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Edge restrainedness in hypergraph

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Abstract
In this paper we have introduced edge restrained set and one maximal edge restrained set in hypergraph. We have also introduced RE – Set and RE – Number in hypergraph. We have given characterization of one maximal edge restrained sets. We have also proved that in a hypergraph with minimum edge degree at least two every one – maximal edge restrained set is an edge h – dominating set. We have proved several results about edge restrainedness in subhypergraphs and partial subhypergraphs also.

Keywords: Hypergraph, subhypergraph, partial subhypergraph, edge restrained set, one maximal edge restrained set, RE – set, RE – number

1. Introduction
The concept of restrained domination was studied by [5]. From this concept a new concept of restrainedness of vertices was introduced by D. K. Thakkar and B. M. Kakrecha [12]. Now we consider the concept of restrainedness of edges of hypergraphs. Here we defined edge restrained sets and one maximal edge restrained sets in hypergraphs. We characterize one maximal edge restrained sets. We also define RE – Sets and RE – Number of a hypergraph. We proved that RE – number of a hypergraph is odd if and only if the number of edges of the hypergraph is odd. We prove several results related to the edge restrainedness in subhypergraphs and partial subhypergraphs.

2. Preliminaries
Definition 2.1 Hypergraph [4]: A *hypergraph* G is an ordered pair $(V(G), E(G))$ where $V(G)$ is a non-empty finite set & $E(G)$ is a family of non-empty subsets of $V(G)$ ∍ their union $= V(G)$. The elements of $V(G)$ are called *vertices* & the members of $E(G)$ are called *edges of the hypergraph* G.
We make the following assumption about the hypergraph.
1. Any two distinct edges intersect in at most one vertex.
2. If e_1 and e_2 are distinct edges with $|e_1|, |e_2| > 1$ then $e_1 \nsubseteq e_2$ & $e_2 \nsubseteq e_1$

Definition 2.2 Edge Degree [4]: Let G be a hypergraph & $v \in V(G)$ then the *edge degree* of v $= d_E(v) =$ the number of edges containing the vertex v. The minimum edge degree among all the vertices of G is denoted as $\delta_E(G)$ and the maximum edge degree is denoted as $\Delta_E(G)$.

Definition 2.3 Dominating Set in Hypergraph [1]: Let G be a hypergraph & $S \subseteq V(G)$ then S is said to be a *dominating set* of G if for every $v \in V(G) - S$ there is $u \in S$ ∍ u and v are adjacent vertices.
A dominating set with minimum cardinality is called *minimum dominating set* and cardinality of such a set is called *domination number* of G and it is denoted as $\gamma(G)$.

Definition 2.4 Edge Dominating Set [8]: Let G be a hypergraph & $S \subseteq E(G)$ then S is said to be an *edge dominating set* of G if for every $e \in E(G) - S$ there is some f in S ∍ e and f are adjacent edges.
An edge dominating set with minimum cardinality is called a *minimum edge dominating set* and cardinality of such a set is called *edge domination number* of G and it is denoted as $\gamma_E(G)$.

Definition 2.5 Minimal Edge Dominating Set [8]: Let G be a hypergraph & $F \subseteq E(G)$ then F is said to be a *minimal edge dominating set* if (1) F is an edge dominating set (2) No proper subset of F is an edge dominating set of G.

Definition 2.6 Sub hypergraph and Partial sub hypergraph [3]: Let G be a hypergraph & $v \in V(G)$. Consider the subset $V(G) - \{v\}$ *of* $V(G)$. This set will induce two types of hypergraphs from G.
(1) First type of hypergraph: Here the vertex set = $V(G) - \{v\}$ and the edge set = $\{ e' / e' = e - \{v\}\ for\ some\ e \in E(G) \}$. This hypergraph is called the *sub hypergraph* of G & it is denoted as $G - \{v\}$.
(2) Second type of hypergraph: Here also the vertex set = $V(G) - \{v\}$ and edges in this hypergraph are those edges of G which do not contain the vertex v.This hypergraph is called the *partial sub hypergraph* of G.

Definition 2.7 Edge Neighbourhood [3]: Let G be a hypergraph & e be any edge of G then

Open edge neighbourhood of e $= N(e) = \{ f \in E(G) / f\ is\ adjacent\ to\ e \}$.

Close edge neighbourhood of e $= N[e] = N(e) \cup \{e\}$.

Definition 2.8 Private Neighbourhood of an edge [3]: Let G be a hypergraph. F be a set of edges & e ∈ F, then the *private neighbourhood of e with respect to set F* = Prn [e, F] = { f ∈ E(G) / N[f] ∩ F = {e}}
Here we are introducing a new concept called edge h – domination in hypergraph.

Definition 2.9 Edge h – Dominating Set [11]: Let G be a hypergraph. A collection F of edges of G is called an *edge h – dominating set* of G if
1. All isolated edges of G are in F.
2. If f is not an isolated edge & f ∉ F then there is a vertex x in f ∍ edge degree of x ≥ 2 & all the edges containing x except f are in F.

An edge h – dominating set with minimum cardinality is called a minimum edge h – dominating set of G & its cardinality is called edge h – domination number of G & it is denoted as γ_h'(G).

3. Main Results
Definition 3.1 (Edge Restrained Set): Let G be a hypergraph. A set F of edges of G is said to be an edge restrained set if for every e ∈ E(G) – F ∃ f ∈ E(G) – F ∍ e and f are adjacent.

Example 3.2: Consider the hypergraph G whose vertex set V(G) = {1, 2, 3, 4, 5, 6} & E(G) = {e_1, e_2, e_3, e_4, e_5, e_6}

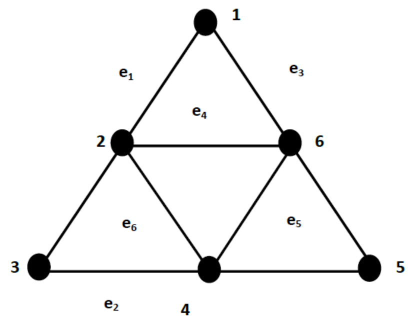

Fig 1: Hypergraph with 6 vertices and 6 edges

Here, F = {e_1, e_2, e_3} is an edge restrained set.

Example 3.3: Consider the hypergraph G whose vertex set V(G) = {1, 2, 3, 4, 5, 6} & E(G) = {e_1, e_2, e_3}

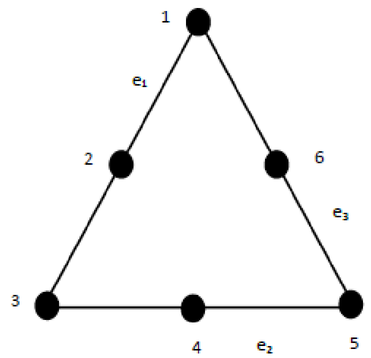

Fig 2: Hypergraph with 6 vertices and 3 edges

Consider $F = \{e_1\}$.
Obviously F is an edge restrained set in hypergraph.
Let $F_1 = \{e_1, e_2\}$. Note that F_1 is not an edge restrained set. Thus, restrainedness is not a super hereditary property.

Example 3.4: Consider the hypergraph G whose vertex set $V(G) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ & $E(G) = \{e_1, e_2, e_3, e_4, e_5\}$

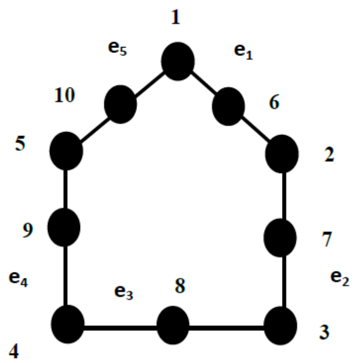

Fig 3: Hypergraph with 10 vertices and 5 edges

Let $F = \{e_1, e_2, e_3\}$ obviously F is an edge restrained set.
Let $F_1 = \{e_1, e_3\}$. Note that F_1 is not an edge restrained set. Thus, restrainedness is not a hereditary property.

Definition 3.5 (One Maximal Edge Restrained Set): Let G be a hypergraph and F be an edge restrained set of G then F is said to be a one maximal edge restrained set if $F \cup \{e\}$ is not an edge restrained set for every $e \in E(G) - F$.

Example 3.6: Consider hypergraph in above example 3.4
Here $F = \{e_1, e_2, e_3\}$ is an edge restrained set.
Now, consider $F_1 = F \cup \{e_4\}$ then it is not an edge restrained set. Similarly, $F_2 = F \cup \{e_5\}$ is also not an edge restrained set.
So, $F = \{e_1, e_2, e_3\}$ is a one maximal edge restrained set.

Example 3.7: Consider the finite projective plane G with $r^2 - r + 1$ vertices & $r^2 - r + 1$ edges ($r \geq 3$).
(1) Any two edges intersect in this hypergraph.
(2) If u and v are distinct vertices than there is a unique edge which contains u and v.
(a) Let F be any set of edges $\ni E(G) - F$ has at least two edges. Let e be any edge of $E(G) - F$ & Let f be any other edge of $E(G) - F$.
As mentioned above e & f intersect in a vertex.
Therefore, e & f are adjacent edges.
Thus, we have proved that F is an edge restrained set of G.
(b) Let F be a set of edges $\ni E(G) - F$ has exactly two distinct edges. It is obvious that F is a one maximal edge restrained set of G.
Thus, we have proved that F is a set of edges $\ni E(G) - F$ has exactly two distinct edges than F is a one maximal edge restrained set.
Conversely suppose F is a one maximal edge restrained set of G. Suppose $E(G) - F$ has at least three edges. Let $e \in E(G) - F$. Than $E(G) - (F \cup \{e\})$ has at least two edges and therefore by (a) $F \cup \{e\}$ is an edge restrained set of G. This contradicts the one maximality of F.
$\therefore$ $E(G) - F$ must have exactly two distinct edges.
Thus, we conclude that a set F of edges is a one maximal edge restrained set of G if and only if $E(G) - F$ has exactly two distinct edges.
We also conclude that all one maximal edge restrained sets have the same cardinality $r^2 - r - 1$.

Definition 3.8 (RE - Set): Let G be a hypergraph. A one maximal edge restrained set with minimum cardinality is called a RE – Set of G. The cardinality of a RE – Set is called the RE – number of G and it is denoted by RE(G).

Example 3.9: Consider the finite projective plane mentioned above. It is obvious that every one maximal edge restrained set is a RE – Set of G.
Therefore, RE – number of this hypergraph is $r^2 - r - 1$.

Characterization of one maximal edge restrained set:
Now, we give a characterization of one maximal edge restrained sets

Theorem 3.10: Let G be a hypergraph and F be an edge restrained set of G. Then F is a one maximal edge restrained set if and only if for every $e \in E(G) - F$ there is an edge f in $E(G) - F$ which is adjacent to only one edge of $E(G) - F$ namely e.

Proof: Suppose F is a one maximal edge restrained set of G. Let $e \in E(G) - F$. Since $F \cup \{e\}$ is not an edge restrained set of G there is an edge f in $E(G) - (F \cup \{e\})$ such that f is not adjacent to any edge of $E(G) - (F \cup \{e\})$.
Now, $f \in E(G) - F$ and therefore f must be adjacent to some edge of $E(G) - F$. This edge must be e.
Thus, f is adjacent to e.
Suppose f is adjacent to some edge $h \ni h \neq e$ and f is adjacent to h. This will imply that f is adjacent to h and h is an edge of $E(G) - (F \cup \{e\})$. This contradicts the earlier statement that f is not adjacent to any edge of $E(G) - (F \cup \{e\})$.
Therefore, f is adjacent to only one edge of $E(G) - F$ namely e.
Conversely suppose the condition is satisfied. Let $e \in E(G) - F$. Let f be an edge of $E(G) - F$ such that f is adjacent to only one edge of $E(G) - F$ namely e. This implies that f is an edge of $E(G) - (F \cup \{e\})$ which is not adjacent to any edge of $E(G) - (F \cup \{e\})$. This proves that $F \cup \{e\}$ is not an edge restrained set of G.
Thus, F is a one maximal edge restrained set of G.
The above theorem can be improved further.

Theorem 3.11: Let G be a hypergraph and F be an edge restrained set of G. Then F is a one maximal edge restrained set of G if and only if for every $e \in E(G) - F$ there is a unique f in $E(G) - F \ni f$ is adjacent to only one edge in $E(G) - F$ namely e.

Proof: First suppose that F is a one maximal edge restrained set of G. Let $e \in E(G) - F$. By the above theorem there is an edge f in $E(G) - F \ni f$ is adjacent to only one edge in $E(G) - F$ namely e.
Now, $f \in E(G) - F$ therefore by the above theorem there is an edge h in $E(G) - F \ni h$ is adjacent to only one edge in $E(G) - F$ namely f.
Thus, f is adjacent to e as well as h and h, $e \in E(G) - F$. Therefore, h = e.
Thus, e is adjacent to only one edge in $E(G) - F$ namely f.
Thus, f is unique for e.
Conversely suppose the condition is satisfied. Let $e \in E(G) - F$ and let f be the unique edge in $E(G) - F$ which is adjacent to only one edge in $E(G) - F$ namely e. By the above theorem the set F is the one maximal edge restrained set.

Corollary 3.12: Let G be a hypergraph and F be a one maximal edge restrained set of G then $E(G) - F$ has even number of edges.

Proof: For every $e \in E(G) - F$ there is a unique edge f_e in $E(G) - F \ni f_e$ is adjacent to only one edge in $E(G) - F$ namely e.
Define the function $H: E(G) - F \rightarrow E(G) - F$ as $H(e) = f_e$
Then H is one – one and onto function from $E(G) - F \rightarrow E(G) - F$
$\therefore$ $|E(G) - F|$ is an even number.
Thus, $E(G) - F$ has even number of edges.

Corollary 3.13: Let G be a hypergraph. Let $m = |E(G)|$ then RE(G) is an odd number if and only if m is an odd number.

Proof: Let F be a RE – Set of G.
Now, $|E(G)| = |F| + |E(G) - F|$
i.e. $m = |F| + |E(G) - F|$
Since $|E(G) - F|$ is always an even number, $|F| = RE(G)$ is an odd number if and only if m is an odd number.
Now, we prove the following theorem.

Theorem 3.14: Let G be a hypergraph with minimum edge degree of $G \geq 2$. If F is a one – maximal edge restrained set of G then it is an edge h – dominating set of G.

Proof: First we recall that a set F is an edge h – dominating set if it satisfies the following condition.
(1) All isolated edges of G are in F.
(2) If f is not an isolated edge & $f \notin F$ then there is a vertex x in $f \ni$ edge degree of $x \geq 2$ & all the edges containing x except f are in F.
Let $e \in E(G) - F$. Let f be an edge of $E(G) - F \ni f$ is adjacent to only one edge in $E(G) - F$ namely e.
Let $f \cap e = \{x\}$.
Now, f is the unique edge in $E(G) - F \ni e$ is adjacent to f. Let y be any vertex of $e \ni y \neq x$. Let h be any edge containing y except e. Then $h \neq f$ and therefore h cannot be an edge of $E(G) - F$.
$\therefore$ $h \in F$.
Thus, every edge which contains y and different from e is in F.
This proves that F is an edge h – dominating set of G.

Corollary 3.15: Let G be a hypergraph with minimum edge degree of $G \geq 2$. Then $\gamma_h'(G) \leq RE(G)$

Proof: Let F be an RE – Set of G. Then F is also an edge h – dominating set of G by above theorem.
$\therefore$ $\gamma_h'(G) \leq |F| = RE(G)$.

Remark 3.16: Let G be a hypergraph and $v \in V(G)$ be $\ni \{v\}$ is not an edge of G. Let F be a one maximal edge restrained set of G. Suppose e_1, e_2, e_3 are three distinct edges of G such that each of them contains the vertex v & $e_i \in E(G) - F$ for i = 1, 2, 3. Since F is one maximal there should be a unique edge h in $E(G) - F \ni h \cap e_1 \neq \phi$ but as mentioned above $e_1 \cap e_2 = \{v\}$ and $e_1 \cap e_3 = \{v\}$ which is a contradiction.

Thus, we conclude that F is a one maximal edge restrained set of G then $E(G) - F$ can include at most two edges containing v. This is true for any vertex v of G.

Therefore, any one maximal edge restrained set of G must contains at least $\Delta_E(G) - 2$ edges. In particular any RE – set of G must contain at least $\Delta_E(G) - 2$ edges.

$\therefore RE(G) \geq \Delta_E(G) - 2$

Let G be a hypergraph and $v \in V(G)$ be $\ni \{v\}$ is not an edge of G. First we will consider the subhypergraph $G - v$ whose vertex set is $V(G) - \{v\}$ & the edge set is $\{e' = e - \{v\} \ni e \in E(G)\}$. Let F be a set of edges of G then F′ will denote the set $\{e' \ni e \in E(G)\}$

Theorem 3.17: Let G be a hypergraph and $v \in V(G)$ be $\ni \{v\}$ is not an edge of G. Let F be a set of edges of G $\ni$ F is an edge restrained set of G then F′ is an edge restrained set of $G - v$ iff for every $e \in E(G) - F$ there is an edge h in $E(G) - F \ni e \cap h = \{w\}$ for some $w \neq v$.

Proof: First suppose that for every $e \in E(G) - F$ there is an edge h in $E(G) - F \ni e \cap h = \{w\}$ for some $w \neq v$.

Let e′ be any edge of $G - v \ni e' \notin F'$ then $e \notin F$. Since the condition is satisfied, there is an edge h in $E(G) - F \ni e \cap h = \{w\}$ for some $w \neq v$ then $e' \cap h' = \{w\}$ and w is a vertex of $G - v$. Also $h' \notin F'$ as $h \notin F$.

This proves that F′ is an edge restrained set of $G - v$.

Conversely suppose F′ is an edge restrained set of $G - v$. Let $e \in E(G) - F$ then $e' \notin F'$. There is an edge h′ such that $h' \notin F'$ & $e' \cap h' = \{w\}$ for some vertex w of $G - v$. Then $e \cap h = \{w\}$ and $w \neq v$. Also $h \notin F$ as $h' \notin F'$.

This proves that the condition is satisfied.

Theorem 3.18: Let G be a hypergraph and $v \in V(G)$ be $\ni \{v\}$ is not an edge of G. Let F be a set of edges of G $\ni$ F is a one maximal edge restrained set of G then F′ is a one maximal edge restrained set of $G - v$ iff for every $e \in E(G) - F$ there is a unique edge h in $E(G) - F \ni e \cap h = \{w\}$ for some $w \neq v$.

Proof: Suppose the condition that for every $e \in E(G) - F$ there is a unique edge h in $E(G) - F \ni e \cap h = \{w\}$ for some $w \neq v$ is satisfied.

Let e′ be any edge of $G - v \ni e' \notin F'$ then $e \notin F$. There is a unique edge h in $E(G) - F \ni e \cap h = \{w\}$ for some $w \neq v$ then $h' \notin F'$ and $e' \cap h' = \{w\}$.

Suppose g′ is any edge of $G - v \ni g' \notin F'$ and $e' \cap g' = \{w'\}$ for some $w' \in G - \{v\}$ then $g \in E(G) - F$ and $g \cap e = \{w'\}$ and $w' \neq v$. By the uniqueness assumption g = h and therefore g′ = h′.

Thus, h′ is unique.

This proves that F′ is a one maximal edge restrained set of $G - v$.

Conversely suppose that F′ is a one maximal edge restrained set of $G - v$. Let $e \in E(G) - F$ then $e' \notin F'$. Since F′ is a one maximal edge restrained set of $G - v$ there is a unique edge h′ of G - v such that $h' \notin F'$ & $e' \cap h' = \{w\}$ for some vertex w of $G - v$. Then $e \cap h = \{w\}$ and $h \in E(G) - F$.

To see that h is unique let g be an edge of $E(G) - F \ni g \cap e = \{w'\}$ for some $w' \neq v$. Then $g' \cap e = \{w'\}$ and $g' \notin F'$. By the uniqueness of h′, g′ = h′ and therefore g = h. Thus, h is unique.

This proves that the condition is satisfied.

Remark 3.19: Let G be a hypergraph and $v \in V(G)$ be $\ni \{v\}$ is not an edge of G. Let F be an RE – Set of G $\ni$ the condition in above theorem is satisfied.

Then $RE(G - v) \leq |F'| = |F| = RE(G)$

$\therefore RE(G - v) \leq RE(G)$

The condition in the above theorem cannot be dropped. This is mentioned in the example given.

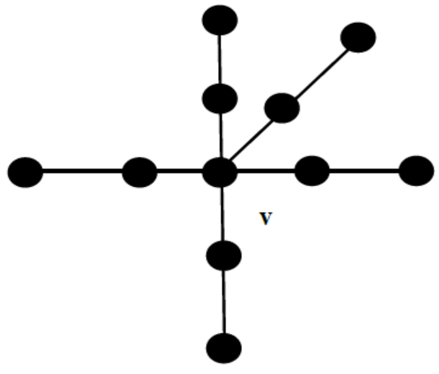

Fig 4: Hypergraph G

For the above hypergraph G, RE(G) = 3
Now, consider the sub hypergraph G – v which is mentioned below.

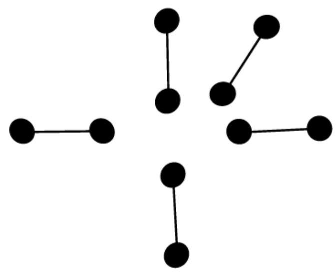

Fig 5: Sub hypergraph G – v

Note that RE(G - v) = 5 & thus, RE(G - v) > RE(G)
Observe that the condition in above theorem is not satisfied in this hypergraph.

Partial Sub hypergraph G – v

Let G be a hypergraph and $v \in V(G)$. The partial sub hypergraph G – v is the hypergraph whose vertex set is V(G) – {v} and the edge set is the set of those edges of G which do not contain the vertex v.
Let v be a vertex of G ∍ edge degree of $v \geq 2$. Let $N_e(v) = \{e \in E(G) \ni v \in e\}$. Let e_1, e_2 be two distinct edges of $N_e(v)$. Let $N_e(v)' = N_e(v) - \{ e_1, e_2 \}$
Now, we prove the following theorem.

Theorem 3.20: Let G be a hypergraph and $v \in V(G)$. Then a set of edges F of G – v is an edge restrained set of G – v iff $F \cup N_e(v)$ is an edge restrained set of G.

Proof: First suppose that F is an edge restrained set of G – v. Let e be an edge of G ∍ $e \notin F \cup N_e(v)$ then e is an edge of G – v & $e \notin F$.
Since F is an edge restrained set of G – v there is an edge h of G – v ∍ $h \notin F$ and h is adjacent to e in G – v. Since $v \notin h$, $h \notin F \cup N_e(v)$ and h is adjacent to e in G.
Thus, $F \cup N_e(v)$ is an edge restrained set of G.
Conversely suppose $F \cup N_e(v)$ is an edge restrained set of G. Let e be an edge of G – v ∍ $e \notin F$. Since $F \cup N_e(v)$ is an edge restrained set of G and $e \notin F \cup N_e(v)$, there is an edge h of G ∍ $h \notin F \cup N_e(v)$ and h is adjacent to e in G. Note that $v \notin h$. Therefore, h is an edge of G – v and h is adjacent to e in G – v.
∴ F is an edge restrained set of G – v.
In fact we can prove the following theorem.

Theorem 3.21: Let G be a hypergraph and $v \in V(G)$. Then a set of edges F of G – v is a maximal edge restrained set of G – v iff $F \cup N_e(v)$ is a maximal edge restrained set of G.

Proof: First suppose that F is a maximal edge restrained set of G – v. Let e be an edge of G ∍ $e \notin F \cup N_e(v)$. Obviously e is an edge of G – v & $e \notin F$.
Since F is maximal in G – v there is a unique edge h of E(G – v) - F ∍ h is adjacent to e in G – v. Then h is an edge of g ∍ $h \in E(G) - F$ and h is adjacent to e in G also. Suppose h′ is an edge of $E(G) - (F \cup N_e(v))$ ∍ h′ is adjacent to e in G. Then h′ belongs to E(G – v) – F and h′ is adjacent to e in G – v also. Since h is unique for e in G – v, h′ = h. Thus, $F \cup N_e(v)$ is a maximal edge restrained set of G.
Converse part of the statement is similar and can be proved in a straight forward manner.
The above theorem gives rise to another theorem which follows from the above theorem.

Theorem 3.22: Let G be a hypergraph, $v \in V(G)$ and F be a set of edges of G ∍ $N_e(v) \subseteq F$. Then F is a maximal edge restrained set of G iff F - $N_e(v)$ is a maximal edge restrained set of G - v.

Proof: Let $F_1 = F - N_e(v)$. Then F_1 is a set of edges of G – v. By the above theorem F_1 is a maximal edge restrained set of G – v iff F is a maximal edge restrained set of G.

Corollary 3.23: Let G be a hypergraph, $v \in V(G)$ and suppose there is a RE - set F of G such that $N_e(v) \subseteq F$. Then
(1) $RE(G - v) \leq RE(G) - d_e(v)$
(2) $RE(G - v) \leq RE(G) - 1$
(3) $RE(G - v) < RE(G)$

Proof: From the above theorem $F_1 = F - N_e(v)$ is a maximal edge restrained set of G – v.
∴ $RE(G - v) \leq |F_1| = |F| - |N_e(v)| = RE(G) - d_e(v) \leq RE(G) - 1$

$\therefore$ RE(G - v) < RE(G)

Example 3.24: Consider the finite projective plane with $r^2 - r + 1$ vertices & $r^2 - r + 1$ edges($r \geq 3$).
Let's denote this hypergraph by G. In an earlier example we have observed that RE(G) = $r^2 - r - 1$.
As we have mentioned earlier for any vertex v | $N_e(v)$ | = r
By the above corollary RE(G - v) ≤ RE(G) – |$N_e(v)$|
$= r^2 - r - 1 - r$
$= r^2 - 2r - 1$

Consider the partial subhypergraph G – v. It can be easily proved that for any maximal edge restrained set F of G – v, E(G – v) – F contains exactly two edges.
$\therefore |F| = (r^2 - r + 1) - r - 2$
$= r^2 - 2r - 1$

Consider two edges e_1 and e_2 of G ∍ v $\notin e_1$ and v $\notin e_2$
Let $F_1 = E(G) - \{e_1, e_2\}$ then F_1 is a RE – set of G and $N_e(v) \subseteq F_1$. Let $F = F_1 - N_e(v)$ then F is a RE – set of G – v(by above argument)
$\therefore$ RE(G - v) = |F| = $r^2 - 2r - 1$

4. Concluding Remark
It may be interesting to study the effect of removing an edge from the hypergraph on the RE – number of the hypergraph. Further edge restrained dominating sets can be considered for the further study.

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