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Optimization of special type double sampling plan using generalized Poisson distribution through minimum angle method

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Abstract
This paper presents a procedure for designing a Special Type Double Sampling (STDS) plan indexed through Minimum Angle Method along with Operating Characteristic (OC) curve. In sampling plan, whenever there is a declined angle θ, the more nearly the OC curve approach an ideal form and it is achieved for the proposed plan using Generalized Poisson Distribution (GPD). Suitable tables are presented for the selection of the plan through Minimum Angle Method with declination in the OC curve. The designing procedures of the plan for various consumer's and producer's quality levels are developed using Minimum Angle Method, making an operating characteristic curve approaching to the ideal form. Theoretical and numerical evaluations are given with suitable illustrations.

Keywords: Special type double sampling plan, generalized poisson model, producer's quality level, consumer's quality level, minimum angle method

1. Introduction

In the present age of increasing globalization, it is a great challenge for industrialists to fulfill the requirements of consumers on customer related products, reliable lead time and finally customer satisfaction. Today, quality control is a statistical programme to prevent free from defects during and after productions with a complete guarantee for the quality product. Sampling inspection in product control is a statistical methodology which generally works with a basic assumption that the production process from which lots are formed is stable and the lot quality defined in terms of fraction non-conforming is a fixed constant. Generally, Poisson distribution is widely applied in acceptance sampling plans since it has the property that its mean and variance are equal, which is called equidispersion. However, in most of the real life situations, the production processes are not always stable and the lots coming from such processes will have quality variations which may occur due to random fluctuations. The most appropriate probability distribution to handle this kind of situation is the Generalized Poisson Distribution (GPD) developed by Consul and Jain (1973) [1]. This is a strong requirement in real life practice that, when the product quality exhibits a dispersed behaviour, there is a need for a distribution that can handle both over dispersed and under dispersed conditions, such as the aforementioned GPD. It can fulfill the property: the variance is greater than the mean (over-dispersion) or the variance is less than the mean (under-dispersion). The density function of GPD is given as follows:

$$p_x(\lambda_1,\lambda_2)=\lambda_1(\lambda_1+x\lambda_2)^{x-1}\frac{e^{-(\lambda_1+x\lambda_2)}}{x!};x=0,1,2,\ldots$$

Where, λ_1 is the scale parameter and λ_2 is the shape parameter which determines the shape of the performance measures. Here we consider, the case $\lambda_2 > 0$, for establishing the tables. It ensures the protection for producer and consumer especially, the STDS plan is applicable in the area of safety related testing like medicine, army weapons, chemicals, etc.
The single sampling plan under the conditions of GPD developed by Kaviyarasu and Devika (2018) [4] exaggerate the need and importance of GPD based sampling plan in the area of

Product control. According to Govindaraju (1984, 1991) [2, 3], the Operating Characteristic (OC) curves of single sampling plan with smaller acceptance numbers results in conflicting interest between the producer and consumer. While single sampling plan with acceptance number Ac=0, which pays more attention towards the consumer but on the other hand acceptance number Ac=1 may favor to producer. This limitation could be overcome to a greater extent when a Special Type Double Sampling (STDS) plan is applied for sentencing the individual lots.

For any sampling plan is considered, the OC curve will determine the discriminating power existing between the producer risk and consumer risk. Here, the risk minimization could be achieved through the minimum angle method, making an OC curve approaching to the ideal form. An idealized OC curve has a greater slope; the greater is the discriminating power. Thus it emphasizes the quality manager to take decision making regarding the lot disposition and also make an alarm sense about risk minimization. Both these conditions can be achieved through the minimum angle method. Here an attempt is made to study the GPD through its twin properties of a distribution which strike out the conditions of GPD in acceptance sampling plan since it possesses the characteristics to deal with over-dispersion as well as under-dispersion of data according to $\lambda_2 > 0$ or $\lambda_2 < 0$ respectively. In this context, this paper is developed to find out its optimum parameters for the sampling plan for various producer's and consumer's quality levels through minimum angle method.

2. Conditions for Application

1. The production process is reasonably steady, so that results on current and preceding lots are broadly indicative of a continuing process.
2. Samples are taken from lot substantially in the order of production so that observed variations in quality of product reflect process performance.
3. The product comes from a source in which consumer has confidence.
4. Inspection is by attributes, with quality measured in terms of fraction defective p.

3. Operating Procedure

According to Schilling and Neubauer (2009) [8], a SSP with zero acceptance number are very useful in the area of compliance and safety related testing. To overcome the pitfalls in SSP with smaller acceptance number, a lot by lot STDS plan is used as an alternative. In the first phase of the sampling inspection the acceptance number are considered as Ac = 0 and in second phase the acceptance number is relaxed for the large sample size which may have Ac = 1. The operating procedure of STDS plan under GPD is very simple as given below:

1. A random sample of size n_1 units are taken from a lot and observe the number of defectives d_1. If $d_1 \geq 1$, reject the entire lot.
2. If $d_1 = 0$, select a second random sample of size n_2 and observe the number of defectives d_2. If $d_2 \leq 1$, accept the lot. Otherwise reject the lot.

The OC function for the GPD based STDS plan is designed and expressed as follows;

$$P_a(p) = P(d_1 = 0/n_1; p) * P(d_2 = 1/n_2; p)$$

Where d_1 is the number of defectives found in the first random sample of size n_1 and d_2 is the number of defectives found in the second random sample of size n_2 for the lot quality p.

That is, $P_a(p) = e^{-x}(1+\phi x e^{-\lambda_2})$

Here, $x = np, \varphi = n_2/n_1 + n_2$ and λ_2 is the dispersion parameter

In the literature, this paper provides a new procedure for designing GPD based STDS plan through Minimum Angle Method. Soundararajan and Christina (1997) [9] developed the single sampling plans by variables based on the minimum angle method. Suresh and Jayalakshmi (2006) [10] deliberate the tightened normal tightened scheme under minimum angle method. Suresh and Sangeetha (2010) [11] have studied one plan suspension system through the minimum angle method. Kaviyarasu and Manikandan (2016) [5] have studied the quick switching multiple deferred sampling systems through minimum angle method.

4. Selection Procedure

Selection procedure for Special Type Double Sampling plan using Minimum Angle method under the condition of Generalized Poisson Distribution

Norman Bush *et al.* (1953) [6] used different techniques to describe the direction of the operating characteristic (OC) curve through Minimum Angle Method. When producer and consumer are negotiating for designing sampling plans, it is important especially to minimize the consumer risk. Further they have considered two points on the OC curve as (AQL, 1 – α) and (IQL, 0.50) for minimizing the consumer risk. Peach and Littauer (1946) have taken two points on the OC curve as (AQL, 1 – α) and (LQL, β) for making the ideal condition. Hence the optimization approach for ideal condition to minimize the consumers risks is carried through minimizing the angle θ between the lines joining the points [(AQL, 1 – α), (AQL, β)] and [(AQL, 1 – α), (LQL, β)] as given in Figure 1. The formula for tan θ is,

$$tan\theta = \frac{opposite\ side}{adjacent\ side}$$

$$tan\theta = \frac{(p_2 - p_1)}{(p_a(p_1) - p_a(p_2))}$$

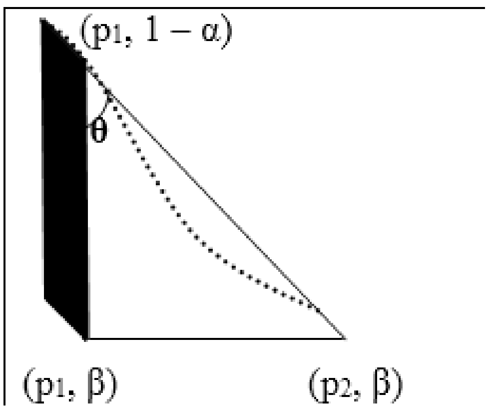

Fig 1: Tangent Angle Minimization

Hence, for two given points on the OC curve, the minimum values of angle, θ can be calculated and tabulated as in Table 1 for the proposed plan using θ = $\tan^{-1}${(ntanθ/n)}.

4.1 Example

Given p_1=0.0001 and p2=0.021

Then OR=210

The associated sets of values corresponding to the computed OR form Table 1are:

$\lambda_2 = 0.7$; $\varphi = 0.75$; $np_1 = 0.01462$; $n\tan\theta = 3.42509$
$\lambda_2 = 0.8$; $\varphi = 0.85$; $np_1 = 0.01462$; $n\tan\theta = 3.44344$
$\lambda_2 = 0.9$; $\varphi = 0.65$; $np_1 = 0.01364$; $n\tan\theta = 3.20474$.

which implies that,

1. $n = np_1/p_1 = 0.01462/0.0001 = 146$
   $\theta = \tan^{-1}\{(n \tan \theta)/n\} = \tan^{-1}(0.02343) = 1.342$.
2. $n = np_1/p_1 = 0.01462/0.0001 = 146$
   $\theta = \tan^{-1}\{(n \tan \theta)/n\} = \tan^{-1}(0.02355) = 1.349$.
3. $n = np_1/p_1 = 0.01364/0.0001 = 136$
   $\theta = \tan^{-1}\{(n \tan \theta)/n\} = \tan^{-1}(0.0235) = 1.346$.

From the above three choices, the minimum angle is θ = 1.342. Hence the optimum plan parameters for the proposed sampling plan is c = 0,1 with n = 146, $p_1 = 0.0001$, $p_2 = 0.021$, $\alpha = 0.01$, β= 0.10, n_1=37, n_2=110 and $\lambda_2 = 0.7$ with minimum angle θ = 1.342.

Table 1: Parametric values for GPD based STDS plan using Minimum Angle Method

λ_2	φ	np_1	np_2	ntan θ	OR	λ_2	φ	np_1	np_2	ntan θ	OR
	0.50	0.0138	2.8852	3.2263	208.681		0.50	0.0129	2.7886	3.1187	215.436
	0.55	0.0144	2.9366	3.2834	204.419		0.55	0.0133	2.8327	3.1679	212.543
	0.60	0.0146	2.9868	3.3395	204.292		0.60	0.0137	2.8760	3.2161	209.390
	0.65	0.0146	3.0356	3.3943	207.632		0.65	0.0142	2.9184	3.2632	205.981
0.6	0.70	0.0163	3.0831	3.4459	189.314		0.70	0.0146	2.9600	3.3094	202.460
	0.75	0.0170	3.1294	3.4970	183.692	0.8	0.75	0.0146	3.0006	3.355	205.24
	0.80	0.0179	3.1745	3.5468	177.757		0.80	0.0146	3.0404	3.3997	207.96
	0.85	0.0188	3.2184	3.5951	171.521		0.85	0.0146	3.0793	3.4434	210.621
	0.90	0.0198	3.2611	3.6420	164.996		0.90	0.0168	3.1174	3.4837	185.201
	0.95	0.0209	3.3028	3.6875	158.192		0.95	0.0175	3.1546	3.5249	180.431
	1	0.0221	3.3433	3.7317	151.122		1	0.0182	3.1911	3.565	175.456
	0.50	0.0133	2.8350	3.1704	212.386		0.50	0.0126	2.7457	3.0709	217.942
	0.55	0.0138	2.8828	3.2235	208.872		0.55	0.0129	2.7865	3.1163	215.565
	0.60	0.0143	2.9294	3.2754	205.047		0.60	0.0133	2.8265	3.1610	212.970
	0.65	0.0146	2.9750	3.3263	203.488		0.65	0.0136	2.8659	3.2047	210.161
0.7	0.70	0.0146	3.0195	3.3763	206.532		0.70	0.0140	2.9045	3.2477	207.142
	0.75	0.0146	3.0629	3.4251	209.504	0.9	0.75	0.0144	2.9423	3.2898	203.918
	0.80	0.0166	3.1054	3.4705	186.670		0.80	0.0146	2.9795	3.3313	203.794
	0.85	0.0173	3.1468	3.5162	181.460		0.85	0.0146	3.0159	3.3722	206.286
	0.90	0.0181	3.1872	3.5608	175.997		0.90	0.0146	3.0516	3.4124	208.730
	0.95	0.0189	3.2267	3.6042	170.291		0.95	0.0163	3.0867	3.4498	188.901
	1	0.0199	3.2652	3.6464	164.349		1	0.0169	3.1210	3.4878	184.748

5. Construction of tables

For STDS plan, the probability of acceptance, $P_a(p)$ for a given lot proportion defective, *p*, is given by,

$$P_a(p) = e^{-x}(1 + \phi x e^{-\lambda_2})$$

Where, $x = np, \varphi = n_2/n_1 + n_2$ and λ_2 is the dispersion parameter.

The minimum angle criteria for the proposed plan can be obtained as

$$tan\theta = \frac{(p_2 - p_1)}{(e^{-np_1(1+\varphi np_1 e^{-\lambda_2})}) - (e^{-np_2(1+\varphi np_2 e^{-\lambda_2})})} \text{ (or)}$$

$$tan\theta = \frac{np_2 - np_1}{1 - \alpha - \beta}$$

Based on the operating ratio value, the optimized plan parameters are obtained through the angle calculated as $\theta = \tan^{-1}\{(n\tan\theta/n)\}$. The Goal seek search procedure has been used to obtain the optimum value of the tangent angle for specific values of np_1 and np_2 by keeping the producer's risk below 1% and consumer's risk below 10%.

6. Conclusion

This paper presents an optimization approach to support the design of Generalized Poisson Distribution (GPD) based Special Type Double Sampling (STDS) plan through Minimum Angle Method. In practice, it is desirable to design a sampling plan with the associated quality levels concerning producer and consumer. This method is useful in the area of testing related to safety and destructive cases. The general framework are studied which makes the OC curve approaching to the ideal form by minimizing the tangent angle joining the points [(AQL, 1 – α), (AQL, β)] and [(AQL, 1 – α), (LQL, β)]. Thus it provides an alarm sense for quality controller towards the lot sentencing and the risks exists under this plan.

The work presented in this paper is mainly related to obtain optimum plan parameters for manufacturing industrialists for inspection procedure. Tables are provided to safeguard both producer and consumer simultaneously for obtaining good quality products in inspection. The proposed sampling plan through its economic aspects may be the future research work.

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