2.6 Minimal Edge Dominating Set [7]: Let G be a hypergraph & $F \subseteq E(G)$ then F is said to be a *minimal edge dominating set* if

1. F is an edge dominating set
2. No proper subset of F is an edge dominating set of G.

Definition 2.7 Subhypergraph and Partial Subhypergraph [3]: Let G be a hypergraph & $v \in V(G)$. Consider the subset $V(G) - \{v\}$ *of* $V(G)$. this set will induce two types of hypergraphs from G.

1. First type of hypergraph: Here the vertex set = $V(G) - \{v\}$ and the edge set = $\{e' / e' = e - \{v\}$ *for some* $e \in E(G)\}$. This hypergraph is called the *subhypergraph* of G & it is denoted as $G - \{v\}$.
2. Second type of hypergraph: Here also the vertex set = $V(G) - \{v\}$ and edges in this hypergraph are those edges of G which do not contain the vertex v. This hypergraph is called the *partial subhypergraph* of G.

Definition 2.8 Edge Neighbourhood [3]: Let G be a hypergraph & e be any edge of G then
Open edge neighbourhood of $e = N(e) = \{ f \in E(G) / f \text{ is adjacent to } e \}$.
Close edge neighbourhood of $e = N[e] = N(e) \cup \{e\}$.

Definition 2.9 Private Neighbourhood of an edge [3]: Let G be a hypergraph. F be a set of edges & e ∈ F, then the *private neighbourhood of e with respect to set F* = Prn [e, F] = {f ∈ E (G) / N[f] ∩ F = {e}}

3. Vertex Removal from the Hypergraph

Let G be a hypergraph & $v \in V(G)$ such that $\{v\}$ is not an edge of G. Consider the partial subhypergraph $G - \{v\}$ whose vertex set is $V(G) - \{v\}$ & edge set = $\{e \in E(G) / v \notin e\}$. The following examples show that the edge domination number of a hypergraph may increase, decrease or remain unchanged when a vertex is removed from the hypergraph.
Example 3.1: Consider the hypergraph whose vertices are {1, 2, 3… 9}

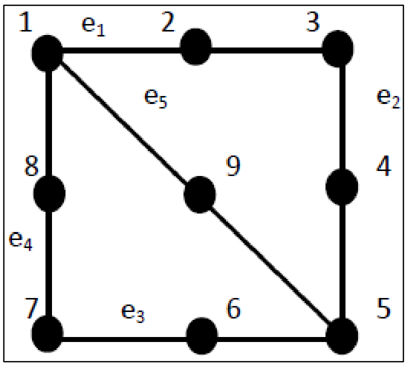

Fig 1

Note that γ_E (G) = 1, also observe that γ_E (G – 9) = 2. Here, γ_E (G – v) > γ_E (G).
In this hypergraph γ_E (G – 2) = 1 = γ_E (G). Here, γ_E (G – v) = γ_E (G).
Now, consider the hypergraph whose vertices are {1, 2, 3… 8}

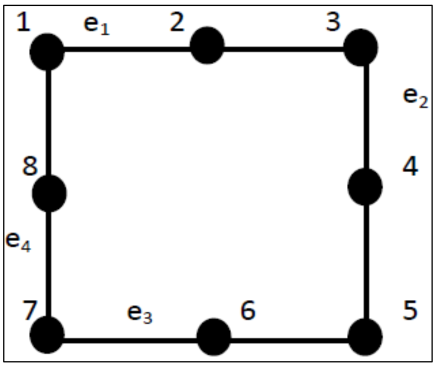

Fig 2

Here, γ_E (G) = 2, also observe that γ_E (G – 2) = 1. Here, γ_E (G – v) < γ_E (G).
Now, we stat & prove a necessary and sufficient condition under which the edge domination number of a hypergraph decreases when a vertex v is removed from the hypergraph.

Theorem 3.2: Let G be a hypergraph & $v \in V(G)$ such that $\{v\}$ is not an edge of G then γ_E(G – v) < γ_E(G) iff there is a minimum edge dominating set F of G ∍ F = F_1 ∪ {e} for some minimum edge dominating set F_1 of G & some edge e of G containing v.

Proof: Suppose γ_E (G – v) < γ_E (G)
Let F_1 be a minimum edge dominating set of G – v & lete be any of G which contains v. Let F = F_1 ∪ {e}. Let h be any edge of G ∍ h ∉ F. If v ∉ h then h is an edge of G – v & h ∉ $F_{1.}$ Since F_1 is an edge dominating set of G – v, there is some g ∈ F_1 ∍ h is adjacent to g, then g ∈ F also. Thus h is adjacent to some member of F also. Suppose v ∈ h then h ∩ e ≠ ϕ because v ∈ h. Therefore, h is adjacent to e & e ∈ F. Thus, F is an edge dominating set of G. Obviously F is a minimum edge dominating set of G.
Conversely, suppose the condition is satisfied.
Let F be a minimum edge dominating set of G such that F = F_1 ∪ {e} for some minimum edge dominating set F_1 of G – v & for some edge e containing v. Then γ_E (G – v) = |F_1| < |F | = γ_E (G)
∴ γ_E (G–v) < γ_E (G)

Corollary 3.3: Let G be a hypergraph & v ∈ V (G) such that {v} is not an edge of G if γ_E (G – v) < γ_E (G) then γ_E (G – v) = γ_E (G) – 1

Proof: Obvious

Remark 3.4: Let G be a hypergraph & v ∈ V (G) such that {v} is not an edge of G & suppose γ_E (G – v) < γ_E (G). From the proof of the above theorem it is clear that every minimum edge dominating set F_1 of G – v & every edge e containing v give rise to a minimum edge dominating set of G namely F_1 = F ∪ {v}.
Suppose there are k distinct minimum edge dominating set of G – v & suppose the edge degree of v = j, then in the above manner they will generate kj minimum edge dominating set of G.
It follows that the number of minimum edge dominating set of G ≥ the number of minimum edge dominating set of G – v.
Now, we stat & prove a set of necessary and sufficient conditions under which the edge domination number of a hypergraph increases when a vertex v is removed from the hypergraph.

Theorem 3.5: Let G be a hypergraph & $v \in V(G)$ such that {v} is not an edge of G then γ_E (G – v) > γ_E (G) iff the following conditions are satisfied

1. V ∈ V (F), for every minimum edge dominating set F of G.
2. There is no subset F of E (G) ∍ |F| ≤ γ_E (G), N_E (v) ∩ (E (G) – F) ≠ ϕ & F edge dominates G – V.

Proof: Suppose γ_E (G – v) > γ_E (G)

1. Suppose there is a minimum edge dominating set F of G ∍ v ∉ V(F) then F is a set of edges of G – v. Let h be any edge of G – v then h is also an edge of G & therefore h ∩ g ≠ ϕ for some g ∈ F. Thus F is an edge dominating set of G – v.
   ∴ γ_E (G – v) ≤ |F| = γ_E (G). Which is a contradiction.

2. Suppose the set F mentioned in statement (2) of the theorem exists.

Then γ_E (G – v) ≤ |F| ≤ γ_E (G). Which is again a contradiction. Thus condition (2) also holds.
Conversely suppose (1) & (2) hold.
First suppose that γ_E (G – v) = γ_E (G).
Let F be a minimum edge dominating set of G – v, then v ∉ V (F). Suppose F is an edge dominating set of G also. Then F is a minimum edge dominating set of G. Thus, F is a minimum edge dominating set of G ∍ v ∉ V (F). This is a contradiction to (1). Therefore, let’s assume that F is not an edge dominating set of G. Therefore there is an edge h ∍ v ∈ h & h is not adjacent with any

member of F. Therefore, $h \notin F$ & $h \in N_E(v)$. Thus F is an edge dominating set of G – v ∍ $|F| \leq \gamma_E(G)$ & $N_E(v) \cap (E(G) - F) \neq \phi$. This contradicts condition – (2).
∴ $\gamma_E(G - v) = \gamma_E(G)$ is not possible.

Suppose that $\gamma_E(G - v) < \gamma_E(G)$.
Let F be a minimum edge dominating set of G – v then F cannot be an edge dominating set of G.
∴ There is an edge h ∍ $v \in h$ & v is not adjacent with any edge of F. Therefore $h \notin F$. Thus, F is an edge dominating set of G – v.
∴ $|F| \leq \gamma_E(G)$, $N_E(v) \cap (E(G) - F) \neq \phi$. This again contradicts condition – (2).
∴ $\gamma_E(G - v) < \gamma_E(G)$ is also not possible.
Hence, $\gamma_E(G - v) > \gamma_E(G)$.

Theorem 3.6 [7]: Let G be a hypergraph and $F \subseteq E(G)$ be an edge dominating set then F is a minimal edge dominating set iff $\forall$ e $\in$ F at least one of the following two conditions is satisfied
1. E is not adjacent to any other edge of F (i.e. e is an isolate in F)
2. There is an edge f in E (G) – F such that f is adjacent to e but f is not adjacent to any other edge of F.

Theorem 3.7: Let G be a hypergraph & F be an edge dominating set of G then F is a minimal edge dominating set iff $\forall$ e $\in$ F, Prn [e, F]$\neq \phi$.

Proof: Suppose F is a minimal edge dominating set of G. Let $e \in F$ then by above theorem one of the following two conditions hold.
1. e is not adjacent to any other edge of F. (2) There is an edge f in E(G) – F such that f is adjacent to e but f is not adjacent to any other edge of F.

If condition (1) holds then $e \in$ Prn [e, F] thus Prn [e, F]$\neq \phi$.
If condition (2) holds then $f \in$ Prn [e, F] thus Prn [e, F]$\neq \phi$.
Thus, in both the cases Prn [e, F]$\neq \phi$.
Conversely suppose the condition holds. Let $e \in F$, then Prn [e, F]$\neq \phi$. So, let $g \in$ Prn [e, F]
First suppose g = e. Then $N[e] \cap F = \{e\}$. This means that e is not adjacent to any other edge of F. Therefore condition (1) of above theorem is satisfied.
Suppose $g \neq e$. Then $g \in E(G) - F$. Since $N[g] \cap F = \{e\}$, g is adjacent to only one member of F namely e. Therefore condition (2) of above theorem is satisfied.
∴ In any case condition (1) or (2) of above theorem is satisfied.
∴ F is a minimal edge dominating set.

Now, we prove a necessary and sufficient condition under which the edge domination number of a hypergraph decreases when a vertex v is removed from the hypergraph.

Theorem 3.8: Let G be a hypergraph & $v \in V(G)$ such that {v} is not an edge of G then $\gamma_E(G - v) < \gamma_E(G)$ iff there is a minimum edge dominating set F & edge e containing v of G ∍ $e \in F$ & the following two conditions are satisfied.
1. $e \in$ Prn [e, F]
2. Prn [e, F] is a subset of $N_E(v)$

Proof: Suppose $\gamma_E(G - v) < \gamma_E(G)$
Let F_1 be a minimum edge dominating set of G – v. Then F_1 cannot be an edge dominating set of G. Therefore, there is an edge e of G ∍ e is not adjacent with any member of F_1. Let $F = F_1 \cup \{e\}$. Let g be any edge of G ∍ $g \notin F$. Suppose $v \in g$ then $g \cap e \neq \phi$. Thus, g intersects a member of F. Suppose $v \notin g$ then g is an edge of the partial subhypergraph G – v. Therefore g is adjacent to some member h of F_1. Thus, g is adjacent to some member of F. Thus, F is an edge dominating set of G. Since $|F| = |F_1| + 1$, F is a minimum edge dominating set of G containing the edge e. Note that, e is not adjacent to any other member of F. Therefore, $e \in$ Prn [e, F].
To prove condition (2) for F, let $g \notin N_E(v)$. Therefore g is an edge of the partial subhypergraph G – v. Therefore g is adjacent with some member of F_1. Thus, g is adjacent with some member h of G which does not contain the vertex v. Thus, $g \notin$ Prn [e, F]. Therefore, Prn [e, F] $\subseteq N_E(v)$. Thus condition (2) is also satisfied by F.
Conversely suppose there is a minimum edge dominating set F of G containing e ∍ (1) & (2) are satisfied.
First we observe that if h is an edge of G ∍ $v \in h$ then $h \notin F$ because $e \in F$ & $e \in$ Prn [e, F]. Thus e is the only edge of F which contains vertex v. Now, let $F_1 = F - \{e\}$ then $|F_1| < |F|$. Let g be any edge of G – v which does not belong to F_1. Then $g \notin N_E(v)$. Therefore $g \notin$ Prn [e, F]. Now, there is an edge h in F ∍ g is adjacent to h. If h = e then g is adjacent to some other edge h′ of F and as mentioned above $v \notin h'$. Suppose g is not adjacent to e, then g is adjacent to some edge h_1 of F ∍ $h_1 \neq e$ & $v \notin h_1$. Thus, g is adjacent to some member of F_1. Hence, F_1 is an edge dominating set of G – v.
∴ $\gamma_E(G - v) \leq |F_1| < |F| = \gamma_E(G)$.
∴ $\gamma_E(G - v) < \gamma_E(G)$.

Example 3.9: Consider the graph in example – 3.1 (figure – 2). Here, $e_1 = \{1, 2, 3\}$, $e_2 = \{3, 4, 5\}$, $e_3 = \{5, 6, 7\}$, $e_4 = \{7, 8, 1\}$. For it γ_E (G) = 2 & γ_E (G – 2) = 1.Therefore, $\gamma_E (G – v) < \gamma_E (G)$. Here $F = \{e_1, e_3\}$ is a minimum edge dominating set of G containing the edge e_1. Note that $e_1 \in$ Prn [e_1, F]. Prn [e_1, F] is a subset of $N_E(2) = \{e_1\}$.

Proposition 3.10: Let G be a hypergraph without isolated vertices. A subset F of E (G) is an edge dominating set of G iff it is a dominating set in G*.

Proof: First suppose that F is an edge dominating set of G. Let e ∈ V (G*) – F then e is an edge of G which is not in F. Since F is an edge dominating set of G, e is adjacent to some member f of F. This means that e is adjacent to f in G* & f ∈ F. Thus F is a dominating set of G*.
Conversely, suppose F ⊆ E (G) is a dominating set of G*. Let e be any edge of G ∍ e ∉ F. Since F is a dominating set of G*, e is adjacent to some member f of F. This means that F is an edge dominating set of G.

Remark 3.11: Let G be a hypergraph & v ∈ V (G) such that {v} is not an edge of G. Consider the sub hypergraph G – {v}. We may again note that the edges of G – v are precisely the set E′ where E′ = E – {v} for some edge e of G. One can easily see that two vertices x & y of G – v are adjacent in G – v iff they are adjacent in G.
Now, we prove a necessary and sufficient condition under which the domination number of a hypergraph increases when a vertex is removed from the hypergraph.

Theorem 3.12: Let G be a hypergraph & v ∈ V (G) such that {v} is not an edge of G then $\gamma (G – v) > \gamma(G)$ iff the following two conditions are satisfied.

1. V ∈ S for every minimum dominating set S of G.
2. There is no subset S of G – v ∍ S ∩ N[v] =ϕ, $|S| \le \gamma(G)$ & S is a dominating set of G – v. (Here G – v is the subhypergraph of G.)

Proof: Suppose $\gamma(G – v) > \gamma(G)$

1. Suppose there is a minimum dominating set S of G ∍ v ∉ S. Let x be any vertex of G – v ∍ x ∉ S. Since S is a dominating set of G, S is adjacent to some vertex y in S in the hypergraph G. Note that y ≠ v because v ∉ S. Thus, x & y are vertices of G – v which are adjacent in G. Therefore, by above remark they are adjacent in G – v also. Thus, S is a dominating set in the sub hypergraph G –v. Therefore $\gamma (G – v) \le |S| = \gamma(G)$, which contradicts our assumption. Therefore, v ∈ S for every minimum dominating set S of G.
2. Suppose there is a subset S of G – v ∍ S ∩ N[v] =ϕ, $|S| \le \gamma(G)$ & S is a dominating set of sub hypergraph G – v. Then $\gamma (G – v) \le |S| = \gamma(G)$, which again contradicts our assumption.

Conversely suppose condition (1) & (2) are satisfied.
First suppose that $\gamma (G – v) = \gamma (G)$. Let S be a minimum dominating set of G – v. Let x ∈ V (G) ∍ x ∉ S & x ≠ v. Now, x is adjacent to some vertex y in S in the sub hypergraph G – v. Therefore, x & y are adjacent in G also. Suppose v is adjacent to some vertex of Sin G then S is a minimum dominating set of G not containing v. This contradicts (1). Suppose v is not adjacent to any vertex of Sin G then S ∩ N[v] =ϕ, $|S| \le \gamma (G)$ & S is a dominating set of G – v. This contradicts (2).
Thus, we have contradiction in both the cases when v is adjacent to some vertex of S or v is not adjacent with any vertex of S. Therefore $\gamma (G – v) = \gamma (G)$ is not possible.
Suppose $\gamma (G – v) < \gamma (G)$. Let S be a minimum dominating set of G – v. Let x ∈ V (G) ∍ x ∉ S & x ≠ v. Then as proved above x is adjacent to some vertex of Sin G. Now, S cannot be a dominating set of G because $|S| \le \gamma (G)$. Therefore v is not adjacent to any vertex of S. This means that S ∩ N[v] =ϕ, $|S| \le \gamma(G)$ & S is a dominating set of G – v. This is again a contradiction. Thus, $\gamma (G – v) < \gamma(G)$ is also not possible. Thus, $\gamma(G – v) > \gamma(G)$.

Theorem 3.13: Let G be a hypergraph & v ∈ V (G) such that {v} is not an edge of G then $\gamma_E (G – v) > \gamma_E (G)$ iff the following conditions are satisfied.

1. If F is a minimum edge dominating set of G then F contains some edge containing v.
2. There is no set F of edges disjoint from $N_E(v)$ with $|F| \le \gamma_E (G)$ such that F is an edge dominating set of G – v.

Proof: Suppose $\gamma_E (G – v) > \gamma_E (G)$

1. Let F be a minimum edge dominating set of G & suppose there is no edge h containing v ∍ h belongs to F then F is an edge dominating set of G – v.

∴ $\gamma_E (G – v) \le |F| = \gamma_E (G)$, which is a contradiction.
∴ There is an edge containing v which is also in F.

2. Suppose there is a set F of edges of G – v ∍ F ∩ N_E (v) =ϕ, $|F| \le \gamma_E (G)$ & F is an edge dominating set of G – v. Then $\gamma_E (G – v) \le |F| \le \gamma_E (G)$, which is again a contradiction. Therefore (2) holds.

Conversely suppose (1) & (2) hold.

First suppose that γ_E (G – v) = γ_E (G). Let F be a minimum edge dominating set of G – v. First suppose that F is an edge dominating set of G. Then F is a minimum edge dominating set of G not containing any edge containing v. This is a contradiction to (1).

Suppose F is not an edge dominating set of G - v. Therefore, no edge containing v can be a member of F. Thus, F $\cap$ N_E (v) =ϕ, |F| $\leq$ γ_E (G) & F is an edge dominating set of G – v. This is a contradiction to (2). Therefore, γ_E (G – v) = γ_E (G) is not possible.

Suppose that γ_E (G – v) < γ_E (G)

Let F be a minimum edge dominating set of G – v then F cannot be an edge dominating set of G. Therefore, no edge containing v can be a member of F. Thus, F $\cap$ N_E (v) =ϕ, |F| $\leq$ γ_E (G) & F is an edge dominating set of G – v. This is a contradiction to (2). Therefore, γ_E (G – v) < γ_E (G) is also not possible.

Hence, γ_E (G – v) > γ_E (G).

Theorem 3.14: Let G be a hypergraph & v $\in$ V(G) such that {v} is not an edge of G if γ_E(G – v) > γ_E(G) & if F is a minimum edge dominating set of G then there is an edge e containing v ∍ e $\in$ F & Prn[e, F] contains two distinct edges.

Proof: Since γ_E (G – v) > γ_E (G), by above theorem there is an edge containing v which is in F. Let e_1, e_2… e_k be all the edges incident at v which are in F.

Claim 1: There is no edge e_j in the set e_1, e_2… e_k ∍ Prn [e_j, F] contains only those edges which are incident at v.

Proof of the Claim 1: If there is some j ∍ Prn[e_j, F] contains only those edges which are incident at v then F – {e_j} is an edge dominating set of G – v.

$\therefore$ γ_E (G – v) < γ_E(G), Which is a contradiction. Then Prn [e_i, F] contains an edge h_i ∍ h_i does not contain the vertex v (for i = 1, 2… k).

Claim 2: It is impossible that $\forall$ i = 1, 2… k Prn [e_i, F] = {h_i}

Proof of the Claim 2: Suppose $\forall$ i Prn [e_i, F] = {h_i} then obviously $h_i \neq h_j$ if i $\neq$ j.

Let F_1 = (F – {e_1, e_2… e_k}) $\cup$ {h_1, h_2… h_k}) then F_1 is a minimum edge dominating set of G ∍ $e_i \notin F_1$, $\forall$ i = 1, 2… k and this contradicts (1) of the previous theorem.

$\therefore$ For some j in 1, 2… k Prn [e_j, F] contains atleast two distinct edges.

4. Concluding Remarks

In this paper we have studied edge domination in hypergraphs and the operations of vertex removal and edge removal. Further a new concept like total edge domination can be introduced in hypergraphs and the effect of above operations can be studied for total edge domination in hypergraphs.

5. References

1. Acharya B, Domination in Hypergraphs, AKCE J. Graphs. Combin. 2007; 4(2):111-126.
2. Behr A, Camarinopoulos L. On the domination of hypergraphs by their edges, Discrete Mathematics. 1998; 187:31-38.
3. Berge Graphs C. Hypergraphs, North-Holland, Amsterdam, 1973.
4. Berge C. Hypergraphs, North-Holland Mathematical Library, New York, 1989, 45.
5. Haynes T, Hedetniemi S, Slater P. Domination in Graphs Advanced Topics, Marcel Dekker, Inc. New York, 1998.
6. Haynes T, Hedetniemi S, Slater P. Fundamental of Domination in Graphs, Marcel Dekker, Inc. New York, 1998.
7. Thakkar D, Dave V. Edge Domination in Hypergraph, International Journal of Mathematics & Statistics Invention. 2017; 5(9):13-17.