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Implementation of zero acceptance number single sampling plan on EOQ with permissible delay in payment with extended payment privileges

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Abstract
Traditionally, all the Economic Ordering Quantity models are developed based on the assumption that the received products are of free of defect. But in reality, the products may contain defects. Acceptance sampling plan helps to discriminate the product with the benefits of less time consuming, economic and very useful when the testing involves destructive. This study implements single sampling plan on the EOQ model with permissible delay in payments. Mathematical models are developed for extended payment privileges. Also numerical examples are provided to validate the results.

Keywords: Economic ordering quantity, permissible delay in payments, single sampling by attributes, payment privileges

Introduction

In the production and assembly industries, testing is one of the crucial instruments in assess the quality of the product. It involves the measurement of quality by compare to the determined standard and specifications. It is a key when there is a call for to create competent decision with the restricted resources. Testing procedures may vary based on the nature of the product. Some products may require destructive testing. Destructive testing is defined as the test which may alters the shape, form or size or structure of the product. The products after destructive testing have no future use. 100% inspection of the product is impossible as the by doing all the products are destructed so nothing will be available for future use. So there is a need for a practical tool.

Acceptance sampling plans assent the predicting the overall quality of a product through the testing of a relatively small number of samples thus curtailing the redundant costs. It is mainly applicable when the sole intention of the inspection is to either accept or reject the lot. It is applied in assessing the quality of the raw materials, finished products etc. Acceptance sampling plans are optimal under the following scenarios; when the cost of inspection is huge; when 100% inspection is tedious & exhausting; when inspection requires destructive testing.

Sampling plans with expensive or destructive testing it is appropriate to use single sampling plan with zero acceptance number. Schilling (1982) indicated that in the field of compliance testing, and principally for safety-related items, zero acceptance number is principally desirable.

Since most of the inspection process involves human inspectors there is possibility oferrors in the inspection. Effect of inspection error on the acceptance sampling plans are studied by different authors for instance, Harris (1968) [6], Collins (1973) [2], Bennet (1974) [1], Hoag (1975) [7], Kenneth (1975) [8], Dorris (1978) [3].

Pradeepa Veerakumari and Aruna (2018) developed zero acceptance number single sampling plans with inspection error on EOQ model with permissible delay in payments. Permissible delay in payment was pioneered by Goyal (1985) [5]. J.T. Teng (2002) amended the Goyal's model by considering varying unit price and unit cost. This study extends Pradeepa Veerakumari and Aruna (2018) [9] model when the vendor is offered extended payment privileges. In the study, it is assumed that the buyer applies the single sampling inspection plan to detect the defects.

The sampling is executed without replacement requiring destructive testing. Total costs associated with the model are ordering costs, holding cost, inspection cost, acceptance cost. The model is designed that the items of the rejected lot are sold at the discounted costs. Cash flow timing plays a crucial factor in the investment as soon as the cash is available, there may be possibility of using it for other purposes. Therefore the arrangement of cash flow is a important factor for an organization. In most of the business transactions, the vendor may incorporate the cash models with the permissible delay in payments. The consumer can acquire cash discount if the payment is paid before the discount period; the buyer will make full payment if paid within permissible delay in payments; otherwise the payment should pay with the interest levied by the vendor. In general, the cash discount period is shorter than the permissible delay in payments which in turn shorter than the interest levying period. For instance, the vendor provides a cash discount for r_1 if paid with in cash discount period M_1,pays full payment if paid within permissible delay in payments M_2; Other pays with interest r_1 if paid after M_2 i.e. M_3.In short $M_1<M_2<M_3$. The value of $r_1$and r_2 lies between 0 to1. It is noted that just like the grace period of the credit card payment paid before or on the permissible delay in payments does not carry any interest charges.

Model Assumptions
1. The demand for the item is known and invariable over time
2. No shortages are granted
3. The retail price of every single product sold during credit period is nest egg with interest I_e on the termination of credit period; buyer settles the credit and begins to settle with the interest I_p on the items in the stock. $I_p>I_e$
4. Time frame is non-finite
5. Single product is considered
6. Imperative replenishment cycle
7. $W<S$, $I_e<I_P$, $M_1<M_2<M_3$

Model Formulation
Consider that the buyer places an order of Q to the vendor. The ordering cost is constant and symbolized as C_o and Demand rate D. After acquiring the lot, sampling is executed on the sample of size n. Proportion defective in the lot is symbolized as p. It is presumed that single sampling by attributes is implemented with some faulty inspection .$e_1$is the probability of committing type I error in that case defective is classified as non-defective. e_2 is the probability of committing type II error i.e. classifying non-defective as defective. The modus operandi of the single sampling plan is followed. If the case involves destructive testing generally zero acceptance number is applied. Then the probability of acceptance becomes,

$$P_a = (1-p)^n \tag{1}$$

Since the inspection process involves the errors, the apparent fraction defective p_e can be represented as a function of the known true fraction defective p,

$$p_e = p(1-e_2) + (1-p)e_1 \tag{2}$$

By substituting in eqn (1), obtaining,

$$\mathrm{P_{a_e}} = [(1-\mathrm{e}_1)(1-\mathrm{p}) + \mathrm{p}(\mathrm{e}_2)]^{\mathrm{n}} \tag{3}$$

Since,process involves zero acceptance sampling plan the sample size n can be obtained by using β and LQL. where β representing consumer risk, LQL representing the probability of accepting reject able quality. It is assumed that the inspection is faulty with inspection error then the LQL=

$$n = \frac{\log \beta}{\log(1-LQL')} \tag{4}$$

Costs associated with the model
If a lot is accepted then there is a shortage of $(DT-n)p_e+n$ in the lot is. Then the cost of accepting the lot is $C_a((DT-n)p_e+n)/T$. Otherwise, if the lot is rejected, then it is sold at discounted rate G per unit. Then the cost associated with the rejection is $G(DT-n)/T$. Then the further, costs related to the model are

i. Annual Ordering cost= $\frac{C_o}{T}$ (5)

ii. Annual Inventory Holding Cost= $\frac{(DT-n)^2 C_h}{2DT}$ (6)

iii. Annual Cost of Inspection:

The expected number of units destructed in a sample of size n, providing the fraction defective p_e is np_e. Then the expected cost of inspection is n $(C_i+C_d.p_e)$

Annual Cost of Inspection = $\dfrac{n(C_i + C_d.p_e)}{T}$ (7)

iv. Annual Interest Paid

Case (1): *when the payment made at time M_1*

Sub- Case 1.1: $T \geq M_1$

Annual cost of interest charges for the items retained in the stock

$$= \frac{w(1-r_1)I_p D(T-M_1)^2}{2T} \quad (8)$$

Sub -Case 1.2: $T \leq M_1$

Considering this case, the interest charges for the items retained in the stock=0

Case (2): *when the payment made at time M_2*
Sub-Case 2.1: $T \geq M_2$

Annual cost of interest charges for the items retained in the stock

$$= \frac{wI_p D(T-M_2)^2}{2T} \quad (9)$$

Sub-Case 2.2: $T \leq M_2$

Considering this case, the interest charges for the items retained in the stock=0

Case (3): *when the payment made at time M_3*
Sub-Case 3.1: $T \geq M_3$

Annual cost of interest charges for the items retained in the stock

$$= \frac{w(1+r_2)I_p D(T-M_3)^2}{2T} \quad (10)$$

Sub-Case 3.2: $T \leq M_3$

Considering this case, the interest charges for the items retained in the stock=0

Annual Interest Earned

1. **When payment is made at M_1**

Subcase 1.1. When $T \geq M_1$

Annual Interest Earned= $\dfrac{S.I_e.DM_1^{\,2}}{2T}$ (11)

Subcase 1.2. When $T \leq M_1$

Annual Interest Earned= $\dfrac{S.I_e.DT}{2} + S.I_e.D(M_1 - T)$ (12)

2. **When payment is made at M_2**

Subcase 2.1. When $T \geq M_2$

Annual Interest Earned= $\dfrac{S.I_e.DM_2^{\,2}}{2T}$ (13)

Subcase 2.2.When T≤M2

Annual Interest Earned= $\dfrac{S.I_e.DT}{2}+S.I_e.D(M_2-T)$ (14)

3. When payment is made at M3

Subcase 3.1.When T≥M3

Annual Interest Earned= $\dfrac{S.I_e.DM_3{}^2}{2T}$ (15)

Subcase 1.2.When T≤M3

Annual Interest Earned= $\dfrac{S.I_e.DT}{2}+S.I_e.D(M_3-T)$ (16)

Since it is assumed that the consumer continues to reorder the lot until, a lot is accepted and sold the rejected lot at discounted lot, the process continues until x times, then the expected total cost, when T>M1

$$E(TC_{1.1})=\left(\begin{array}{l} P_{ae}\sum_{x=0}^{\infty}[(x+1)(\frac{C_o}{T}+\frac{(C_i+C_d.p_e)n}{T})-x\frac{G(DT-n)}{T} \\ +\frac{(DT-n)^2c_h}{2}+\frac{C_a((DT-n)p_e+n)}{T} \\ -\frac{S.I_e.DM_1{}^2}{2T}+\frac{W(1-r_1).D.I_p(T-M_1)^2}{2T}],(1-P_{ae})^x \end{array}\right) \quad (17)$$

The series can be rewritten as,

$$E(TC_{1.1})=P_{ae}\left(\begin{array}{l} \sum_{x=0}^{\infty}(1-P_{ae})^x(x+1)(\frac{C_o}{T}+\frac{(C_i+C_d.p_e)n}{T}) \\ -\sum_{x=0}^{\infty}x(1-P_{ae})^x\frac{G(DT-n)}{T}+ \\ \sum_{x=0}^{\infty}(1-P_{ae})^x\left(\begin{array}{l}\frac{(DT-n)^2c_h}{2}+\frac{C_a((DT-n)p_e+n)}{T} \\ -\frac{S.I_e.DM_1{}^2}{2T}+\frac{W(1-r_1).D.I_p(T-M_1)^2}{2T}\end{array}\right) \end{array}\right) \quad (18)$$

The results of this series expansion are converging,

$$\sum_{x=0}^{\infty}[(1+x)(1-P_{ae})^x]=\frac{1}{P_{ae}^2} \quad (19)$$

$$\sum_{x=0}^{\infty}x(1-P_{ae})^x=\frac{1-P_{ae}}{P_{ae}^2} \quad (20)$$

$$\sum_{x=o}^{\infty}(1-P_{ae})^x=\frac{1}{P_{ae}} \quad (21)$$

Substituting the values of the eqn 19 to 21 in 18, the total expected cost function becomes,

$$E(TC_{1.1}(T))=\left(\begin{array}{l} (\frac{C_o}{T}+\frac{(C_i+C_d.p_e).n}{T})\frac{1}{P_{ae}}-\frac{G(DT-n)}{T}\frac{1-P_{ae}}{P_{ae}}+\frac{(DT-n)^2c_h}{2DT} \\ -\frac{S.I_e.DM_1{}^2}{2T}+\frac{C_a((DT-n)p_e+n)}{T}+\frac{W.D.I_p(T-M_1)^2}{2T} \end{array}\right) \quad (22)$$

Since the zero acceptance number sampling plan is applied, then $P_{ae}=(1-p_e)^n$, thus reducing the cost function to,

$$TC_{1.1}(T)=\begin{pmatrix} (\frac{C_o}{T}+\frac{(C_i+C_d.p_e).n}{T})\frac{1}{(1-p_e)^n}-\frac{G(DT-n)}{T}\frac{1-(1-p_e)^n}{(1-p_e)^n}+\frac{(DT-n)^2C_h}{2DT}+ \\ \frac{C_a((DT-n)p_e+n)}{T}-\frac{S.I_e.DM_1^{\,2}}{2T}+\frac{W(1+r_1).D.I_p(T-M_1)^2}{2T} \end{pmatrix} \quad (23)$$

Similarly, total cost of $TC_{1.2}$ (T) when $T\leq M_1$,is obtained as,

$$\mathrm{TC}_{1.2}(\mathrm{T})=\begin{pmatrix} (\frac{C_o}{T}+\frac{(C_i+C_d.p_e).n}{T})\frac{1}{(1-p_e)^n}-\frac{G(DT-n)}{T}\frac{1-(1-p_e)^n}{(1-p_e)^n}+\frac{(DT-n)^2C_h}{2DT} \\ +\frac{C_a((DT-n)p_e+n)}{T}-\frac{S.I_e.DT}{2}-S.I_eD(M_1-T) \end{pmatrix} \quad (24)$$

the total cost for $T>M_2$,is obtained as,

$$TC_{2.1}(T)=\begin{pmatrix} (\frac{C_o}{T}+\frac{(C_i+C_d.p_e).n}{T})\frac{1}{(1-p_e)^n}-\frac{G(DT-n)}{T}\frac{1-(1-p_e)^n}{(1-p_e)^n} \\ +\frac{(DT-n)^2C_h}{2DT}+\frac{C_a((DT-n)p_e+n)}{T}-\frac{S.I_e.DM_2^{\,2}}{2T}+\frac{WD.I_p(T-M_2)^2}{2T} \end{pmatrix} \quad (25)$$

Total cost of TC_2(T) when $T\leq M_2$,is obtained as,

$$TC_{2.2}(T)=\begin{pmatrix} (\frac{C_o}{T}+\frac{(C_i+C_d.p_e).n}{T})\frac{1}{(1-p_e)^n}-\frac{G(DT-n)}{T}\frac{1-(1-p_e)^n}{(1-p_e)^n} \\ +\frac{(DT-n)^2c_h}{2DT}+\frac{C_a((DT-n)p_e+n)}{T}-\frac{S.I_e.DT}{2}-S.I_eD(M_2-T) \end{pmatrix} \quad (26)$$

The total cost for $T>M_3$,is obtained as,

$$TC_{3.1}(T)=\begin{pmatrix} (\frac{C_o}{T}+\frac{(C_i+C_d.p_e).n}{T})\frac{1}{(1-p_e)^n}-\frac{G(DT-n)}{T}\frac{1-(1-p_e)^n}{(1-p_e)^n} \\ +\frac{(DT-n)^2C_h}{2DT}+\frac{C_a((DT-n)p_e+n)}{T}-\frac{S.I_e.DM_3^{\,2}}{2T}+\frac{W(1+r_2)D.I_p(T-M_3)^2}{2T} \end{pmatrix} \quad (27)$$

Total cost of TC_2(T) when $T\leq M_3$,is obtained as,

$$TC_{3.2}=\begin{pmatrix} (\frac{C_o}{T}+\frac{(C_i+C_d.p_e).n}{T})\frac{1}{(1-p_e)^n}-\frac{G(DT-n)}{T}\frac{1-(1-p_e)^n}{(1-p_e)^n} \\ +\frac{(DT-n)^2C_h}{2DT}+\frac{C_a((DT-n)p_e+n)}{T}-\frac{S.I_e.DT}{2}-S.I_eD(M_3-T) \end{pmatrix} \quad (28)$$

In order to optimize the cost function and to determine the optimal replenishment cycle time, first order and second order derivative function of total cost functions with respect to T are obtained.

$$TC_{1.1}^{'}(T)=\frac{1}{2DT^2(1-p_e)^n}\begin{pmatrix} -2D(C_o+(C_i+C_d.p_e).n)-2DGn(1-(1-p_e)^n) \\ +n^2(C_h)(1-p_e)^n+2DC_a((p_e-1)+n)(1-p_e)^n \\ -D^2M_1^{\,2}(W(1-r_1).I_p-S.I_e)(1-p_e)^n \\ +D^2T^2(C_h+W(1-r_1).I_p)(1-p_e)^n \end{pmatrix} \quad (29)$$

$$TC''_{1.1}(T)=\frac{1}{DT^3(1-p_e)^n}\begin{pmatrix}2D(C_o+(C_i+C_d.p_e).n)-2DGn(1-(1-p_e)^n)+n^2C_h(1-p_e)^n\\+2D(C_a(-n)p_e+n)(1-p_e)^n+D^2M_1{}^2(W(1-r_1).I_p-S.I_e)(1-p_e)^n\end{pmatrix} \tag{30}$$

$$TC'_{1.2}(T)=\frac{1}{2DT^2(1-p_e)^n}\begin{pmatrix}-2D(c_o+(C_i+C_d.p_e).n)-2DGn(1-(1-p_e)^n)\\-2D(C_a(((p_e-1)+n)(1-p_e)^n+n^2.C_h(1-p_e)^n\\+D^2T^2(C_h+S.I_e)(1-p_e)^n\end{pmatrix} \tag{31}$$

$$TC''_{1.2}(T)=\frac{1}{DT^3(1-p_e)^n}\begin{pmatrix}2D(C_o+(C_i+C_d.p_e).n)+2GDn(1-(1-p_e)^n)\\+2D(c_a((-n)p_e+n)(1-p_e)^n+n^2C_h(1-p_e)^n\end{pmatrix} \tag{32}$$

$$TC'_{2.1}(T)=\frac{1}{2DT^2(1-p_e)^n}\begin{pmatrix}-2D(C_o+(C_i+C_d.p_e).n)-2DGn(1-(1-p_e)^n)\\+n^2(C_h)(1-p_e)^n+2Dc_a((p_e-1)+n)(1-p_e)^n\\-D^2M_2{}^2(W.I_p-S.I_e)(1-p_e)^n\\+D^2T^2(C_h+W.I_p)(1-p_e)^n\end{pmatrix} \tag{33}$$

$$TC''_{2.1}(T)=\frac{1}{DT^3(1-p_e)^n}\begin{pmatrix}2D(C_o+(C_i+C_d.p_e).n)+2DGn(1-(1-p_e)^n)+n^2C_h(1-p_e)^n\\+2D(C_a(-n)p_e+n)(1-p_e)^n+D^2M_2{}^2(W.I_p-S.I_e)(1-p_e)^n\end{pmatrix} \tag{34}$$

$$TC'_{2.2}(T)=\frac{1}{2DT^2(1-p_e)^n}\begin{pmatrix}-2D(C_o+(C_i+C_d.p_e).n)-2DGn(1-(1-p_e)^n)\\-2D(C_a(((p_e-1)+n)(1-p_e)^n-n^2.C_h(1-p_e)^n\\+D^2T^2(C_h+S.I_e)(1-p_e)^n\end{pmatrix} \tag{35}$$

$$TC''_{2.2}(T)=\frac{1}{DT^3(1-p_e)^n}\begin{pmatrix}2D(C_o+(C_i+C_d.p_e).n)+2DGn(1-(1-p_e)^n)\\+2D(c_a((-n)p_e+n)(1-p_e)^n+n^2c_h(1-p_e)^n\end{pmatrix}>0 \tag{36}$$

$$Tc'_{3.1}(T)=\frac{1}{2DT^2(1-p_e)^n}\begin{pmatrix}-2D(C_o+(C_i+C_d.p_e).n)-2DGn(1-(1-p_e)^n)\\-n^2(C_h)(1-p_e)^n+2DC_a((p_e-1)+n)(1-p_e)^n\\-D^2M_3{}^2(W(1+r_2).I_p-S.I_e)(1-p_e)^n\\+D^2T^2(C_h+W(1+r_2).I_p)(1-p_e)^n\end{pmatrix} \tag{37}$$

$$Tc''_{3.1}(T)=\frac{1}{DT^3(1-p_e)^n}\begin{pmatrix}2D(C_o+(C_i+C_d.p_e).n)+2DGn(1-(1-p_e)^n)-n^2C_h(1-p_e)^n\\+2D(C_a(-n)p_e+n)(1-p_e)^n+D^2M_3{}^2(W.I_p-S.I_e)(1-p_e)^n\end{pmatrix} \tag{38}$$

$$TC'_{3.2}(T)=\frac{1}{2DT^2(1-p_e)^n}\begin{pmatrix}-2D(C_o+(C_i+C_d.p_e).n)-2DGn(1-(1-p_e)^n)\\-2D(C_a(((p_e-1)+n)(1-p_e)^n-n^2.C_h(1-p_e)^n\\+D^2T^2(C_h+S.I_e)(1-p_e)^n\end{pmatrix} \tag{39}$$

$$TC''_{3.2}(T)=\frac{1}{DT^3(1-p_e)^n}\begin{pmatrix}2D(C_o+(C_i+C_d.p_e).n)+2DGn(1-(1-p_e)^n)\\+2D(C_a((-n)p_e+n)(1-p_e)^n-n^2C_h(1-p_e)^n\end{pmatrix} \tag{40}$$

The optimal replenishment cycle time associated with least possible cost is obtained when; the non-negative term of first order derivative is equated to zero. Thus obtaining,

$$T_{1.1}^{*}=\left(\begin{array}{l}\frac{1}{D^{2}(C_{h}+W(1-r_{1}).I_{p})(1-p_{e})^{n}}(2D((C_{o}+(C_{i}+C_{d}.p_{e}).n)+Gn(1-(1-p_{e})^{n})\\+(C_{a}(((p_{e}-1)+n)(1-p_{e})^{n}))-n^{2}C_{h}(1-p_{e})^{n}-D^{2}M^{2}(W(1-r_{1}).I_{p}-S.I_{e})(1-p_{e})^{n})\end{array}\right)^{1/2} \quad (41)$$

$$T_{1.2}^{*}=\left(\begin{array}{l}\frac{1}{D^{2}(C_{h}+S.I_{e})(1-p_{e})^{n}}(2D((C_{o}+(C_{i}+C_{d}.p_{e}).n)+Gn(1-(1-p_{e})^{n})\\+(C_{a}((p_{e}-1)+n)(1-p_{e})^{n}))\end{array}\right)^{1/2} \quad (42)$$

$$T_{2.1}^{*}=\left(\begin{array}{l}\frac{1}{D^{2}(C_{h}+WI_{p})(1-p_{e})^{n}}(2D((C_{o}+(C_{i}+C_{d}.p_{e}).n)+Gn(1-(1-p_{e})^{n})\\+(C_{a}(((p_{e}-1)+n)(1-p_{e})^{n}))-n^{2}C_{h}(1-p_{e})^{n}-D^{2}M^{2}(W.I_{p}-S.I_{e})(1-p_{e})^{n})\end{array}\right)^{1/2} \quad (43)$$

$$T_{2.2}^{*}=\left(\begin{array}{l}\frac{1}{D^{2}(C_{h}+S.I_{e})(1-p_{e})^{n}}(2D((C_{o}+(C_{i}+C_{d}.p_{e}).n)+Gn(1-(1-p_{e})^{n})\\+(C_{a}((p_{e}-1)+n)(1-p_{e})^{n}))\end{array}\right)^{1/2} \quad (44)$$

$$T_{3.1}^{*}=\left(\begin{array}{l}\frac{1}{D^{2}(C_{h}+W(1+r_{2}).I_{p})(1-p_{e})^{n}}(2D((C_{o}+(C_{i}+C_{d}.p_{e}).n)+2Gn(1-(1-p_{e})^{n})\\+(C_{a}(((p_{e}-1)+n)(1-p_{e})^{n}))-n^{2}C_{h}(1-p_{e})^{n}-D^{2}M^{2}(W.(1+r_{2}).I_{p}-P.I_{e})(1-p_{e})^{n})\end{array}\right)^{1/2} \quad (45)$$

$$T_{3.2}^{*}=\left(\begin{array}{l}\frac{1}{D^{2}(C_{h}+P.I_{e})(1-p_{e})^{n}}(2D((C_{o}+(C_{i}+C_{d}.p_{e}).n)+Gn(1-(1-p_{e})^{n})\\+(C_{a}((p_{e}-1)+n)(1-p_{e})^{n}))\end{array}\right)^{1/2} \quad (46)$$

It is noted from the equations, 42, 44 and 46 that, despite of the payment period $T_{1.2}^{*},T_{2.2}^{*} and T_{3.2}^{*}$ are same. Let it is considered that C= $2D((c_{o}+(c_{i}+c_{d}.p_{e}).n)+Gn(1-(1-p_{e})^{n})+(c_{a}((-n)p_{e}+n)(1-p_{e})^{n})+n^{2}c_{h}(1-p_{e})^{n}$.The optimality condition for $T_{1.1}^{*}$ is obtained by equating the Eqn70 to the inequality $T_{1.1}^{*}\le M_{1}$ Thus, getting $\Delta_{1}=2C-DM_{1}^{2}(c_{h}+S.I_{e})\ge 0$ similarly, for $T_{1.2}^{*}$, the optimality condition is $\Delta_{1}\le 0$. I.e. The *Optimal value of* $T_{1.1}^{*}$ *is obtained, If and only if* the $\Delta_{1}\ge 0$ and for $T_{1.2}^{*}$ is if, $\Delta_{1}\le 0$ Similarly consider, $\Delta_{2}=2C-DM_{2}^{2}(C_{h}+S.I_{e})\ge 0$, then optimal condition is if $\Delta_{2}\ge 0$, the optimal value of $T_{2.1}^{*}$ is obtained or otherwise if $\Delta_{2}\le 0$, the optimal value of $T_{2.2}^{*}$ is obtained. In similar fashion, considering, $\Delta_{3}=C-DM_{3}^{2}(C_{h}+S.I_{e})\ge 0$, the optimal value of $T_{3.1}^{*}$ is obtained only if $\Delta_{3}\ge 0$ and the optimal value of $T_{3.2}^{*}$ is obtained only if $\Delta_{3}\le 0$.

Axioms

i. With the condition that, $\Delta_{1}\ge 0,\ \Delta_{2}\le 0,\Delta_{3}\le 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time $T_{1.1}^{*} or T_{2.2}^{*} or T_{3.2}^{*}$

ii. With the condition that, $\Delta_{1}\ge 0,\ \Delta_{2}\ge 0,\Delta_{3}\le 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time $T_{1.1}^{*} or T_{2.1}^{*} or T_{3.2}^{*}$

iii. With the condition that, $\Delta_{1}\ge 0,\ \Delta_{2}\ge 0,\Delta_{3}\ge 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time is $T_{1.1}^{*} or T_{2.1}^{*} or T_{3.1}^{*}$

iv. With the condition that, $\Delta_{1}\ge 0,\ \Delta_{2}\le 0,\Delta_{3}\ge 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time is $T_{1.1}^{*} or T_{2.2}^{*} or T_{3.1}^{*}$.

v. With the condition that, $\Delta_{1}\le 0,\ \Delta_{2}\ge 0,\Delta_{3}\ge 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time is $T_{1.2}^{*} or T_{2.1}^{*} or T_{3.1}^{*}$.

vi. With the condition that, $\Delta_1 \leq 0,\ \Delta_2 \leq 0, \Delta_3 \geq 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time is $T_{1.2}^{*} or T_{2.2}^{*} or T_{3.1}^{*}$.

vii. With the condition that, $\Delta_1 \leq 0,\ \Delta_2 \leq 0, \Delta_3 \leq 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time is $T_{1.2}^{*} or T_{2.2}^{*} or T_{3.2}^{*}$

viii. With the condition that, $\Delta_1 \leq 0,\ \Delta_2 \geq 0, \Delta_3 \leq 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time is $T_{1.2}^{*} or T_{2.1}^{*} or T_{3.2}^{*}$

ix. With the condition that, $\Delta_1 \leq 0,\ \Delta_2 \leq 0, \Delta_3 \geq 0$, then the optimal payment cycle time is minimum of (M_1, M_2, M_3) with the minimized cost associated with the optimal replenishment cycle time is $T_{1.2}^{*} or T_{2.2}^{*} or T_{3.1}^{*}$

Numerical Illustrations

To illustrate the results, proposed method is applied to hypothetical numerical examples where the associated costs and parameters are, c_0=50, c_i=0.1perunit; c_d=0.2perunit; c_h=0.5;D=1000; G=0.05; W=5 per unit; S=7 per unit; I_e=.10; I_p=.15; M_1=0.34; M_2=0.38; M_3=0.4; r_1=0.05; r_2=0.06; p=0.01 in appropriate units. By applying the theorem, for various values of LQL,e_1 and e_2: the optimal replenishment cycle interval associated with the least cost is derived.

Table 1: Optimal replenishment interval associated with total cost for different values of LQL,e_1, e_2.

LQL	e_1%	e_2%	n	T^*	$TC(T^*)$
2.0	0	0	114	$T_{3.1}^*$=0.5031	679.32
2.0	1	1	77	$T_{3.1}^*$=0.4700	726.85
2.0	1	10	82	$T_{3.1}^*$=0.4755	764.08
2.0	5	1	33	$T_{3.1}^*$=04350	820.77
2.5	0	0	91	$T_{3.1}^*$=0.4830	581.22
2.5	1	1	66	$T_{3.1}^*$=0.4555	627.85
2.5	1	10	71	$T_{3.1}^*$=0.4621	661.47
2.5	5	1	31	$T_{3.1}^*$=0.4303	752.84
3.0	0	0	76	$T_{3.1}^*$=0.4612	520.48
3.0	1	1	58	$T_{3.1}^*$=0.4429	564.33
3.0	1	10	62	$T_{3.1}^*$=0.4493	588.42
3.0	5	1	29	$T_{3.1.}^*$=0.4250	691.65

It is noted from the table that with the increase in inspection error leads to the decrease in optimal replenishment cycle time and increase in the total cost. By reducing the inspection error the total cost will be minimized.

Conclusion

As a result of the permissible delay in payments, there is a decrease in the total annual costs. When the customer is offered with the payment privileges, it enables the customer to select better option depend on the situation. On the other hand, it enables the vendor to retain the customer. The study can be extended to other sampling plans for instance, Skip lot sampling plan, Quick Switching System.

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