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Characteristics and special role of matrices of class 5

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Abstract
We have classified square matrices based on the 'libra value' * property in six** different classes. In this note we unfold what we have found and establish about dominating properties of matrices of class 5. It has been shown that some sets of standard form of class5 matrices satisfy commutative property under matrix multiplication. In addition to this, we describe different characteristics of higher order of class5 and also the different methods of constructions of higher order matrices of the same class 5.
Libra value: *Libra value of a given square matrix is that constant value for the given matrix obtained by adding the all the entries of each row or column or diagonal (both leading or non-leading) as the case may be depending on the classification.
** [six classes are class 1, 2, 3, 4A, 4B, 5]

Keywords: Class – 4A, Class 4B, Libra Value, Eigen values

1. Introduction
In this introductory phase we describe the classification of square matrices according to the properties – characteristic properties, Each class is identified by the libra value property that exists either in each row or column or each type of diagonal. -p which is a real or complex constant. We give their general form of each class.
In the class 5 type of square matrices we have included only those square matrices for which the property p5 holds true. The property p5 describes that in any member matrix of this class the sum of all the elements of each row, each column, and each diagonal (leading and non-leading) remains the same constant = p (real or complex). This constant value, as mentioned earlier, is called its libra value. In the context of the matrix A it is denoted as L(A) = p.
We give some illustrations.

$A = \begin{pmatrix} 19 & -2 & 13 \\ 4 & 10 & 16 \\ 7 & 22 & 1 \end{pmatrix}$ In this case the, as clarified in the above paragraph, the sum remains a constant.

We write this sum = L (A) = p = 30; which is a libra value of the matrix.
We quote one example of the next higher order.

$A = \begin{pmatrix} 18 & 11 & 8 & 27 \\ 26 & 9 & 12 & 17 \\ 15 & 20 & 23 & 6 \\ 5 & 24 & 21 & 14 \end{pmatrix}$ in this case L (A) = 64

We can write the matrices of higher order also with the property p5.

1.1 Some Standard Forms
In this section we mention some standard formats of matrices of order 3x3 and 4x4. We will take some standard forms for further mathematical work.

(1) Form 1

For some real p and k, the first form is

$$A = \begin{pmatrix} p+3k & p-4k & p+k \\ p-2k & p & p+2k \\ p-k & p+4k & p-3k \end{pmatrix} \in CJ5\ (3X3),\ L\ (A) = 3p) \quad (1)$$

(2) Form 2

For some real p and k, the first form is

$$A = \begin{pmatrix} p+7k & p & p+5k \\ p+2k & p+4k & p+6k \\ p+3k & p+8k & p+k \end{pmatrix} \in CJ5\ (3X3),\ L\ (A) = 3(p + 4k)) \quad (2)$$

[We would like to share the outcome of our efforts to handle 3x3 matrices of class 5.
(1) We begin with 8 terms of an arithmetic progression.
Let these be 'a, a+k, a+2k… +a+8k 'where 'a' and 'k' are real values.
Arrange them as shown by 9 integers through 1 up to 9 as arranged by numbers shown.
In the case shown above, the first position is occupied by the first term of an A.P. = a and the common difference = k.

(3) Form 3

We give a general form of class 5 matrix of order 4x4.

$$A = \begin{pmatrix} q+4k & p+2k & p+k & q+7k \\ p+7k & q+k & q+2k & p+4k \\ q+3k & p+5k & p+6k & q \\ p & q+6k & q+5k & p+3k \end{pmatrix} \in CJ5\ (4X4), L(A)= 2p +2q + 14k) \quad (3)$$

For p, q, and k are real numbers. We give an illustration. Let p = 4, q = 2, and k = 5. The 4x4 matrix of class 5 is as follows.

$$A = \begin{pmatrix} 22 & 14 & 9 & 37 \\ 39 & 7 & 12 & 24 \\ 17 & 29 & 34 & 2 \\ 4 & 32 & 27 & 19 \end{pmatrix} \in CJ5\ (4X4),\ L(A) = 2p(=4) +2q(=2) + 14k(=5) =82)$$

[We, at this point of time, reveal the underlying logic. We have two different arithmetic sequences with 8 terms in each one or one arithmetic sequence with 16 terms. If two sequences are considered (may be with two different first terms) then care is being taken that their common difference remains same in each sequence.
We write the first 8 terms (first 4 terms in the first row and remaining 4 terms in the next/second row from right to left or from left to right. We follow the same procedure for the 8 terms (4 + 4) of the second A.P. for the third and the fourth row.
Now, considering the pattern of A = (a_{ij}) for i, and j = 1, 2, 3, and 4, interchange the terms as follows.
In the above format we have considered two arithmetic sequences of 8 numbers. For the first A.P. first term =p and common difference = k; while the other one with the first term = q and the same common difference = k.
(1) a_{11} and a_{44} (2) a_{14} and a_{41} (3) a_{22} and a_{33} and (4) a_{23} and a_{32}.
In this way the resultant matrix given by (3) matrix falls in class5.]

1.2 Three Fundamental Tenets

In order to continue further in the set CJ5 we shall accept the three fundamental tenets which are already accepted in general.

(1) Equality of Two Matrices

Two matrices A = (a_{ij})∈ CJ5(nxn), L(A)= p1) and B =(b_{ij}) ∈ CJ5(nxn), L(A)= p2)with p1 and p2 being real values.
They are said to be equal, i.e. A = B ⇔ $a_{ij} = b_{ij}$ for all i and j.

(2) Additional of Two Matrices

If there are any two matrices of same order of class 5, then this rule allows you to add the two matrices and the addition is of corresponding elements. This will result in to a matrix of the class 5only.
A = (a_{ij})∈ CJ5(nxn), L(A)= p1) and B =(b_{ij}) ∈ CJ5(nxn), L(A)= p2)with p1 and p2 being real values. Their addition denoted as A + B is also a matrix, say C.
C = (c_{ij} = a_{ij} + b_{ij})∈ CJ5(nxn), L(A)= p3) = A + B.
We also note the fact that L(C) = L (A) + L (B), i.e. p3 = p1 + p2

(4) Multiplication by a Scalar

For any matrix A = (a_{ij})∈ CJ5(nxn), L(A)= p) and c ∈ R we define multiplication of A by the scalar c denoted symbolically as cA. It is matrix such that cA = c (a_{ij}) = (caij).

cA = (caij) ∈ CJ5(nxn), L(cA)= cp) The matrix cA is obtained by multiplying each element of A by the factor c. We write L(cA) = c L(A)

A = $\begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 3p = 3(5) = 15)

Let c = 2 so we have $\therefore cA \begin{pmatrix} 16 & 2 & 12 \\ 6 & 10 & 14 \\ 8 & 18 & 4 \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 2p = 2(15) = 30)

This fundamental rule hold true for the matrices of higher order of the class CJ5.
Now, in the next topic we discuss structural algebra of class 5.

(2) Abelian Structure

In order to prove that matrices of class5 of order 3x3 given by general format (1) line up with Abelian structure we proceed as follows.
We have with us,

A = $\begin{pmatrix} p+3k & p-4k & p+k \\ p-2k & p & p+2k \\ p-k & p+4k & p-3k \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 3p), where both p and k are real.

Let p1, p2, p3, k1, k2, and k3 be real values; taking these values we construct three matrices denoted as A, B, and C.

A = $(p_1\, k_1) * = \begin{pmatrix} p1+3k1 & p1-4k1 & p1+k1 \\ p1-2k1 & p1 & p1+2k1 \\ p1-k1 & p1+4k1 & p1-3k1 \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 3p1) [*Abbreviation (4)]

In the same way we denote B = (p2k2), and C = (p3k3).

2.1 Closure: Any two member matrices, say A and B, of the class CJ5 (3X3), L (A) = 3p) when operated with regular addition ' +' results in to a matrix of the same class CJ5.
This is obvious as p1, p2, k1, and k2 being real values their sum can be arranged in the same form as that of the elements of original matrix. It is by the virtue of closure property of members of the set R of real numbers. i.e. For A and B ∈ CJ5(3X3), L(A) = 3p), we have A + B ∈ CJ5(3X3), L(A) = 3p

2.2 Associative Property: This refers to any three member matrices A, B, and C of the class 5.
We wish to check whether A+ (B + C) = (A+ B) + C holds in Class CJ5.
Let A, B, and C ∈ CJ5 (3X3), L (A) = 3p1, L (B) = 3p2, and L(C) = 3p3)
Also all A, B, and C are of the general form given by (1),

$\begin{pmatrix} p+3k & p-4k & p+k \\ p-2k & p & p+2k \\ p-k & p+4k & p-3k \end{pmatrix}$ ∈ CJ5 (3X3), L = 3p)

Let A = $(p_1\, k_1)$, B = $(p_2\, k_2)$, and C = $(p_3\, k_3)$

As p_i and k_i for i = 1, 2, and 3 are real values and the members of the set R satisfy associative property for corresponding elements of the matrices; we have

A + (B + C) = (A+ B) + C

2.3 Existence of Identity: One of the structures that we have taken for establishing mathematical structure (group, ring etc.) is as follows.

A = $\begin{pmatrix} p+3k & p-4k & p+k \\ p-2k & p & p+2k \\ p-k & p+4k & p-3k \end{pmatrix}$ ∈ CJ5 ((3X3), L(A) = 3p)

where p and k are real values. If we take both p and k equal to zero then the given matrix 'A' becomes an identity matrix generally denoted as I.

We know that I = $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ ∈ CJ5 ((3X3), L (A) = 3p = 0) as p and k both = 0

So we have A + I = I + A = A; this establishes existence of Identity in the class 5 = CJ5

2.4 Existence of additive Inverse: In part (3) of section 1.2 we have considered multiplication of a matrix by a scalar 'c'. If we take c = -1 then it becomes a matrix '–A'.
For this matrix '-A' we have A + (-A) = (-A) + A = I.
This establishes Existence of additive Inverse for a given matrix.
We note that the matrix –A ∈ CJ5 ((3X3), L (A) = -3p) for p ∈ R.

2.5 Group and Abelian group
By the virtue of above sub-sections through 2.1 up to 2.4, we claim that the set CJ5 along with '+' as regular additive process on matrices is a group.
Therefore, (CJ5, '+') is a group. As in this group (CJ5, '+') of matrices of class 5, the constituent elements are in terms of p and k, being real numbers, commutative property for '+' is satisfied; i.e. for

A = $(p_1\, k_1) * = \begin{pmatrix} p1+3k1 & p1-4k1 & p1+k1 \\ p1-2k1 & p1 & p1+2k1 \\ p1-k1 & p1+4k1 & p1-3k1 \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 3p1)

[*Abbreviation (4)]
In the same way we denote B = (p2k2), as p1, p2, k1, and k2 being real values, we write (p1 + p2) + (k1 + k2) = (p2 + p1) + (k2 + k1); this establishes commutative property with addition as binary operation.
In general, we have A + B = B + A.
It implies that group (CJ5, '+') of matrices of class 5 is an Abelian group.

3. Additional Features: In this section we will study additional algebraic operations and these new in along with the previous ones will help us check further properties.

3.1 Binary Operation
We will now carry out matrix multiplication process on the two matrices of class 5. We again take the form defined by (1).

$\begin{pmatrix} p+3k & p-4k & p+k \\ p-2k & p & p1+2k1 \\ p-k & p+4k & p-3k \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 3p)

Let us take two matrices of this class, say A and B.

A = $(p_1\, k_1) * = \begin{pmatrix} p1+3k1 & p1-4k1 & p1+k1 \\ p1-2k1 & p1 & p1+2k1 \\ p1-k1 & p1+4k1 & p1-3k1 \end{pmatrix}$ ∈ CJ5 ((3X3), L (A) = 3p1)

B = $(p_2\, k_2) * = \begin{pmatrix} p2+3k2 & p2-4k2 & p2+k2 \\ p2-2k2 & p2 & p2+2k2 \\ p2-k2 & p2+4k2 & p2-3k2 \end{pmatrix}$ ∈ CJ5 ((3X3), L (A) = 3p2)

We find the product of matrix A by matrix B, i.e. AB.
The result of the product which is again a matrix [shown in the annexure-1] but it leaves the class 5 and becomes a member of class 4A.
AB ∈ CJ4A ((3X3), L (AB) = 9p1p2)
From this we conclude that matrix multiplication on the matrices of class 5 is not a binary operation.
For A, and B ∈ CJ5 (3X3), L (A) = 3p), AB ∉ ∈ CJ5 (3X3), L (A) = 3p)
But AB ∈ CJ4A*****((3X3), L (AB) = 3p)
[*class4A is the class of the square matrices in which the sum of entries of (1) each column, (2) each row, and (3) non-leading diagonal remains the same constant. It excludes the sum of all the entries of the principal diagonal.]

3.2 Associative and Distributive Property
For the three matrices, say A, B, and C of class 5, under the process of matrix multiplication, we do have the result A (BC) = (AB) C; which can be easily established simply because of virtue of associative property in the set of real numbers- R.
The only issue that it brings forth is the end result of A (BC), and (AB) C since they are the members of class 4A.
Proceeding on the same lines we have doubly distributive laws which do hold good in the class 5 but the same snag of class preservation comes along. For the three matrices, say A, B, and C of class 5 we have

A (B + C) = AB + AC and (B+ C) A = BA + CA

3.3 Commutative property: The points are, to some extent, clear that we have both AB and BA both well-defined and in all the cases they are equal but again both AB and BA leave the class and admit in the previous class4A.

For A = $(p_1\ k_1)* = \begin{pmatrix} p1+3k1 & p1-4k1 & p1+k1 \\ p1-2k1 & p1 & p1+2k1 \\ p1-k1 & p1+4k1 & p1-3k1 \end{pmatrix}$ ∈ CJ5 ((3X3), L (A) = 3p1)

And B = $(p_2\ k_2)* = \begin{pmatrix} p2+3k2 & p2-4k2 & p2+k2 \\ p2-2k2 & p2 & p2+2k2 \\ p2-k2 & p2+4k2 & p2-3k2 \end{pmatrix}$ ∈ CJ5 ((3X3), L (A) = 3p2)

Both AB and BA are equal; i.e. AB = BA but as said earlier both AB and BA ∈ CJ4A ((3X3), L (AB) = L (BA) = 9p1p2) also we have L (AB) = L (BA) = L (A) L (B) We give an illustration by assigning the real values to the variables p1, p2, k1, and k2.

Let A = $\begin{pmatrix} 3 & -11 & -1 \\ -7 & -3 & 1 \\ -5 & 5 & -9 \end{pmatrix}$ for p = -3 and k = 2 in the standard form.

And B = $\begin{pmatrix} -1 & 6 & 1 \\ 4 & 2 & 0 \\ 3 & -2 & 5 \end{pmatrix}$ for p = 2 and k = -1 in the standard form.

We have both A and B ∈ CJ5 (3X3), L (A) = -9, and L (B) = 6)

We have AB = $\begin{pmatrix} -50 & -2 & -2 \\ -2 & -50 & -2 \\ -2 & -2 & -50 \end{pmatrix}$ = BA

As said earlier both AB = BA and they ∈ CJ4A ((3X3), L (AB) = 9(-3) (2) = -54 L (BA) = L (A) L (B))

3.4 Identity Matrix and Inverse

First of all let us be clear that this class5 cannot possess an identity matrix. The identity matrix belongs to the class 4A. The identity matrix does not satisfy all the constraints of the class5.
For the matrix,

A = $\begin{pmatrix} p+3k & p-4k & p+k \\ p-2k & p & p+2k \\ p-k & p+4k & p-3k \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 3p), its determinant value

Denoted as $|A| = -72\ k^2\ p = p\ (-72k^2)$ *
[We have proved in our previous paper of class4B, that the determinant value of any class of matrix **A** through 1 to class 5 is always a multiple of the libra value of that class.]
$|A| = -72\ k^2\ p = p\ (-72k^2)$ * = 0 if any one or both p and k is zero.
If p = 0 then in determinant of the matrix A, i.e. |A|, the operation R1 R1→ (R2 + R3) makes all the elements of the first row equal to zero.
On the other hand, if k = 0 then matrix is a scalar matrix with each member equal to p.
On the condition that |A|≠0, we have its inverse, denoted as A^{-1}, is as shown below.

$$A^{-1} = \frac{1}{72kp}\begin{pmatrix} 8k+9p & 4(2k-3p) & 8k+3p \\ 2(-3p+4k) & 8k & 2(3p+4k) \\ 8k-3p & 4(3p-2k) & 8k-9p \end{pmatrix}$$

We have two eye-catching points to be noted.
(1) A^{-1} ∈ CJ5 (3X3), L (A^{-1}) = 1/3p) with p≠0
(2) L (A^{-1}) = 1/L (A) = 1/ (3p)
[At this junction we would like to mention that the class 5 does not contain the identity matrix but its inverse exists and it is a member of the same class 5.]
We find inverse of the above given matrix B.

B = $\begin{pmatrix} -1 & 6 & 1 \\ 4 & 2 & 0 \\ 3 & -2 & 5 \end{pmatrix}$ for p = 2 and k = -1 in the standard form.

$|B| = -72(-1)^2\ (2) = -144 \neq 0$
$B^{-1} = \frac{-1}{144}\begin{pmatrix} 10 & -32 & -2 \\ -20 & -8 & 4 \\ -14 & 16 & -26 \end{pmatrix}$ ∈ CJ5 (3X3), L (B^{-1}) = 1/6)

Also L (B^{-1}) = 1 / L (B) = 1/(3p)

3.5 Eigen Values and Eigen Vector

Search for eigen values and corresponding eigen vectors is the most important part of matrix algebra. These two inter-mingled entities are associated with any given square matrix. We try to find them in our case of matrices of class 5. For a given matrix A if we find a number λ such that for some vector X ≠ 0, we have AX = λX then λ is called the eigen value (latent value or characteristic value) and its corresponding companion vector is called eigen vector.

For the matrix A = $\begin{pmatrix} p+3k & p-4k & p+k \\ p-2k & p & p+2k \\ p-k & p+4k & p-3k \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 3p)

The eigen values are (1) 3p, (2) 2√6 k, and (3) --2√6 k where k non-zero is preferred or else the matrix is a singular matrix and eigen values are (1) 3p, (2) 0, and (3) 0. Again we recall the fact that for the matrix of any class through 1 to class5, its libra value is necessarily one of the eigen values. The eigen vector corresponding to the eigen value 3p is $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

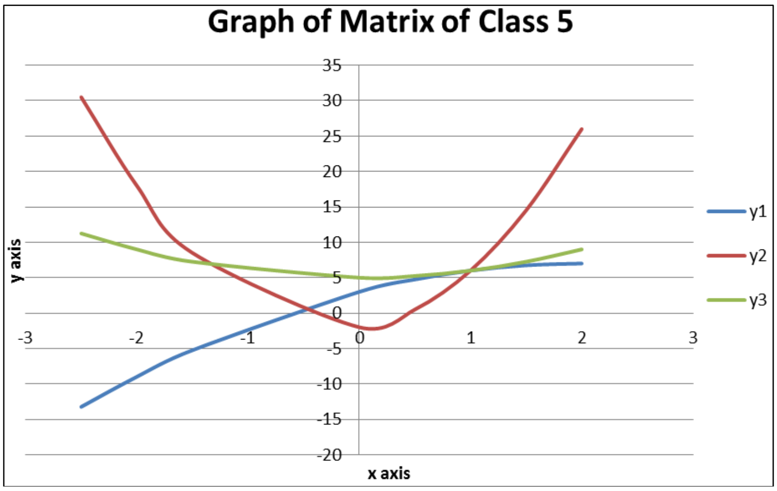
From this we conclude that for a given matrix of class5 if we know the values of p and k, the eigen values are easy to find.

4. Graph of Matrix of Class 5

In this section we take a new concept; considering each column vector of the matrix of class 5 in the form of a quadratic curve. Then for each column vector we draw its corresponding
Graph. All these graphs intersect at a point y = L (A) = libra value for x =1 in each case. We take an illustration as follows.

A = $\begin{pmatrix} -1 & 6 & 1 \\ 4 & 2 & 0 \\ 3 & -2 & 5 \end{pmatrix}$ ∈ CJ5 (3X3), L (A) = 3(2) = 6)

We represent all the column vectors as quadratic curves.
$y1 = -x^2 + 4x + 3$, $y2 = 6x^2 + 2x - 2$, and $y3 = x^2 + 5$,
We plot their graphs which are shown as follows.



Graph 1

For this graph we have y1 ∩ $y2$ ∩ $y3$ = (x=1, y=6) = L (A) = libra value.
We mean that all the column entries when considered as quadratic curves have their point of concurrence at the libra value.

5. Conclusion

This is the last class of the six classes we have designed for the square matrices. This class contains all the libra value properties that exists for each column, each row, and different diagonals that decides the category of the class. This class inherits the properties of different classes that dominate in the set of classes – class 3, and class4A.

6. Vision: Matrices of this class 5 exhibit different results when they undergo different types of product like (1) Jordan product, (2) Lie Product (3) Hadamard Product. Establishing their internal relationship and their algebra is a part of on-going work.

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