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Efficiency comparison of system GMM estimators through Kantorovich inequality upper bounds

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Abstract
This paper compares the efficiency of system generalized method of moments (GMM) estimator and the new system GMM estimator and also assesses the potential loss of efficiency of one-step system GMM estimator and new one-step system GMM estimator compared to their respective two-step GMM estimators by computing the Kantorovich Inequality Upper Bounds (KIUB). The KIUB is computed using the weight matrices of the one-step and the two-step GMM estimators. Here, the weight matrix of two-step system GMM estimator is computed using the one-step system GMM estimator without using limiting property. Through Monte-Carlo simulation we observe that the system GMM estimator involving new initial weight matrix has a minimum loss of efficiency compared to the system GMM estimator involving conventional initial weight matrix.

Keywords: Autoregressive panel data model, system GMM estimator, Kantorovich inequality, Monte Carlo simulation

1. Introduction

Magness and McGuire (1962) [4] used the Kantorovich Inequality Upper Bounds (KIUB) to determine the efficiency of OLS estimator compared to the best linear unbiased estimators for the vector-regressor case in the general linear model. The KIUB for matrix-regressor case including single positive definite matrix is given by Marshall and Olkin (1990) [5] and Baksalary and Puntanen (1991) [1]. Later, the KIUB with two positive definite matrices is presented by Wang and Shao (1992) [6].

In the panel data context Liu and Neudecker (1997) [3] present the new multivariate versions of the KIUB with two positive definite matrices for Minimum distance (MD) estimator, Generalized Method of Moments (GMM) estimator and Weighted Least Square (WLS) estimator and are applied to compare the inefficient and efficient estimators.

Windmeijer (2000) [7] has used the approach of Liu and Neudecker (1997) [3] and computed KIUB using the weight matrices of one-step and its respective two-step system GMM estimator, where the weight matrix of two-step system GMM estimator is computed based on the asymptotic properties ($\psi_s = plim\, W_{s2}^{-1}$, where W_{s2}^{-1} is the weight matrix of one-step system GMM estimator) and it is observed that KIUB values increases for the case $T \geq 4$.

In this paper, the KIUB are used to obtain the efficiency loss of one-step system and new one-step system GMM estimators compared to their respective efficient two-step estimators. The KIUB are computed for the one-step system GMM estimator with conventional weight matrix and the new one-step system GMM estimator with new weight matrix. The weight matrix of two-step system GMM estimator is computed without using the asymptotic property ($\psi_s = W_{s2}^{-1}$).

The rest of the paper is as follows. Section 2 provides the AR(1) panel data model, its assumptions, system GMM estimator and New system GMM estimator. Efficiency loss of system and new system GMM estimators is presented in Section 3. Monte Carlo simulation design, results and discussions are given in Section 4. Section 5 presents some concluding remarks.

2. AR(1) panel data model and assumptions

The first-order autoregressive panel data model is given by,

$$y_{it} = \delta y_{it-1} + u_{it}, i = 1,2,\dots,N; t = 2,3,\dots,T. \quad (1)$$
$$u_{it} = \eta_i + v_{it}$$

Where, η_i are individual effects, v_{it} are idiosyncratic errors, N is number of individuals, T is time period.
The initial value is given by,

$$y_{i1} = \frac{\eta_i}{1-\delta} + w_{i1}, \text{ for } i = 1,2,\dots,N. \quad (2)$$

Where, $w_{i1} = \sum_{j=0}^{\infty} \delta^j v_{i1-j}$ and independent of η_i. It is assumed that,

$$E(\eta_i) = E(v_{it}) = E(\eta_i v_{it}) = E(y_{i1} v_{it}) = 0, i = 1,2,\dots,N; t = 2,3,\dots,T.$$

$$E(v_{it} v_{is}) = 0, i = 1,2,\dots,N; t \neq s.$$

And

$$V(\eta_i) = \sigma_\eta^2, V(v_{it}) = \sigma_v^2, V(u_{it}) = \sigma_u^2, i = 1,2,\dots,N; t = 2,3,\dots,T.$$

2.1 System GMM Estimator

Under the above-mentioned assumptions, Blundell and Bond (1998) [2] considered system GMM estimator in which the moment conditions of system GMM estimator are the combination of the moment condition of first-difference and level GMM estimators and is given by,

$$E(Z'_{si} u_{si}) = 0 \quad (3)$$

Where,

$$Z_{si} = \begin{bmatrix} Z_{di} & 0 \\ 0 & Z_{li} \end{bmatrix}; \; u_{si} = \begin{bmatrix} \Delta u_i \\ u_i \end{bmatrix}$$

With,

$$Z_{di} = \begin{bmatrix} y_{i1} & 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & y_{i1} & y_{i2} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & y_{i1} & \dots & y_{iT-2} \end{bmatrix}; \; Z_{li} = \begin{bmatrix} \Delta y_{i2} & 0 & \dots & 0 \\ 0 & \Delta y_{i3} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Delta y_{iT-1} \end{bmatrix}$$

And

$$\Delta u_i = \begin{bmatrix} \Delta u_{i3} \\ \Delta u_{i4} \\ \vdots \\ \Delta u_{iT} \end{bmatrix}; \; u_i = \begin{bmatrix} u_{i3} \\ u_{i4} \\ \vdots \\ u_{iT} \end{bmatrix}$$

The one-step system GMM estimator is given by,

$$\hat{\delta}_{1sys} = (s'_{-1} Z_s W_{s1} Z'_s s_{-1})^{-1} (s'_{-1} Z_s W_{s1} Z'_s s) \quad (4)$$

Where, $s'_{i,-1}$ is the $1 \times 2(T-2)$ vector $(\Delta y_{i2}, \dots, \Delta y_{iT-1}, y_{i2}, \dots, y_{iT-1})$, s'_i is the $1 \times 2(T-2)$ vector $(\Delta y_{i3}, \dots, \Delta y_{iT}, y_{i3}, \dots, y_{iT})$, s_{-1} and s are stacked across individuals. $W_{s1} = \left(N^{-1} \sum_{i=1}^{N} Z'_{si} A_s Z_{si}\right)^{-1}$, is a $\left(\frac{(T-2)(T-1)}{2}\right) + (T-2) \times \left(\frac{(T-2)(T-1)}{2}\right) + (T-2)$ weight matrix. Where, $A_s = \begin{bmatrix} A_d & 0 \\ 0 & A_l \end{bmatrix}$ with A_d is a $(T-2) \times (T-2)$ matrix with 2's on the main diagonal, -1's on the first sub-diagonals and zeros otherwise and A_l is a $(T-2) \times (T-2)$ diagonal matrix.
The efficient two-step system GMM estimator is based on the following weight matrix:

$$W_{s2} = \left(\frac{1}{N} \sum_{i=1}^{N} Z'_{si} \hat{u}_{si} \hat{u}'_{si} Z_{si}\right)^{-1} \quad (5)$$

Where, $\hat{u}_{si}$ is a residual obtained from the one-step system GMM estimator $\hat{\delta}_{1sys}$.

2.2 New System GMM Estimator

The new one-step system GMM estimator considers the new weight matrix (W_{ns1}), instead of conventional weight matrix (W_{s1}) and is given by,

$$W_{ns1} = \left(N^{-1} \sum_{i=1}^{N} Z'_{si} A_{ns} Z_{si}\right)^{-1} \quad (6)$$

Where, $A_s = \begin{bmatrix} A_d & 0 \\ 0 & A_l \end{bmatrix}$ with A_{nl} is a $(T-2) \times (T-2)$ matrix with σ_u^2 on the main diagonal and σ_η^2 otherwise.

The new two-step system GMM estimator is based on the following weight matrix:

$$W_{ns2} = \left(\frac{1}{N} \sum_{i=1}^{N} Z'_{si}\, \hat{u}_{nsi}\, \hat{u}'_{nsi}\, Z_{si}\right)^{-1} \quad (7)$$

where, $\hat{u}_{nsi}$ is a residual obtained from the new one-step system GMM estimator.

3. Efficiency Comparison

To detect which initial weight matrix is to be used to obtain the efficient one-step system GMM estimator, without calculating its variances, the KIUBs are calculated to compare the efficiency difference between the one-step and two-step system GMM estimators.

Let us define,

$$\gamma = \frac{\sigma_\eta^2}{(1-\delta)^2} + \frac{\delta \sigma_v^2}{1-\delta^2};\ \sigma_y^2 = \frac{\sigma_\eta^2}{(1-\delta)^2} + \frac{\sigma_v^2}{1-\delta^2}$$

3.1 Efficiency comparison of the one-step system GMM estimator compared to its efficient two-step estimator

Expression for the efficiency comparison between one-step system GMM estimator and its efficient two-step estimator is obtained using the initial weight matrix W_{s1}, which is similar to the expression derived by Liu and Neudecker (1997) [3] and is given by,

$$(G_0' W_{s1} G_0)^{-1} G_0' W_{s1} \psi_s W_{s1} G_0 (G_0' W_{s1} G_0)^{-1} \leq \frac{(\lambda_{B_1} + \lambda_{B_m})^2}{4 \lambda_{B_1} \lambda_{B_m}} (G_0' \psi_s G_0)^{-1} \quad (2)$$

Where $G_0' = (s'_{-1} Z_s)$ and $\lambda_{B_1} \geq \cdots \geq \lambda_{B_m}$ are the eigenvalues of the matrix $\psi_s W_{s1}$ with $m = (\frac{(T-2)(T-1)}{2}) + (T-2)$. The two-step estimator is asymptotically efficient but not the one-step estimator, since ψ_s is based on the one-step estimator, we do not calculate it by using asymptotic property ($\psi_s^{-1} = \text{plim}_{N\to\infty} W_{s2}$).

Now ψ_s is given by,

$$\psi_s = \left(\frac{1}{N} \sum_{i=1}^{N} Z'_{si}\, \hat{u}_{si}\, \hat{u}'_{si}\, Z_{si}\right)$$

$$\psi_s = \begin{bmatrix} \sigma_y^2 E(\Delta\hat{u}_{i3})^2 & \sigma_y^2 E(\Delta\hat{u}_{i3}\Delta\hat{u}_{i4}) & \gamma E(\Delta \hat{u}_{i3}\Delta\hat{u}_{i4}) & -\frac{\sigma_v^2}{1+\delta} E(\hat{u}_{i3}\Delta\hat{u}_{i3}) & -\frac{\delta\sigma_v^2}{1+\delta} E(\hat{u}_{i4}\Delta\hat{u}_{i3}) \\ \sigma_y^2 E(\Delta\hat{u}_{i3}\Delta\hat{u}_{i4}) & \sigma_y^2 E(\Delta\hat{u}_{i4})^2 & \gamma E(\Delta\hat{u}_{i4})^2 & -\frac{\sigma_v^2}{1+\delta} E(\hat{u}_{i3}\Delta\hat{u}_{i4}) & -\frac{\delta\sigma_v^2}{(1+\delta)} E(\hat{u}_{i4}\Delta\hat{u}_{i4}) \\ \gamma E(\Delta \hat{u}_{i3}\Delta\hat{u}_{i4}) & \gamma E(\Delta\hat{u}_{i4})^2 & \sigma_y^2 E(\Delta\hat{u}_{i4})^2 & \frac{\sigma_v^2}{(1+\delta)} E(\hat{u}_{i3}\Delta\hat{u}_{i4}) & -\frac{\sigma_v^2}{1+\delta} E(\hat{u}_{i4}\Delta\hat{u}_{i4}) \\ -\frac{\sigma_v^2}{1+\delta} E(\hat{u}_{i3}\Delta\hat{u}_{i3}) & -\frac{\sigma_v^2}{1+\delta} E(\hat{u}_{i3}\Delta\hat{u}_{i4}) & \frac{\sigma_v^2}{1+\delta} E(\hat{u}_{i3}\Delta\hat{u}_{i4}) & \frac{2\sigma_v^2}{1+\delta} E(\Delta\hat{u}_{i3})^2 & \frac{(\delta-1)\sigma_v^2}{1+\delta} E(\hat{u}_{i3}\hat{u}_{i4}) \\ -\frac{\delta\sigma_v^2}{1+\delta} E(\hat{u}_{i4}\Delta\hat{u}_{i3}) & -\frac{\delta\sigma_v^2}{(1+\delta)} E(\hat{u}_{i4}\Delta\hat{u}_{i4}) & -\frac{\sigma_v^2}{1+\delta} E(\hat{u}_{i4}\Delta\hat{u}_{i4}) & \frac{(\delta-1)\sigma_v^2}{1+\delta} E(\hat{u}_{i3}\hat{u}_{i4}) & \frac{2\sigma_v^2}{1+\delta} E(\Delta\hat{u}_{i4})^2 \end{bmatrix}$$

Further,

$$W_{s1} = \left(N^{-1} \sum_{i=1}^{N} Z'_{si} A_s Z_{si}\right)^{-1}$$

$$W_{s1} = \begin{bmatrix} \frac{2}{3\sigma_y^2} & \frac{1}{3\sigma_y^2} & 0 & 0 & 0 \\ \frac{1}{3\sigma_y^2} & \frac{(4\sigma_y^4 - \gamma^2)}{6\sigma_y^2(\sigma_y^4 - \gamma^2)} & -\frac{\gamma}{2(\sigma_y^4 - \gamma^2)} & 0 & 0 \\ 0 & -\frac{\gamma}{2(\sigma_y^4 - \gamma^2)} & \frac{\sigma_y^2}{2(\sigma_y^4 - \gamma^2)} & 0 & 0 \\ 0 & 0 & 0 & \frac{(1+\delta)}{2\sigma_v^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{(1+\delta)}{2\sigma_v^2} \end{bmatrix}$$

Now, the matrix $B = \psi_s W_{s1}$ is given by,

$$\left[\begin{matrix} \frac{2\,E(\Delta\hat{u}_{i3})^2 + E(\Delta\hat{u}_{i3}\Delta\hat{u}_{i4})}{3} & \frac{E(\Delta\hat{u}_{i3})^2 + 2E(\Delta\hat{u}_{i3}\Delta\hat{u}_{i4})}{3} \\ \frac{2E(\Delta\hat{u}_{i3}\Delta\hat{u}_{i4}) + E(\Delta\hat{u}_{i4})^2}{3} & \frac{E(\Delta\hat{u}_{i3}\Delta\hat{u}_{i4}) + 2E(\Delta\hat{u}_{i4})^2}{3} \\ \frac{\gamma[2E(\Delta\hat{u}_{i3}\Delta\hat{u}_{i4}) + E(\Delta\hat{u}_{i4})^2]}{3\sigma_y^2} & \frac{\gamma[2E(\Delta\hat{u}_{i3}\Delta\hat{u}_{i4}) + E(\Delta\hat{u}_{i4})^2]}{6\sigma_y^2} \\ \frac{-\sigma_v^2[2E(\hat{u}_{i3}\Delta\hat{u}_{i3}) + E(\hat{u}_{i3}\Delta\hat{u}_{i4})]}{3\sigma_y^2(1+\delta)} & \left[\frac{E(\hat{u}_{i3}\Delta\hat{u}_{i3})}{3\sigma_y^2} + \frac{(4\sigma_y^4 - \gamma^2 + 3\gamma\sigma_y^2)E(\hat{u}_{i3}\Delta\hat{u}_{i4})}{6\sigma_y^2(\sigma_y^4 - \gamma^2)}\right]\left(-\frac{\sigma_v^2}{1+\delta}\right) \\ \frac{-\delta\sigma_v^2[2E(\hat{u}_{i4}\Delta\hat{u}_{i3}) + E(\hat{u}_{i4}\Delta\hat{u}_{i4})]}{3\sigma_y^2(1+\delta)} & \left[\frac{E(\hat{u}_{i4}\Delta\hat{u}_{i3})}{3\sigma_y^2} + \frac{(4\delta\sigma_y^4 - \delta\gamma^2 - 3\gamma\sigma_y^2)E(\hat{u}_{i4}\Delta\hat{u}_{i4})}{6\sigma_y^2(\sigma_y^4 - \gamma^2)}\right]\left(-\frac{\sigma_v^2}{1+\delta}\right) \end{matrix}\right.$$

$$\left.\begin{matrix} 0 & -\frac{E(\hat{u}_{i3}\Delta\hat{u}_{i3})}{2} & -\frac{\delta E(\hat{u}_{i4}\Delta\hat{u}_{i3})}{2} \\ 0 & -\frac{E(\hat{u}_{i3}\Delta\hat{u}_{i4})}{2} & -\frac{\delta E(\hat{u}_{i4}\Delta\hat{u}_{i4})}{2} \\ \frac{E(\Delta\hat{u}_{i4})^2}{2} & \frac{E(\hat{u}_{i3}\Delta\hat{u}_{i4})}{2} & -\frac{E(\hat{u}_{i4}\Delta\hat{u}_{i4})}{2} \\ \frac{\sigma_v^2 E(\hat{u}_{i3}\,\Delta\hat{u}_{i4})}{2\,(1+\delta)(\sigma_y^2 - \gamma)} & E(\Delta\hat{u}_{i3})^2 & \frac{(\delta - 1)E(\hat{u}_{i3}\hat{u}_{i4})}{2} \\ \frac{\sigma_v^2(\delta\gamma - \sigma_y^2)E(\hat{u}_{i4}\,\Delta\hat{u}_{i4})}{2\,(1+\delta)(\sigma_y^4 - \gamma^2)} & \frac{(\delta - 1)E(\hat{u}_{i3}\hat{u}_{i4})}{2} & E(\Delta\hat{u}_{i4})^2 \end{matrix}\right]$$

Now, compute the eigen values of matrix B and substitute in the expression $\frac{(\lambda_{B_1} + \lambda_{B_m})^2}{4\lambda_{B_1}\lambda_{B_m}}$, which is the required KIUB.

3.2 Efficiency comparison of the new one-step system GMM estimator compared to its efficient two-step estimator

Expression for the efficiency comparison between new one-step system GMM estimator and its efficient two-step estimator is obtained using the initial weight matrix W_{ns1} is derived and is given by,

$$(G_0' W_{ns1} G_0)^{-1} G_0' W_{ns1} \psi_{ns} W_{ns1} G_0 (G_0' W_{ns1} G_0)^{-1} \leq \frac{(\lambda_{B_{n1}} + \lambda_{B_{nm}})^2}{4\lambda_{B_{n1}}\lambda_{B_{nm}}} (G_0' \psi_{ns} G_0)^{-1} \quad (9)$$

Where, $\lambda_{B_{n1}} \geq \cdots \geq \lambda_{B_{nm}}$ are the eigenvalues of the matrix $\psi_{ns} W_{ns1}$.

Now ψ_s is given by,

$$\psi_{ns} = \left(\frac{1}{N}\sum_{i=1}^{N} Z_{si}'\,\hat{u}_{nsi}\,\hat{u}_{nsi}'\,Z_{si}\right)$$

$$\psi_{ns}=\begin{bmatrix} \sigma_y^2\,E(\Delta\hat{u}_{ni3})^2 & \sigma_y^2\,E(\Delta\hat{u}_{ni3}\Delta\hat{u}_{ni4}) & \gamma\,E(\Delta\,\hat{u}_{ni3}\Delta\hat{u}_{ni4}) & -\frac{\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni3}\Delta\hat{u}_{ni3}) & -\frac{\delta\sigma_v^2}{1+\delta}E(\hat{u}_{ni4}\Delta\hat{u}_{ni3}) \\ \sigma_y^2\,E(\Delta\hat{u}_{ni3}\Delta\hat{u}_{ni4}) & \sigma_y^2\,E(\Delta\hat{u}_{ni4})^2 & \gamma\,E(\Delta\hat{u}_{ni4})^2 & -\frac{\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni3}\Delta\hat{u}_{ni4}) & -\frac{\delta\sigma_v^2}{(1+\delta)}E(\hat{u}_{ni4}\Delta\hat{u}_{ni4}) \\ \gamma\,E(\Delta\,\hat{u}_{ni3}\Delta\hat{u}_{ni4}) & \gamma\,E(\Delta\hat{u}_{ni4})^2 & \sigma_y^2\,E(\Delta\hat{u}_{ni4})^2 & \frac{\sigma_v^2}{(1+\delta)}\,E(\hat{u}_{ni3}\Delta\hat{u}_{ni4}) & -\frac{\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni4}\Delta\hat{u}_{ni4}) \\ -\frac{\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni3}\Delta\hat{u}_{ni3}) & -\frac{\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni3}\Delta\hat{u}_{ni4}) & \frac{\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni3}\Delta\hat{u}_{ni4}) & \frac{2\sigma_v^2}{1+\delta}\,E(\Delta\hat{u}_{ni3})^2 & \frac{(\delta\,-1)\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni3}\hat{u}_{ni4}) \\ -\frac{\delta\sigma_v^2}{1+\delta}E(\hat{u}_{ni4}\Delta\hat{u}_{ni3}) & -\frac{\delta\sigma_v^2}{(1+\delta)}E(\hat{u}_{ni4}\Delta\hat{u}_{ni4}) & -\frac{\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni4}\Delta\hat{u}_{ni4}) & \frac{(\delta\,-1)\sigma_v^2}{1+\delta}\,E(\hat{u}_{ni3}\hat{u}_{ni4}) & \frac{2\sigma_v^2}{1+\delta}\,E(\Delta\hat{u}_{ni4})^2 \end{bmatrix}$$

Further,

$$\mathrm{W_{ns1}}=\left(\mathrm{N}^{-1}\sum_{\mathrm{i=1}}^{\mathrm{N}}\mathrm{Z'_{si}A_{ns}\,Z_{si}}\right)^{-1}$$

$$=\begin{bmatrix} \frac{2}{3\sigma_y^2} & \frac{1}{3\sigma_y^2} & 0 & 0 & 0 \\ \frac{1}{3\sigma_y^2} & \frac{(4\sigma_y^4-\gamma^2)}{6\sigma_y^2(\sigma_y^4-\gamma^2)} & -\frac{\gamma}{2(\sigma_y^4-\gamma^2)} & 0 & 0 \\ 0 & -\frac{\gamma}{2(\sigma_y^4-\gamma^2)} & \frac{\sigma_y^2}{2(\sigma_y^4-\gamma^2)} & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\delta)\sigma_\eta^2\sigma_u^2}{\sigma_v^2[4\sigma_u^2-\sigma_\eta^2(\delta-1)^2]} & \frac{(1-\delta^2)\sigma_\eta^4}{\sigma_v^2[4\sigma_u^2-\sigma_\eta^2(\delta-1)^2]} \\ 0 & 0 & 0 & \frac{(1-\delta^2)\sigma_\eta^4}{\sigma_v^2[4\sigma_u^2-\sigma_\eta^2(\delta-1)^2]} & \frac{2(1+\delta)\sigma_\eta^2\sigma_u^2}{\sigma_v^2[4\sigma_u^2-\sigma_\eta^2(\delta-1)^2]} \end{bmatrix}$$

Now, the matrix $B_n=\psi_{ns}W_{ns1}$ is given by,

$$B_n=\begin{bmatrix} \frac{2\,E(\Delta\hat{u}_{ni3})^2+E(\Delta\hat{u}_{ni3}\Delta\hat{u}_{ni4})}{3} & \frac{E(\Delta\hat{u}_{ni3})^2+2E(\Delta\hat{u}_{ni3}\Delta\hat{u}_{ni4})}{3} & 0 \\ \frac{2E(\Delta\hat{u}_{ni3}\Delta\hat{u}_{ni4})\ +E(\Delta\hat{u}_{ni4})^2}{3} & \frac{E(\Delta\hat{u}_{ni3}\Delta\hat{u}_{ni4})\ +2E(\Delta\hat{u}_{ni4})^2}{3} & 0 \\ \frac{\gamma[2E(\Delta\hat{u}_{ni3}\Delta\hat{u}_{ni4})\ +E(\Delta\hat{u}_{ni4})^2]}{3\sigma_y^2} & \frac{\gamma[2E(\Delta\hat{u}_{ni3}\Delta\hat{u}_{ni4})\ +E(\Delta\hat{u}_{ni4})^2]}{6\sigma_y^2} & \frac{E(\Delta\hat{u}_{ni4})^2}{2} \\ \frac{-\sigma_v^2[2E(\hat{u}_{ni3}\Delta\hat{u}_{ni3})\ +E(\hat{u}_{ni3}\Delta\hat{u}_{ni4})\,]}{3\sigma_y^2(1+\delta)} & \left[\frac{E(\hat{u}_{ni3}\Delta\hat{u}_{ni3})}{3\sigma_y^2}+\frac{(4\sigma_y^4-\gamma^2+3\gamma\sigma_y^2)E(\hat{u}_{ni3}\Delta\hat{u}_{ni4})}{6\sigma_y^2(\sigma_y^4-\gamma^2)}\right]\left(-\frac{\sigma_v^2}{1+\delta}\right) & \frac{\sigma_v^2E(\hat{u}_{ni3}\,\Delta\hat{u}_{ni4})}{2\,(1+\delta)(\sigma_y^2-\gamma)} \\ \frac{-\delta\sigma_v^2[2E(\hat{u}_{ni4}\Delta\hat{u}_{ni3})\ +E(\hat{u}_{ni4}\Delta\hat{u}_{ni4})\,]}{3\sigma_y^2(1+\delta)} & \left[\frac{E(\hat{u}_{ni4}\Delta\hat{u}_{ni3})}{3\sigma_y^2}+\frac{(4\delta\sigma_y^4-\delta\gamma^2-3\gamma\sigma_y^2)E(\hat{u}_{ni4}\Delta\hat{u}_{ni4})}{6\sigma_y^2(\sigma_y^4-\gamma^2)}\right]\left(-\frac{\sigma_v^2}{1+\delta}\right) & \frac{\sigma_v^2(\delta\gamma-\sigma_y^2)E(\hat{u}_{ni4}\,\Delta\hat{u}_{ni4})}{2\,(1+\delta)(\sigma_y^4-\gamma^2)} \end{bmatrix}$$

$$-\frac{\sigma_\eta^2\left[2\sigma_u^2E(\hat{u}_{ni3}\Delta\hat{u}_{ni3})+\sigma_\eta^2\delta(1-\delta)E(\hat{u}_{ni4}\Delta\hat{u}_{ni3})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}$$
$$-\frac{\sigma_\eta^2\left[2\sigma_u^2E(\hat{u}_{ni3}\Delta\hat{u}_{ni4})+\sigma_\eta^2\delta(1-\delta)E(\hat{u}_{ni4}\Delta\hat{u}_{ni4})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}$$
$$\frac{\sigma_\eta^2\left[2\sigma_u^2E(\hat{u}_{ni3}\Delta\hat{u}_{ni4})-\sigma_\eta^2(1-\delta)\,E(\hat{u}_{ni4}\Delta\hat{u}_{ni4})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}$$
$$\frac{\sigma_\eta^2\left[4\sigma_u^2E(\hat{u}_{ni3})^2-\sigma_\eta^2(1-\delta)^2E(\hat{u}_{ni3}\hat{u}_{ni4})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}$$
$$\frac{2(\delta-1)\sigma_\eta^2\left[\sigma_u^2E(\hat{u}_{ni3}\hat{u}_{ni4})-\sigma_\eta^2E(\hat{u}_{ni4})^2\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}$$

$$\left.\begin{matrix}\frac{\sigma_\eta^2\left[\sigma_\eta^2(\delta-1)E(\hat{u}_{ni3}\Delta\hat{u}_{ni3})-2\delta\sigma_u^2E(\hat{u}_{ni4}\Delta\hat{u}_{ni3})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}\\ \frac{\sigma_\eta^2\left[\sigma_\eta^2(\delta-1)E(\hat{u}_{ni3}\Delta\hat{u}_{ni4})-2\delta\sigma_u^2E(\hat{u}_{ni4}\Delta\hat{u}_{ni4})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}\\ \frac{\sigma_\eta^2\left[\sigma_\eta^2(1-\delta)E(\hat{u}_{ni3}\Delta\hat{u}_{ni4})-2\sigma_u^2E(\hat{u}_{ni4}\Delta\hat{u}_{ni4})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}\\ \frac{\sigma_\eta^2\left[2\sigma_\eta^2(1-\delta)E(\hat{u}_{ni3})^2-2(\delta-1)\sigma_u^2E(\hat{u}_{ni3}\hat{u}_{ni4})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}\\ \frac{\sigma_\eta^2\left[4\sigma_u^2E(\hat{u}_{ni4})^2-\sigma_\eta^2(1-\delta)^2E(\hat{u}_{ni3}\hat{u}_{ni4})\right]}{\left[4\sigma_u^4-\sigma_\eta^4(\delta-1)^2\right]}\end{matrix}\right]$$

Now, compute the eigen values of matrix B and substitute in the expression $\frac{(\lambda_{B_{n1}}+\lambda_{B_{nm}})^2}{4\lambda_{B_{n1}}\lambda_{B_{nm}}}$, which is the required KIUB.

4. Simulation Design

To investigate the finite sample performance of the above-mentioned estimators we carry out the Monte Carlo simulation. The data generating process for the autoregressive panel data model is as follows:

$$y_{it}=\delta y_{it-1}+\eta_i+v_{it}$$

$$y_{i1}=\frac{\eta_i}{1-\delta}+w_{i1}$$

$$w_{i1}\sim N\left(0,\frac{\sigma_v^2}{1-\delta^2}\right);\ \eta_i\sim N(0,\sigma_\eta^2);\ v_{it}\sim N(0,\sigma_v^2)$$

The number of individuals $N=50,100$, time period $T=4$, variance of idiosyncratic error term $\sigma_v^2=1$, values of parameter $\delta=\{0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9\}$ and variance of individual effects $\sigma_\eta^2=\left\{\frac{1}{4},1,4\right\}$ and the results are based on 10000 replications.

5. Results and Discussion

Table 1: Kantorovich inequality upper bounds of one-step system GMM estimator

		δ								
vr	***N***	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1/4	50	2.34	2.17	2.03	1.91	1.80	1.71	1.62	1.53	1.48
	100	2.36	2.20	2.05	1.94	1.83	1.75	1.67	1.57	1.49
1	50	1.93	1.82	1.73	1.66	1.60	1.55	1.53	1.52	1.64
	100	1.94	1.83	1.73	1.66	1.61	1.56	1.53	1.52	1.58
4	50	2.47	2.32	2.17	2.03	1.91	1.78	1.71	1.72	1.81
	100	2.58	2.44	2.29	2.20	2.09	1.94	1.83	1.75	1.95

Table 2: Kantorovich inequality upper bounds of new one-step system GMM estimator

		δ								
vr	N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1/4	50	2.31	2.14	2.01	1.88	1.76	1.66	1.57	1.46	1.38
	100	2.33	2.17	2.03	1.91	1.80	1.71	1.63	1.52	1.40
1	50	1.63	1.54	1.47	1.41	1.36	1.31	1.27	1.25	1.25
	100	1.63	1.55	1.49	1.43	1.38	1.33	1.29	1.25	1.24
4	50	1.22	1.19	1.16	1.15	1.13	1.12	1.11	1.11	1.10
	100	1.22	1.19	1.17	1.15	1.13	1.12	1.11	1.11	1.10

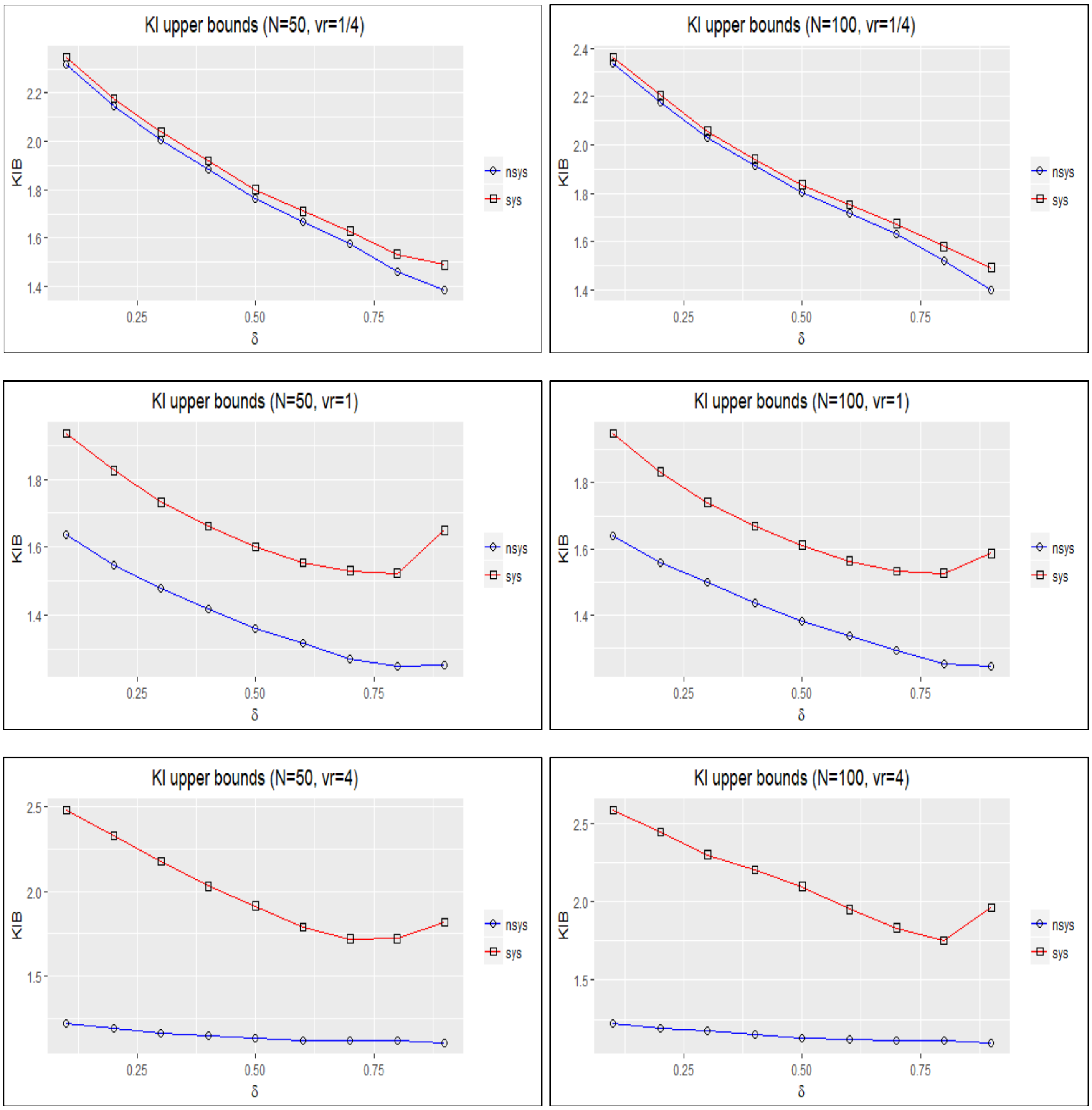

Fig 1: Efficiency comparison of one-step system GMM estimator and new one-step system GMM estimator with compared to its respective two-step estimators.

Table 1 and 2 provides the KIUBs of the one-step system GMM estimator (SYS) with the initial weight matrix W_{s1} and the new one-step system GMM estimator (NSYS) with the new initial weight matrix W_{ns1} respectively. Figure 1 presents the KIUB values of SYS and NSYS compared to their respective two-step system GMM estimators.

From Table 1 it is observed that, when $vr = 1$ ($vr =$ variance ratio i.e., $vr = \frac{\sigma_\eta^2}{\sigma_v^2}$), the KIUB values of SYS is almost same for both $N = 50$ and 100. When vr is less than or greater than 1, the KIUB values of SYS differs for both $N = 50$ and 100. Same can be observed in the case of NSYS (Table 2). From Table 2 it is also observed that as vr increases KIUB decreases.

From both Table 1 and 2 it is observed that for different values of vr, KIUB decreases with δ. For all the values of δ, vr and N, the KIUB values of NSYS are lesser than the values of SYS.

The one-step GMM estimator is said to be efficient when the KIUB value is near to one. From the above two table it is found that the KIUB values of NSYS estimator is near to 1 than the KIUB values of SYS estimator. When $vr = 4$, $\delta = 0.5$ and$N = 100$, the KIUB value of NSYS is 1.13215, indicating that the one-step NSYS estimator is 13% inefficient than its two-step NSYS estimator.

Figure 1 shows that as vr increases the difference between KIUB of SYS and NSYS increases and the same pattern is observed for both $N = 50$ and 100. For all the values of δ, vr and N, the NSYS has a minimum KIUB value than the SYS.

6. Conclusion

From the earlier study, it is known that the system GMM estimator is efficient for the autoregressive panel data model than the other GMM estimators and also known that the two-step estimator is efficient than the one-step estimator.

In the above study, we computed the KIUBs for one-step system and new system GMM estimators compared to their respective efficient two-step GMM estimators.

It is observed that the KIUB values of NSYS estimator is near to one than the SYS estimator, which means one-step NSYS estimator is efficient than the one-step SYS estimator. We conclude that it is better to use the new weight matrix $W_{ns1}$to compute one-step system GMM estimator instead of using conventional weight matrix W_{s1}.

7. References

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