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Design of six sigma quick switching variables sampling system of type indexed by six sigma AQL and six sigma AOQL

Dr. D Senthilkumar and Dr. B Esha Raffie

Abstract
This article gives designing procedure of Six Sigma Quick Switching Variables Sampling System (SSQSVSS ($n_{T\sigma}$, $n_{N\sigma}$; k_{σ})) indexed by Six Sigma AQL and Six Sigma AOQL. The design procedure constructed tables for easy selection of system given indexed by six sigma quality levels by known and unknown σ respectively.

Keywords: quick switching sampling system, six sigma, AOQ, AOQL, six sigma AQL and six sigma AOQL

Introduction

The designing procedure for the SSQSVSS ($n_{T\sigma}$, $n_{N\sigma}$; k_{σ}) indexed by SSAQL and SSAOQL is based on Govindaraju (1990) [3] procedures and tables, developing for the selection of single sampling plan for variables indexed by AQL and AOQL. Soundararajan (1981) [9] has developed procedures and tables for the selection of single sampling plans for attributes for given AQL and AOQL. Govindaraju (1990) [3] has developed procedures and tables for the selection of single sampling plans for variables indexed by AQL and AOQL. Later Soundarajan and Palanivel (2000) [8] have developed procedures and tables for the selection of quick switching single sampling variables systems indexed by AQL and AOQL. Based on above article Senthilkumar and Esha Raffie (2017) [6] have constructed SSQSVSS (n_{σ}; $k_{T\sigma}$, $k_{N\sigma}$) indexed by Six Sigma AQL and Six Sigma AOQL. The extension work of SSQSVSS ($n_{N\sigma}$, $n_{T\sigma}$; k_{σ}) indexed by Six Sigma AQL and Six Sigma AOQL are given in this article. This concept can be extended to variable quality characteristics of the study, the resulting plan would be designated as SSQSVSS and would be applied under the following conditions:

- The production is steady, so that results on current and preceding lots are broadly indicative of a continuous process.
- Lots are submitted substantially in the order of production.
- Inspection is by variables, with the quality being defined as the fraction of non-conforming.
- The sample units are selected from a large lot and production is continuous.
- The production process should depend on automation and human handling is very negligible.
- The industry may adopt system method with decision makers having an experience in adopting the six sigma quality initiatives.

Basic Assumptions

- The quality characteristic is represented by a random variable X measurable on a continuous scale.
- Distribution of X is normal with mean and standard deviation.
- An upper limit U, has been specified and a product is qualified as defective when $X > U$. [when the lower limit L is Specified, the product is a defective one if $X < L$]
- The Purpose of inspection is to control the fraction defective, p in the lot inspected.

The condition for application above basic assumptions of SSQSVSS ($n_{T\sigma}$, $n_{N\sigma}$; k_σ) system are the same as that of the SSQSVSS (n_σ; $k_{T\sigma}$, $k_{N\sigma}$) system.
The fraction defective in a lot will be

$p = 1-F(v) = F(-v)$ with $v = (U-\mu)/\sigma$ and

$$F(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \qquad (1)$$

Where z ~ N (0, 1). Under the σ- method plan, the lot would be accepted if $\overline{X} + k\sigma \leq U$, where U is the upper specification limit or $\overline{X} + k\sigma \geq L$, where L is the lower specification limit.

Procedure of SSQSVSS ($n_{T\sigma}$, $n_{N\sigma}$; k_σ) with known σ variables plan as the reference plan
The Six Sigma Quick Switching Variables Sampling System with known σ variables plan as the reference plan has following operating procedure

Operating procedure
Step 1: Under normal inspection, draw a sample of size $n_{N\sigma}$ from the lot, inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean $\overline{X}$. Where $\overline{X} = \frac{\sum x_i}{n_{N\sigma}}$

Step 2: i) If $\overline{X} + k\sigma \leq U$ or $\overline{X} + k\sigma \geq L$ accept the lot and repeat Step 1 for the next lot.

ii) If $\overline{X} + k\sigma > U$ or $\overline{X} + k\sigma < L$, reject the lot go to Step 3.

Step 3: Under tightened inspection, draw a sample of size $n_{T\sigma}$ from the next lot inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean $\overline{X}$. Where $\overline{X} = \frac{\sum x_i}{n_{T\sigma}}$

Step 4: i) If $\overline{X} + k\sigma \leq U$ or $\overline{X} + k\sigma \geq L$ accept the lot and repeat Step 1, for the next lot.

ii) If $\overline{X} + k\sigma > U$ or $\overline{X} + k\sigma < L$, reject the lot and repeat Step 3.

Where $n_{N\sigma}$ and $n_{T\sigma}$ are the sample sizes of normal and tightened single sampling variable plans respectively, and k is the acceptance constant under σ-method. Where $\overline{X}$ and σ are the average quality characteristic and standard deviation respectively. The Six Sigma Quick Switching Variables Sampling System as SSQSVSS (n_T, n_N; k_σ). As the tightened plan sample size n_T is greater than the normal plan sample size n_N, for designing the SSQSVS system n_T is fixed as a multiple of n_N i.e. $n_T = m\, n_N$, where m >1. Thus the QSS can also be designated as SSQSVSS (mn_N, n_N; k_σ).
Based on Romboski (1969), the OC function of SSQSVSS ($n_{T\sigma}$, $n_{N\sigma}$; k_σ) can be written as

$$P_a(p) = \frac{P_T}{1-P_N+P_T} \qquad (2)$$

The fraction nonconforming in a given lot has been given in (1) with v= (U-μ)/σ and its probability of acceptance will be

$$P_a(p) = F(w_N, w_T)$$

With

$$w_N = (v - k_\sigma)\sqrt{n_{N\sigma}} \qquad (3)$$

$$w_T = (v - k_\sigma)\sqrt{n_{T\sigma}} \qquad (4)$$

If the quality of the accepted lot is p and all defective units found in the rejected lots are replaced by non-defective units in a rectifying inspection plan, the Six Sigma average outgoing quality (SSAOQ) can be approximated as

$$SSAOQ = pP_a(p) \qquad (5)$$

where $P_a(p)$ is defined in equation (2). If p_m is the proportion nonconforming items at which SSAOQ is maximum, one has

$$SSAOQL = p_m P_a(p_m) \qquad (6)$$

If SSAQL (p_1) is prescribed, then the corresponding value of v_{SSAQL} or v_1 will be fixed and if $P_a(p)$ is fixed at 99.99966%, that is (1-α). Where, $\alpha = 0.0000034 \times 10^{-6}$. Hence we have

$$P_a(p_1) = (1-\alpha)$$

So that for given values of n_σ, w_N, w_T and SSAQL, $k_{N\sigma}$, $k_{T\sigma}$ are determined.

Selection of known σ SSQSVSS ($n_{T\sigma}$, $n_{N\sigma}$; k_σ) for given SSAQL and SSAOQL

Table 1 is used for selection of σ- method SSQS Variables Sampling System. For example, if the SSAQL is fixed at 0.000005 and the SSAOQL is fixed at 0.000006, Table 1 yields $n_{T\sigma}$ = 7731, $n_{N\sigma}$ = 3865 and k_σ =3.941, which is associated with 4.6 sigma level of SSQSVSS ($n_{T\sigma}$, $n_{N\sigma}$; k_σ). The sample size n_{Ts} = m n_{Ns} = (2) (3865) = 7731. Thus, for the requirement, the SSQSVSS ($n_{T\sigma}$, $n_{N\sigma}$; k_σ) is specified by the parameters $n_{T\sigma}$ = 7731, $n_{N\sigma}$ = 3865 and k_σ = 3.941.

The user of Table 1 should understand the limitations of plans indexed by SSAOQL. Sampling with rectifying of rejected lots on the one hand reduces the average percentage of nonconforming items in the lots, but on the other hand introduces non-homogeneity in the series of lots finally accepted. That is, any particular lot will have a quality of p% or 0% nonconforming depending on whether the lot is accepted or rectified. Thus the assumption underlying the SSAOQL principle is that the homogeneity in the qualities of individual lots is unimportant and only the average quality matters. For plans listed in Table 1, if the individual lot quality happens to be the product quality p_m at which SSAOQL occurs, then the associated probability of acceptance will be poor. Table 2 gives $P_a(p_m)$ values of plans given in Table 1. For example, for SSAQL is 0.000004 and SSAOQL is 0.00001, Table 2 gives $P_a(p_m)$ = 0.71. Then p_m = SSAOQL/ $P_a(p_m)$ = 0.000014. In order to avoid such inconvenience, the producer should maintain the process quality more or less at the SSAQL. The higher rate of rejection of lots at p = p_m will also indirectly put pressure on the producer to improve the submitted quality.

Procedure of unknown σ SSQSVSS for given SSAQL and SSLQL

If the population standard deviation σ is unknown, then it is estimated from the sample standard deviation S (n-1 as the divisor). If the sample size of the unknown sigma variables system (S-method) is n_{Ts}, n_{Ns} and the acceptance constant is k_s, then the operating procedure is as follows

Step 1: Draw a sample of size n_s from the lot, inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean $\overline{X}$ and sample standard deviation S_N.

Where $\overline{X}=\dfrac{\sum x_i}{n_N}$, and $S_N=\sqrt{\dfrac{\sum (x_i-\bar{x})^2}{n_N-1}}$

Step 2: i) If $\overline{X}+k_s S_N \le U$ or $\overline{X}+k_s S_N \ge L$ accept the lot and repeat Step 1 for the next lot.

ii) If $\overline{X}+k_s S_N > U$ or $\overline{X}+k_s S_N < L$ reject the lot go to Step 3.

Step 3: Draw a sample of size n_s from the next lot inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean $\overline{X}$ and sample standard deviation S_T.

Where $\overline{X}=\dfrac{\sum x_i}{n_T}$, and $S_T=\sqrt{\dfrac{\sum (x_i-\bar{x})^2}{n_T-1}}$

Step 4: i) If $\overline{X}+k_s S_T \le U$ or $\overline{X}+k_s S_T \ge L$ accept the lot and repeat Step 1 for the next lot.

ii) If $\overline{X}+k_s S_T > U$ or $\overline{X}+k_s S_T < L$ reject the lot and repeat Step 3.

Selection of unknown σ SSQSVSS (n_{Ts}, n_{Ns}; k_s) for Given SSAQL and SSAOQL

Table 1 also gives such matched s-method plan. For example, for given SSAQL is 0.00001 and SSAOQL is 0.00004, one obtains the parameters of the s-method plan from Table 1 to be n_{Ts} = 5030, n_{Ns} = 2515 and k_s = 3.620, which is associated with 4.5 sigma level of SSQSVSS (n_{Ts}, n_{Ns}; k_s). The sample size n_{Ts} = m n_{Ns} = (2) (2515) = 5030. Thus, for the requirem-ent, the SSQSVSS (n_{Ts}, n_{Ns}; k_s) id specified by the parameters, n_{Ts} = 5030, n_{Ns} = 2515 and k_s = 3.620.

Construction of Table 1

For constructing Table 1, a trial value of p_m is assumed and the probability of acceptance at p_m is found using (6) as

$$P_a(p_m) = SSAOQL / p_m \tag{7}$$

The auxiliary variables v_m, w_{Nm} and w_{Tm} corresponding to the values of p_m and $P_a(p_m)$ respectively, are found using (1), (2), (3) and (4). For given values of (p_1, α), determine the values of v_1, w_{N1} and w_{T1} using the approximation (Abramwitz and Stegun (1972)) for the ordinate of the cumulative normal distribution. With the values of v_m, w_{Nm} and w_{Tm}, the following equation is used for calculating $n_{N\sigma}$.

$$\sqrt{n_{N\sigma}} = (\text{-AOQL}) / (p_m^2((1-P_N+P_T)\sqrt{(\exp(v_m^2-w_T^2))}) -$$

$$P_T(\sqrt{(\exp(v_m^2-w_N^2))}-\sqrt{(\exp(v_m^2-w_T^2))})) / (1-P_N+P_T)^2) \tag{8}$$

Where

$$P_N = \phi(w_N) = pr[(U-\bar{x})/\sigma > k_{N\sigma}]$$

and $P_T = \phi(w_T) = pr[(U-\bar{x})/\sigma > k_{T\sigma}]$

Equation (8) is the formula for finding the sample size of a known σ SSQSVS system. With the values of n obtained from (14), it is then checked to see whether the assumed value of p_m corresponds to the proportion non-conforming at which the SSAOQL occurs or not. That is, it is checked to see whether or not the trial value of p_m satisfies the following condition.

$$AOQL - p_m^2((1-P_N+P_T)\sqrt{(n_{T\sigma}\exp(v_m^2 - w_T^2))} -$$

$$P_T(\sqrt{(n_{N\sigma}\exp(v_m^2 - w_N^2))} - \sqrt{(n_{T\sigma}\exp(v_m^2 - w_T^2))})/(1-P_N+P_T)^2)] = 0 \quad (9)$$

Equation (8) was obtained from the following relation

$$\frac{d(SSAOQ)}{dp} = P_a(p) + p\frac{dP_a(p)}{dp} = 0 \quad (10)$$

In which

$$\frac{dP_a(P)}{dp} = (1-P_N+P_T)\sqrt{(n_{T\sigma}\exp(v_m^2 - w_T^2))} -$$

$$P_T(\sqrt{(n_{\sigma}\exp(v_m^2 - w_N^2))} - \sqrt{(n_{T\sigma}\exp(v_m^2 - w_T^2))})/(1-P_N+P_T)^2 \quad (11)$$

If the assumed value of p_m does not satisfy (10), then another trial value of p_m is obtained from (10) by numerical methods. The methods of successive substation is often found to give good results and (10) is rewritten for this purpose as

$$p_m = (-AOQL)/(p_m((1-P_N+P_T)\sqrt{(n_{T\sigma}\exp(v_m^2 - w_T^2))} -$$

$$P_T(\sqrt{(n_{\sigma}\exp(v_m^2 - w_N^2))} - \sqrt{(n_{T\sigma}\exp(v_m^2 - w_T^2))})/(1-P_N+P_T)^2) \quad (12)$$

After determining the next trial value of p_m, again the values of v_m, w_{Nm}, w_{Tm} and n_σ are found and the condition (14) rechecked. This iterative procedure continues until the convergence of p_m is achieved. Then the value of k_σ is obtained from (3) and (4).

For obtaining the values of v_1, w_N and w_T, the approximation for the ordinate of the cumulative normal distribution available in Abramowitz and Stegun (1972) was used.

The S-method plans matching the σ-method plans were obtained using computer search routine through C++ programme. For selected combinations of SSAQL and SSAOQL, Table 1 was constructed following the above iterative procedure.

The iterative procedure given above may also be used to determine a Six Sigma Quick Switching Sampling System for given Six Sigma Indifference Quality Level (SSIQL) and SSAOQL as well as for given Six Sigma Limiting Quality Level(SSLQL) and SSAOQL with appropriate auxiliary variables v, w_N and w_T.

Table 1: SSQSVSS with known and unknown σ indexed by SSAQL and SSAOQL ($n_T = mn_N$, m=2)

SSAQL	SSAOQL	n_T	n_N	k	σ – level	n_T	n_N	k	σ - level
0.000001	0.000002	2138	1069	4.491	4.1	23700	11850	4.491	4.9
	0.000003	1532	766	4.448	4.0	16688	8344	4.448	4.8
	0.000004	1448	724	4.418	4.0	15580	7790	4.418	4.8
	0.000005	1218	609	4.389	3.9	12950	6475	4.389	4.7
	0.000006	996	498	4.368	3.9	10498	5249	4.368	4.6
	0.000007	784	392	4.352	3.8	8209	4104	4.352	4.6
	0.000008	758	379	4.337	3.8	7887	3944	4.337	4.6
	0.000009	556	278	4.323	3.7	5752	2876	4.323	4.5
	0.00001	346	173	4.311	3.5	3561	1781	4.312	4.5
0.000002	0.000003	1685	843	4.336	4.1	17528	8764	4.336	4.8
	0.000004	1593	796	4.306	4.0	16360	8180	4.306	4.8
	0.000005	1340	670	4.277	4.0	13595	6797	4.277	4.7
	0.000006	1096	548	4.256	3.9	11019	5509	4.256	4.7
	0.000007	862	431	4.240	3.8	8615	4307	4.240	4.6
	0.000008	834	417	4.225	3.8	8276	4138	4.225	4.6
	0.000009	612	306	4.211	3.7	6034	3017	4.211	4.5
	0.00001	381	190	4.199	3.5	3736	1868	4.200	4.5

0.000003	0.000004	2230	1115	4.224	4.2	22124	11062	4.224	4.9
	0.000005	1876	938	4.194	4.1	18373	9187	4.194	4.8
	0.000006	1534	767	4.165	4.0	14838	7419	4.165	4.8
	0.000007	1207	604	4.144	4.0	11575	5787	4.144	4.7
	0.000008	1167	584	4.128	4.0	11114	5557	4.128	4.7
	0.000009	856	428	4.113	3.8	8099	4049	4.113	4.6
	0.00001	533	266	4.099	3.7	5009	2505	4.100	4.5
0.000004	0.000005	4314	2157	4.082	4.4	40259	20129	4.082	5.0
	0.000006	2761	1380	4.053	4.3	25439	12719	4.053	4.9
	0.000007	2173	1087	4.032	4.2	19839	9920	4.032	4.8

Table 1: (continued…)

SSAQL	SSAOQL	n_T	n_N	k	σ – level	n_T	n_N	k	σ – level
0.000004	0.000008	2101	1051	4.016	4.2	19046	9523	4.016	4.8
	0.000009	1541	771	4.001	4.1	13878	6939	4.001	4.8
	0.00001	959	480	3.987	3.9	8583	4291	3.987	4.6
	0.00002	316	158	3.975	3.5	2813	1406	3.976	4.5
0.000005	0.000006	7731	3865	3.941	4.6	67767	33884	3.941	5.2
	0.000007	6085	3043	3.920	4.5	52840	26420	3.920	5.1
	0.000008	5883	2942	3.904	4.5	50720	25360	3.904	5.1
	0.000009	4315	2158	3.889	4.4	36951	18476	3.889	5.0
	0.00001	2686	1343	3.875	4.3	22849	11424	3.875	4.9
	0.00002	885	442	3.863	3.9	7487	3744	3.863	4.6
	0.00003	714	357	3.781	3.8	5818	2909	3.781	4.5
	0.00004	605	303	3.732	3.7	4822	2411	3.732	4.5
	0.00005	481	241	3.695	3.7	3766	1883	3.696	4.5
0.00001	0.00002	973	487	3.751	3.9	7821	3910	3.751	4.6
	0.00003	785	393	3.669	3.9	6073	3036	3.669	4.5
	0.00004	666	333	3.620	3.8	5030	2515	3.620	4.5
	0.00005	529	265	3.583	3.7	3927	1964	3.584	4.5
0.00005	0.00006	1608	804	3.377	4.1	10779	5390	3.377	4.7
	0.00007	1272	636	3.355	4.1	8430	4215	3.355	4.7
	0.00008	1003	501	3.335	4.0	6580	3290	3.335	4.6
	0.00009	659	330	3.301	3.8	4252	2126	3.302	4.5
	0.0001	505	252	2.969	3.8	2729	1365	2.970	4.5

Table 2: $P_a\,(p_m)$ Values of known σ plans

SSAOQL	SSAQL						
	0.000001	0.000002	0.000003	0.000004	0.000005	0.00001	0.00005
0.000002	0.91						
0.000003	0.89	0.91					
0.000004	0.84	0.86	0.89				
0.000005	0.81	0.83	0.86	0.89			
0.000006	0.78	0.80	0.83	0.86	0.89		
0.000007	0.76	0.78	0.81	0.84	0.87		
0.000008	0.72	0.74	0.77	0.80	0.83		
0.000009	0.71	0.73	0.76	0.79	0.82		
0.00001	0.63	0.65	0.68	0.71	0.74		
0.00002	0.61	0.63	0.66	0.69	0.72	0.75	
0.00003	0.58	0.60	0.63	0.66	0.69	0.72	
0.00004	0.54	0.56	0.59	0.62	0.65	0.68	
0.00005	0.51	0.53	0.56	0.59	0.62	0.65	
0.00006		0.41	0.44	0.47	0.50	0.53	0.58
0.00007			0.38	0.41	0.44	0.47	0.52
0.00008				0.35	0.38	0.41	0.46
0.00009					0.32	0.35	0.40
0.0001					0.29	0.32	0.37
0.0002					0.23	0.26	0.31
0.0003						0.22	0.27
0.0004							0.25
0.0005							0.21

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