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# Role of sylow's theorem on permutation group and automorphic character

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**Abstract**
In the permutation group. Every group is isomorphic to a permutation group. In permutation group the smallest subgroup of $S_n$ containing (12) and (1, 2, 3, .........., n) is $S_n$. The converse of lagranges theorem does not hold, in general, as $O(A_n)$ is divisible by 6 but it has no sub group of order 6. It is also notable that quotient group of cyclic (abilian) groups are cyclic (abelian) but converse do not hold.

**Keywords:** Role of sylow's theorem on permutation group every group is isomorphic to a permutation group

## Introduction
We start by recalling that by an Automorphism we mean an isomorphism of a group G to itself. Also under permutation groups are noticed that the set of all permutation (1-1 onto maps) forms a group. We show the set of all automorphisms also forms a group, the two being closely elated. We intend studying a few result pertaining to these groups.

**Example:** Show that if G is a group of order 60 and has more than one sylow 5–subgroup then G is simple.

**Soln.** O (G) = 60 = $2^2 \times 3 \times 5$. The number of sylow 5–subgroup is 1+5k st 1+5k/12 $\Rightarrow$ k = 0, 1 if K=0, then $\exists$ a unique normal sylow 5–subgroup. Since G has more than one sylow 5–subgroup k=1 than 1+5k = 1+5–1 = 6 sylow 5–subgroup each of order 5 and hence these are 6×4 = 24 elements of order 5.
Suppose G has a non trivial normal subgroup H i.e. G is not simple. Then possible values of (H). 2, 3, 4, 5, 6, 10, 12, 15, 20, 30

**Case–1 :** O(H) = 5, 10, 15, 20 or 30 i.e. 5/O(H) Then since $5^{1+1}\times$ O(H), We find H has a sylow 5–subgroup $p$. Then $p \subseteq H \subseteq G$ if Q is any conjugate of p then Q = $\subseteq$ $gHg^{-1}$ = H as $H \subseteq G$. All the six conjugate (6 sylow 5– subgroup of H are conjugate) are contained in H $\Rightarrow$ all the 24 elements of order 5 are in H and also e $\in$ H. So O (H) > 25 or that O(H)=30 is the only possibility. But a group of order 30 as has unique normal sylow 5–subgroup.

**Case–2:** O(H) = 2, 3, 4 let $\overline{G} = \frac{G}{H}$ then O(G') = $\frac{O(G)}{O(H)} = \frac{60}{2,3,4}$ - 30, 20, 15,

But the group of order 15, 20, 30 have a unique normal sylow 5–subroup O($\overline{k}$) =5, O($\overline{k}$) / O(k) = 5/O(k). Which is not possible because of (1) Thus, Case (2) is also rouled out.
Finally suppose O(H) = 6 or 12 then H contains a unique normal sylow subgroup, say N. The N is characteristic subgroup of H $\Rightarrow$ $N \subseteq G$.
Since N is a sylow subgroup of H and O(H) = 2×3 or O(H) = $2^2 \times 3$. Sylow subgroups are of order 2, 3 or 4. So O(N) = 2, 3, 4 which is not possible as given $A_5$ is simple.

**Theorem 1:** Show that $A_5$ is simple.

**Soln:** O ($A_5$) = 60 Let σ = (12345), η = (13245)
Then H = ⟨σ⟩ = ⟨σ, $\sigma^2$, $\sigma^3$, $\sigma^4$, $\sigma^5$=1⟩ = {(12345), (13524), (14253), (15432), I} and k = ⟨η⟩ = {(13245), (12534), (14352), (15423), I} = ⟨η, $\eta^2$, $\eta^3$, $\eta^4$, $\eta^5$⟩
are two sylow 5–subgroups of $A_5$ as we know that $A_5$ is simple.

**Theorem 2:** Let G be a finite group and H⊆G. Suppose p is a prime dividing O(G). Let P be a sylow p–subgroup of H contained in some sylow p–subgroup S of G then P = S∩H.

**Problem 1:** If Q is another sylow p–subgroup of St. Q ⊆T we T = sylow p–subgroup of G then show S≠T if P≠Q As we know P = S∩H, Q = T∩H if S=T then P = Q. Proved.

**Aim:** A p-group is a group in which every element has order $p^r$. Where *P*=Prime, Hence *P* is same for all elements and r may vary.
Fore example the group $K_4$ = {1, (12) (34), (13) (24), (14) (23)} and the quaternion group are examples of finite *p*-groups. Here p=2 as me know in an finite group there is an element of finite order thus $S_3$ is not a *p*-group.

**Theorem 3:** We now project a method of constructing Sylow *p*-subgroups Inductively in the symetric group $S_p{}^k$.
Suppose P= Prime St. $P^r$/¬n and p $^{r+1}$×¬n.

Then r = $\sum_{j=1}^{\infty}\left[\frac{n}{p^r}\right]$, where [x] represents greatest not greater then x. In particular of n = $p^k$ then r= $p^{k-1}$+$P^{k-2}$ +.....+. We denote r by n (k) to the highest power of p dividing p¬k. when k=1, then clearly p of (Sp) = ¬p. and $p^2$×¬p ⇒p/(p-1)....2.1
⇒p/(p-r), 1≤r≤p-1 ⇒p≤p-r,
which is a contradiction.
Therefore order of Sylow *p*-subgroup in S*p* is *p* and group generated by (1, 2.......p) is a Sylow *p*-subgroup. So we have constructed Sylow *p*-subgroup when k=1. Assume that we have constructed it for k-1. Consider $S_p{}^k$.
Divide the set of $p^k$ letters 1,2,........$p^k$ into p set each consisting of $p^{k-1}$ letters as follows.
{1, 2, ..................... $p^{k-1}$}, ($p^{k-1}$+1..........2$p^{k-1}$}
{(p-1)} $p^{k-1}$+1...........$p^{k-1}$ p=$p^k$}
Let σ = (1$p^{k-1}$+1............ (p-1) $p^{k+1}$+1) (2$p^{k-1}$+2.....
(p+1) $p^{k-1}$+2$p^{k-1}$............$p^k$)
= product of $p^{k-1}$ disjoint cycles each of Length p clearly $\sigma^p$ = I as disjoint cycles commute.
Let A = {(t∈$s_p{}^k$/t(i)= *i* for all i>$p^{k-1}$}
∴ i∈A ie. A ≠ ϕ. let t, t' ∈A
⇒ t, t' (i) = i for all i > $p^{k-1}$
⇒ N∈ $tt^1$ (o) = i for all i> $p^{k-1}$
⇒ t, t' C-A≤$S_p{}^k$. But t∈A ⇒t is per mutation on $p^{k-1}$ letters and so A ∈ $S_p{}^{k-1}$
→ By induction hypothesis $S^{p-1}$ has sylow p-subgroup.

**Conclusion**
p-subgroup $P_3$ O($P_1$)= $P_n$ $^{(k-1)}$+......+ $p^{i-2}$
let $p_2$ = σ$p^1$$\sigma^{-1}$, $p_3$= $r^2$=$p_1$$r^{-2}$,........, $p_p$=$\sigma^{n-1}$$p_i$
$p_p$= r $p^{-1}$ $P_1$ σ ($p^{-1}$).
Each $p_i$ ≤ $S_p{}^k$. St. Pi ≅ $P_1$ (when x∈$p_1$ is ont $\sigma^{-1}$∈x$\sigma^{-1}$
∴ O ($p_i$) = P ($p_1$)
= $p^n$ (k-1)
Also r takes letters of first set into second. Set. Letter and so on.

So *t* ∈A ⇒ σT$\sigma^{-1}$ consists of letters from second set on T∈A means *T*(i) =*i* for all i > $p^{k-1}$. Similarly $\sigma^2$ *t* $\sigma^{-2}$ will consists of letter from third second then so on. Here T=$p_1$ $p_2$, ....., pp ≤$S_p{}^k$.
Also O (T) = O ($P_1$)O ($P_2$)......O ($P_p$)= $P_p$ ($n^{(k-1)}$).
= $p^{1+n}$ ($k^{-1}$). Thus O(p) = O (<σ> O(T)
= $p^{p2m}$ ($k^{-1}$)+1 = $p^p$(1+p+.......+p($^{(k-2)}$+1) = $p^{pm}$ ($k^{-1}$)
= $p^p$($n^{(k-1)}$1+1= pp1+P...........+p($k^{-2}$)+1
= $P^1$+P+........+ $P^{k-1}$ = $P^{n(k)}$. So P is required Sylow *p*-subgroup of G.

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