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A reliability analysis of 8hp-pml gold engine coupled locally fabricated cassava grinding machine

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Abstract
This work models the failure rate of an 8HP-PML Gold Engine coupled locally fabricated cassava grinding machine whose operational efficiency degrades with time and usage to obtain its performance measures. The failure distribution of the machine was shown to follow a Weibull distribution with shape parameter, $\hat{\alpha} = 0.793$ and scale parameter, $\hat{\beta} = 3675.085$. Some probability functions of the machine such as the failure density function, failure distribution function, the reliability function and the hazard function were obtained. Also, reliability indices obtained for the machine shows that the cassava grinding machine is in a good and sustainable working condition. These include; the mean time to repair (MTTR=99 hours), the mean time between failure (MTBF = 4190 hours) and the availability factor, A = 97.7% while the maintainability factor is M=2.3%.

Keywords: Reliability, cassava grinding machine, failure rate, probability function, Weibull distribution

1. Introduction

A reliability study is concerned with random occurrences of undesirable events of failures during the lifetime of a physical system, Kapur (1941) [5]. Reliability is generally regarded as the likelihood that a system is functional during a certain period of time under a specified operating environment. It is always considered as one of the most important characteristics for industrial products and systems. Reliability studies life data and subsequently uses it to estimate, evaluate and control the capability of component, products and systems. The theories and tools of reliability is applied in widespread fields such as electronic and manufacturing products, aerospace equipment, earthquake and volcano forecasting, communication systems, navigation and transportation control, medical treatment to the survival analysis of human being or biological species and so on (Weibull, 1977; and Lawless 1982).

The reliability period of any system is measured within the durability period of that system. The probabilistic models used are generally called "Time to Event" models, where an event is a failure. Moreover, with the increased complexity of component structure and the continuous requirements of high quality and reliability products, the role of reliability of a product is more important to both the producers and the consumers nowadays.

A repairable system is a system which can be restored to its original working condition by any maintenance action after failure (Lindqvist, 2006) [7]. It can be repaired once failure happens, but it is vital to assess the risk of unexpected failures in order to improve the system reliability, Khan and Haddara (2003) [6] and Wang, Cheng, Hu and Wu (2012) [12].

Maintenance is any activity that is carried out on any facility either to restore or to retain the facility in good and acceptable working conditions. It involves all technical and other procedures performed in order to retain the satisfactory working condition of a machine or part or restoring it to an acceptable working condition so that the set tasks can be performed at the scheduled time and under given conditions. In developing countries, maintenance is often not given the priority it deserves in the overall operating strategy of a facility. Maintenance action can be preventive or corrective (repair). Generally there are two main types or assumptions of maintenance: perfect or "as good as new" and worse than or "as bad as old" maintenance, but in reality the equipment after maintenance lies somewhere in between these two conditions, which is called an imperfect maintenance or "better than old, but worse than new"; Doyen, (2005) [2].

The first two extreme assumptions or types of maintenance are impracticable compared to the imperfect maintenance. This is because the failure nature of repairable systems depends on the repair history of the system, Muhammad, Majid, Amin and Ibrahim, (2009) [9].

Therefore, our interest is on the reliability function and measures of a mechanically repairable cassava grinding machine to provide information on the state of the machine as well as provide time to repair and time to perform other maintenance actions on the system to avoid failure and wastage. A grinding machine does not exhibit constant failure rate. Its operational efficiency degrades with time and usage. In other words, it deteriorates or wears with usage and time. These types of failures are modelled using the Weibull distribution, see Mann and Sinpurwalla (1973) [8].

The particular form of the hazard function which comprises three parts is depicted in Figure 1. The "bathtub curve" of hazard function is a combination of a decreasing hazard of early failure and an increasing hazard of wear-out failure, plus some constant hazard of random failure. The bathtub curve is widely used in reliability engineering.

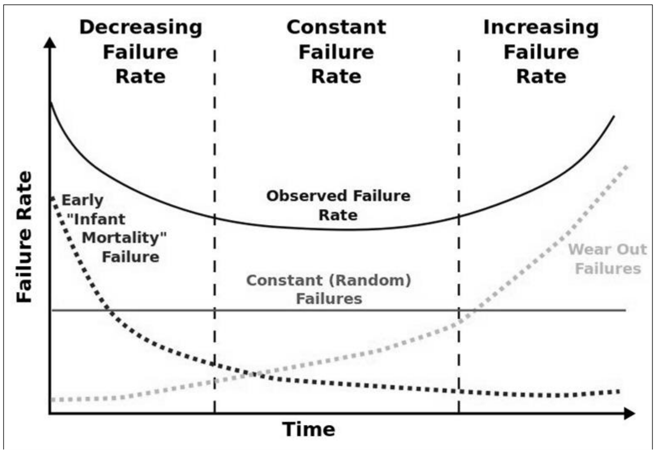

Fig 1: The 'Bathtub Curve' of Hazard Function Graph of Repairable system

The early stage or infant mortality period is the first stage, where the hazard function decreases overtime. The failures occur due to errors in design or due to mistakes caused by inexperienced personnel or user. The infant mortality period is followed by a nearly constant failure rate period known as useful life. In this region failure rate is low and approximately constant and the failures occur by pure chance. In contrast to early failure caused by the inherent weakness of the machine, the failure during this stage occur mostly due to external reasons such as overloading, collision with another object, hidden defect and mistakes of the personnel. Additionally, as the failure rate is constant, this is the only region, in which the exponential distribution can be valid and thereby the time between failures is exponentially distributed. On the other hand, the failure rate during this period follows a Weibull distribution with the shape parameter, $\beta = 1$. While the wear-out period is characterized by a rapid increasing failure rate with time. The failure in this stage is caused by wear, fatigue, cohesion or gradual deterioration of the machine. From Billington's (*1992*) [1] point of view, this period is more evident than the other two periods as in this region, the failure density function will increase firstly, and then decrease to zero for the obsolescence of components. In the wear-out period, the failure density function can often be represented or approximated by normal distribution, gamma, and Weibull distribution among other distributions with shape parameters, whose variation can create significantly different characteristic shape.

1.1 Assumptions of the Study

The basic assumptions of this study are:

1. Failures in the machine occur at random.
2. Given the life time distribution of the machine f(t), it is assumed that failure occurs at the end of time t.
3. The failures that occur at each time t are independent and is continuously distributed.

1.2 The 8HP-PML Gold Engine Coupled Locally Fabricated Cassava Grinding Machine

This is a mechanically repairable machine which does not exhibit constant failure rate. Its operational efficiency degrades with time and usage. In other words, it deteriorates or wears with usage and time. Therefore, our interest is on the reliability function and measures of this machine to provide information on the state of the machine as well as provide time to repair and perform other maintenance actions to avoid failure and wastage. These types of failures are modelled using the Weibull distribution, see Mann and Sinpurwalla (1973) [8].

2. Methodology

2.1 The Weibull life function

A handful of parametric models exist which have been successfully used as population models for failure times both for repairable and non-repairable systems, arising from a wide range of product and failure mechanism. Examples of such models are exponential, Weibull, lognormal, gamma models, etc. These distributions are exhibited by systems according to their mode of failure and failure mechanism. Sometimes, there are probabilistic arguments based on the physics of the failure mode that tends to justify the choice of models. Other times, the models are used solely because of its empirical success in fitting actual failure data. Thus, the choice of our model is based on a combination of both its failure mechanism and empirical success.

2.2 Goodness-of-Fit Test for Weibull Distribution

In this study, the chi-square test would be used to investigate whether the distribution of the failure of the cassava grinding machine differ from expected Weibull distribution. The expected frequencies of failure of the cassava grinding machine at a given intervals of time, t is given by;

$$E_{ij} = N\int_0^t \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}} dt$$

Table 1 shows the observed and expected frequencies of failure of the cassava grinding machine at a given intervals of time, t.

Table 1: Table of pooled expected frequencies

Intervals Of Time t	O_i	E_i
T < 1600	11	10.95
1600 < t < 4800	9	9.1
4800 < t < 8000	10	10.0
Total	30	30

Where O_i is the observed frequency of class *i =1, 2, 3, 4, 5.*

The test showed that the distribution of failure rate of the cassava grinding machine follows the Weibull distribution.

2.3 The Mean and Variance of a Two Parameter Weibull Distribution

The probability density function of the two parameter Weibull Distribution is given by;

$$f(y;\alpha,\beta)=\frac{\beta}{\alpha}\left(\frac{y}{\alpha}\right)^{\beta-1} e^{-\left(\frac{y}{\alpha}\right)^{\beta}} ;\ \alpha,\beta>0;\ \ 0<y<\infty \tag{1}$$

$$E(y)=\int_0^\infty yf(y)dy = \int_0^\infty \frac{\beta}{\alpha} y\left(\frac{y}{\alpha}\right)^{\beta-1} e^{-\left(\frac{y}{\alpha}\right)^{\beta}} dy$$

Let $x=\left(\frac{y}{\alpha}\right)^{\beta} \Rightarrow y=\alpha x^{\frac{1}{\beta}}$ and $dy=\alpha\frac{1}{\beta}x^{\frac{1}{\beta}-1}dx$

Then, $E(y)=\alpha\int_0^\infty x^{\frac{1}{\beta}}e^{-x}dx$

By definition of gamma function, $\Gamma\left(\frac{1}{\beta}\right)=\int_0^\infty x^{\frac{1}{\beta}-1}e^{-x}dx$ and $\int_0^\infty x^{\frac{1}{\beta}}e^{-x}dx=\Gamma\left(1+\frac{1}{\beta}\right)$

Therefore, $E(y)=\alpha\Gamma\left(1+\frac{1}{\beta}\right)$ (2)

Similarly, $E(y^2)=\alpha^2\Gamma\left(\frac{2}{\beta}+1\right)$

Hence, $Var(y)=\alpha^2\Gamma\left(\frac{2}{\beta}+1\right)-\alpha^2\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2 = \alpha^2\left[\Gamma\left(\frac{2}{\beta}+1\right)-\Gamma\left(\frac{1}{\beta}+1\right)^2\right]$ (3)

2.4 Reliability Function of Weibull Distribution

Reliability, R(t) is defined as the probability in which an item or an entity performs its intended function over a period of time under stated conditions. It is given by;

$$R(t)=-P(T\le t)=P(T>t)=1-\int_0^t f(t)dt$$

For Weibull distribution with parameter α and β ;

$$R(t)=1-\int_0^t\left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1}e^{-\left(\frac{t}{\alpha}\right)^{\beta}}dt=1-\left(\frac{\beta}{\alpha}\right)\int_0^t\left(\frac{t}{\alpha}\right)^{\beta-1}e^{-\left(\frac{t}{\alpha}\right)^{\beta}}dt=1-\left(1-e^{-\left(\frac{t}{\alpha}\right)^{\beta}}\right)$$

Hence, $R(t)=1-F(t)=1-\left(1-e^{-\left(\frac{t}{\alpha}\right)^{\beta}}\right)\ \therefore R(t)=e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$ (4)

2.5 Determination of Failure Rate or Hazard Function, $(h(t))$

The failure rate during a given interval of time $t=[t_1,t_2]$ shows the probability that a failure per unit time occurs in the interval (t_1,t_2), conditioned on the event that no failure has occurred at or before time, t_1. This means that $t>t_1$. The failure rate can be defined as follows:

$$h(t)=\frac{R(t_1)-R(t_2)}{(t_2-t_1)R(t_1)}=\frac{F(t_2)-F(t_1)}{(t_2-t_1)R(t_1)} \tag{5}$$

Taking the limit of the failure rate at the interval, $(t_1,\Delta t+1)$ as Δt approaches zero, where $t=t_1$ and $(t+\Delta t)=t_2$ gives the hazard function $h(t)$:

$$h(t)=\lim_{\Delta_t\to 0}\frac{R(t)-R(t+\Delta t)}{\Delta t\times R(t)}=\lim_{\Delta t\to 0}\frac{F(t+\Delta t)-F(t)}{\Delta t}\times\frac{1}{R(t)} \tag{6}$$

But the limit: $h(t)=\lim_{\Delta t\to 0}\frac{F(t\times\Delta t)-F(t)}{\Delta t}=f(t)$

$$h(t)=\frac{f(t)}{R(t)}=\frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1}e^{-\left(\frac{t}{\alpha}\right)^{\beta}}}{e^{-\left(\frac{t}{\alpha}\right)^{\beta}}}=\beta\left(\frac{1}{\alpha}\right)^{\beta}t^{\beta-1}\ \therefore h(t)=\beta\alpha^{-\beta}t^{\beta-1} \tag{7}$$

2.6 Reliability Indices

2.6.1 The mean time between failures (MTBF): The mean time between failures (MTBF) is the length of time within which failure occurs in a machine. It is given by;

$$MTBF=\int_0^t Tf(t)dt=\int_0^t TR(t)dt \tag{8}$$

$$=\int_0^t e^{-\left(\frac{t}{\alpha}\right)^{\beta}}=\alpha\Gamma\left(1+\frac{1}{\beta}\right) \tag{9}$$

2.6.2 The mean time to repair (MTTR): The mean time to repair (MTTR) is the average time required to repair a failed component or device. It is given by; $\frac{u}{c}$ (10)

Where u is the total down time of the machine given by $\sum_{i=1}^{N}u_i$ and c is the total number of failures in the machine.

2.6.3 Availability factor: It is the probability that a system will work as required during a particular period of time. It is given by:

$$A_f=\frac{MTBF}{MTTR+MTBF}\times 100\%. \tag{11}$$

2.6.4 Maintainability factor: It is the ease with which maintenance of a functional unit can be performed in accordance with prescribed requirement. It is given by;

$$M_f = \frac{MTTR}{MTBF + MTTR} \times 100\% \qquad (12)$$

2.7 Estimating the Parameters of Weibull Distribution using Method of Least Squares
The cumulative distribution function is given as;

$$F(y_t) = 1 - e^{-\left(\frac{y_t}{\alpha}\right)^{\beta}} \quad \Rightarrow 1 - F(y_t) = e^{-\left(\frac{y_t}{\alpha}\right)^{\beta}} \quad and \quad \frac{1}{1-F(y_t)} = e^{\left(\frac{y_t}{\alpha}\right)^{\beta}}$$

Taking the natural log of both sides;

$$\ln\left[\frac{1}{1-F(y_t)}\right] = \left(\frac{y_t}{\alpha}\right)^{\beta}$$

And taking natural log again; we obtain

$$= \ln\left[\ln\left(\frac{1}{1-F(y_t)}\right)\right] = \beta \ln y_t - \beta \ln \alpha \qquad (13)$$

Where y_t represent the order statistic, such that $y_{(1)} < y_{(2)} < \ldots < y_{(n)}$

Let $y = \ln\left[\ln\left(\frac{1}{1-F(y_t)}\right)\right]$, $X = \ln y_t$ *and* $\beta_0 = -\beta \ln \alpha$

Then, it can be represented as;

$$y = \beta_0 + \beta_1 X \qquad (14)$$

We obtain the following normal equations by ordinary least square method;

$$\sum Y = n\beta_0 + \beta_1 \sum X$$
$$\sum XY = \beta_0 \sum X + \beta_1 \sum X^2$$

By solving the normal equation simultaneously, we obtain;

$$\hat{\beta} = \hat{\beta}_1 = \frac{n\sum XY - \left(\sum X\right)\left(\sum Y\right)}{n\sum X^2 - \left(\sum X\right)^2}$$

Substituting for X and Y

$$\hat{\beta} = \hat{\beta}_1 = \frac{n\sum \ln Y_t \left[\ln\left(\ln\left(\frac{1}{1-F(Y_t)}\right)\right)\right] - \sum \ln Y_t \sum \ln\left[\ln\left(\frac{1}{1-F(Y_t)}\right)\right]}{n\sum (\ln Y_t)^2 - \left(\sum \ln Y_t\right)^2} \qquad (15)$$

$$\beta_0 = -\beta \ln \alpha$$

$$and \;\; \hat{\alpha} = e^{-\frac{\beta_0}{\beta_1}} \qquad (16)$$

$$\therefore \hat{\beta}_0 = \frac{1}{n}\sum \ln\left[\ln\left(\frac{1}{1-F(Y_t)}\right)\right] - \hat{\beta}_1 \sum \ln Y_t$$

3. Analysis and Results

3.1 Evaluation of the Weibull Parameters

Let $Int = X \;\; and \;\; Y = In\left(In\left(\frac{1}{1-F(Y_t)}\right)\right)$

The estimate of $\hat{\beta}_1$ is given by;

$\hat{\beta}_1 = \frac{n\sum XY - \left(\sum X\right)\left(\sum Y\right)}{n\sum X^2 - \left(\sum Y\right)^2}, n = 30$; $\sum XY = (-77.174); \sum X = 225.9588$; $\sum Y = (-16.08662968)$; $\sum X^2 = 175.323$

From (15); $\hat{\beta} = 3675.09$ (17)

And from (16); $\hat{\alpha} = 0.793$ (19)

3.2 Evaluation of Reliability Indicesof Cassava Grinding Machine

From (2), (10) – (12), the reliability indices; MTBF, MTTR, A_f and M_f were respectively obtained as 4190.33 hours, 99.2 hours, 97.7% and 2.3%.

3.3 Evaluation of Reliability Functions of the Cassava Grinding Machine using Weibull Distribution

By using the estimated parameters of Weibull distribution from our data; $\alpha = 3675.085,\ \beta = 0.793.$ The reliability function, R(t), the hazard function, h(t), the failure cumulative function, F(t) and the failure density function, f(t) were obtained and plotted in Figures 2, 3, 4 and 5.

4 Discussion of Results

4.1 The reliability function: With reference to Figure 2, the reliability of the cassava grinding machine reduces gradually within 4000 hours between the reliability value 0.98 and 0.3. Its reliability reduces rapidly within the time interval of 4000 hours to 8000 hours which shows a sudden decrease.

4.2 The hazard function or failure rate: Figure 3 is a combination of a decreasing hazard rate of early failure and constant hazard rate of random failure which is the first and second stage of the ‘bathtub curve’ of the hazard function. The failure rate decreases from the failure rate value of 0.00054 to 0.00024 within 3000 hours of operation and maintains a constant failure from 4000 hours to 8000 hours at point 0.0002.

4.3 The failure distribution function: Figure 4 shows that the distribution of failure increases from point 0 to 0.65 within the time interval of 0 hour to 4000 hours. It increases continually at a rapid rate within 4000 hours to 8000 hours at point 0.65 and 0.8 respectively.

4.4 The failure density function: Figure 5 shows that the likelihood of occurrence of failure in the cassava grinding machine decreased sharply from point 0.00050 to 0.00030 within 0 hours to 400 hours of operation and started reducing gradually as time increases. This pattern is similar to the density function of a typical Weibull distribution.

4.5 The reliability indices: With reference to subsection 3.2, the mean time between failure of the cassava grinding machine MTBF=4190. This implies that the grinding machine is operational for 4190 hours, which is about 175 days before failure occurs. Also, the mean time to repair MTTR=99.2 hours implies that the machine is restored to normal working condition within 4 days on the average after failure.

The availability factor; A_f =97.7% implies that the cassava grinding machine is in a working state for 97.7% of the time while the maintainability of the cassava grinding machine; M_f =2.3%, implies that 2.3% out of 100% availability time of the grinding machine is used for repair and other maintenance actions.

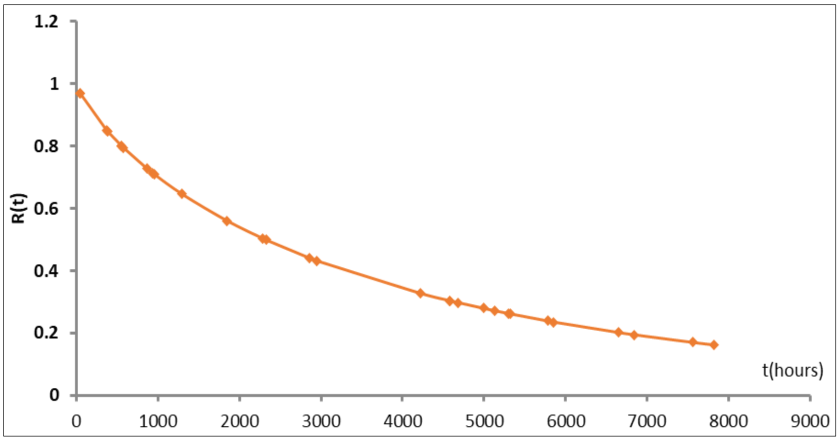

Fig 2: A graph of Reliability function against time of the Cassava Grinding Machine

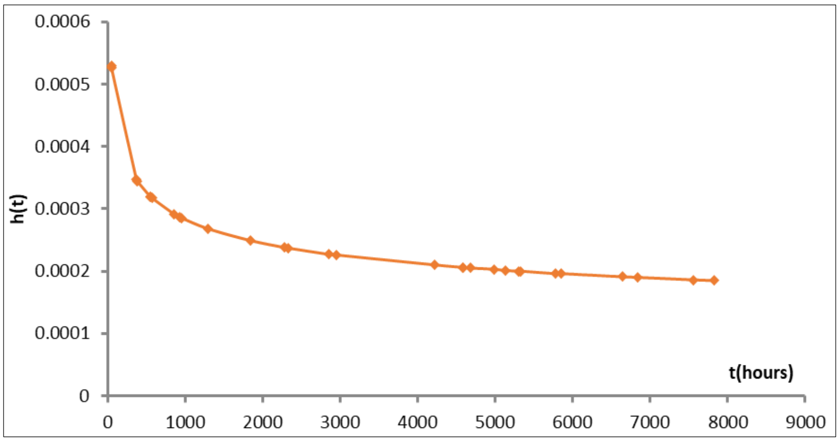

Fig 3: A graph of hazard function against time of the Cassava Grinding Machine

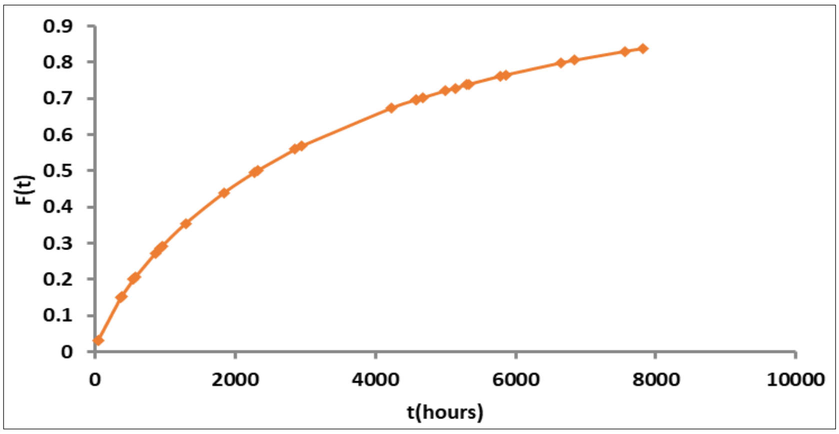

Fig 4: A graph of cumulative distribution against time of the Cassava Grinding Machine

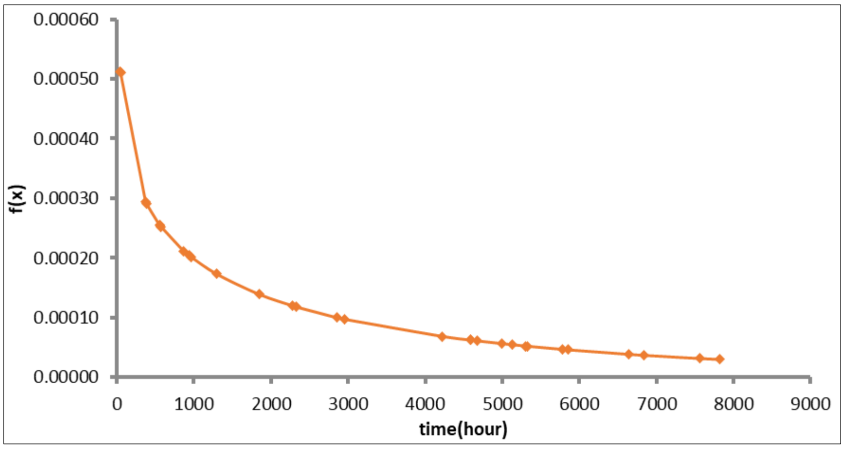

Fig 5: A graph of density function against time of the Cassava Grinding Machine

5. Conclusion

The chi-squared goodness-of-fit test confirmed that the empirical failure data of the 8HP-PML Gold Engine coupled locally fabricated cassava grinding machine follows a Weibull distribution with shape parameter: $\hat{\alpha} = 0.793$ and scale parameter $\hat{\beta} = 3675.085$ and mean time between failures as 4190 hours. The mean time to repair of the machine implies that it takes about 99 hours (3 days) to repair the cassava grinding machine after failure. Also, the availability factor, A = 97.7% implies that the machine is in a working state for about 97.7% of the time while 2.3% out of 100% availability of the machine time is used for repair/maintenance actions.

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