International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(6): 148-149

www.mathsjournal.com
Received: 26-09-2018
Accepted: 28-10-2018

Dr. Satya Prakash Gupta
Department of Mathematics,
Visiting Teacher of Mathematics,
Vidya Vihar College, M.M.H.A &
P University, Patna, Bihar,
India

Corresponding Author:
Dr. Satya Prakash Gupta
Department of Mathematics,
Visiting Teacher of Mathematics,
Vidya Vihar College, M.M.H.A &
P University, Patna, Bihar,
India

Prolongation and prologational limit sets its topological dynamics

Dr. Satya Prakash Gupta

Abstract
In this short paper we try to state and prove some Impovtent theorem which we are given as an exercise to N.P. Bhatia and G.P. Szego and V.V. Stepanove & V.V. Nemytskii.
Here we prove that every non-empty compact invariant set MCX contains some minimal sets. Now we also prove that a set MCX is minimal and only if for each $x \in M, xR = M$ from these two rsults.
We also prove that the omega limit set Ωx is minimal if for any two poihts $y, z \in \Omega x$ and any index $a \in A$ There exists $a\tau \in R$ such that $\pi(y\tau) \in V_a(z)$.

Keywords: Limits sets, prolongation and prolongational limit, minimal sets

Introduction

Main Results

Limit Set

A point $y \in X$ is called positive (omega) limit point of a point $x(\gamma(x) or\ \pi_x)$ if there exists a net $\{t_n\}$ in R^+ with $t_n \to \infty$ and $\pi(x, t_n) \to y$.
The set of all omega limit points of a point $x \in X$ is called omega limit set of X and is usually denoted by Ωx or $\Lambda^+(x)$.

i.e., $\Omega x = \{y \in X$: there exists a net $\{t_n\}$ in R^+

Such that $t_n \to \infty$ and $\pi(x, t_n) \to y\}$.
The set of all omega limit points of all points $x \in B$ c X will be called the omega limit set of B and is denoted by $\pi(B)$, i. e., $\Omega_{(B)} = U\{\Omega x : X \in B\}$. Similarly, we define negative limit (alpha limit) points of a point x ($\gamma(x)$ or π_x) and their set is denoted by a_x or $\Lambda(x)$

i.e., $a_x = \{y \in X$: there exists a net $\{t_n\}$ in R^-

Such that $t_n \to -\infty$ and $\pi(x, t_n) \to y\}$.
Now we mention some basic properties of limit sets without proof.

Theorem 1: For every For every $x \in X$
1. Ω_x is closed and invariant
2. $xR^+ = xR \cup \Omega_x$
3. If Ω_x is compact it is connected, hence it is a continuum.
4. If Ω_x is not compact none of its components is compact.

For the proof we refer to Bhatia [1].

Theorem 2: If closure of $\gamma^+(x)$ is compact Ω_x is non-empty, compact and annected. Hence it is continuum.

Proof: Now we first prove that if $\gamma^+(x)$ is compact then Ω_x is non-empty and compact. For, this we write $\Omega_x = \cap \gamma^+(\pi(x, t))$.

$t > 0$

But $\gamma^+(\pi(x,\tau))c\gamma^+(x)$ for $\tau > 0$. This implies that $\{\gamma^+(\pi(x,\tau)): \tau \in R^+\}$ is a decreasing family of closed. Subsets of $\gamma^+(x)$. Weting that it has finite intersection property and $\gamma^+(x)$. Noting that it has intersection property and $\gamma^+(x)$ is compact. As given from the characterization of compactness by finite intersection property, we have $\gamma^+(\pi(x,r)) \neq \emptyset$. Therefore Ω_x is non-empty and $\Omega_x\, c\gamma^+(x)$ is compact, closed subset of a compact space. Furthermore the compactness of Ω_x implies its connectedness [Theorem 1]. Consequently the omega limit set is a continuum, whenever it is compact.

Prolongation and prolongational limit sets

Let X be a topological space. For any point $x \in X$ the sets $D^+(x) \to (D(x))$ and $J^+(x)(J^-(x))$ defined as below are respectively called the first prolongation and prolongational limit sets of x. The '+' sign denotes the positive prolongation and the '-' sign stands for the negative protongation. These sets plays an important role in the theory of dynamical system, Particularly, in the stability theory and the theory of dispersive system. Their formal definitions run as follows:

i) $D^+(x) = \{y \in X$: there exists a net $\{x_n\}$ is X and there is a net $\{t_n\}$ in R^+ with $x_n \to x$ such that $\pi(x_n, t_n) \to y\}$.
ii) $D^-(x) = \{y \in X$: there exists a net $\{x_n\}$ is X and a net $\{t_n\}$ in R^- with $x_n \to x$ such that $\pi(x_n, t_n) \to y\}$.
iii) $J^+(x) = \{y \in X$: there exists a net $\{x_n\}$ is X and a net $\{t_n\}$ in R^+ with $x_n \to x$ and $t_n \to \infty$ such that $\pi(x_n, t_n) \to y\}$.
iv) $J^-(x) = \{y \in X$: there exists a net $\{x_n\}$ is X and a net $\{t_n\}$ in R^- with $x_n \to x$ and $t_n \to \infty$ such that $\pi(x_n, t_n) \to y\}$.

From above it is obivious that D^+ and D^- maps for X to 2^x. It is clear from the above definitions that for any $x \in X, \gamma^+(x) \subset D^+(x)$ and $\gamma^-(x) \subset D^-(x)$. For this we take $x_n = X$ and $t_n = t$ for all n in the above definitions. Also $+(x) \supset \Omega x$ and $J^-(x) \supset \alpha_x$

Minimal Set: A set $M \subset X$ is called minimal if it is non-empty closed and invariant and does not have any proper subset with these three properties that $\overline{xR} \subset N \subset M$, as N being closed and invariant this implies $xR \neq M$, is a contradiction to our assumption. Therefore, M is minimal.

Theorem 3: If a minimal set $M \subset X$ has an interior point then all its. points interior points.

Proof: Let $x \in M$ be in interior point of M, then there exists an index a such $V_1(x) \subset M$. Since M is minimal, for each $t \in R, \pi(V_a(x), t) \subset M$ this with the homomorphic property of π $(V_a(x), t)$ is a knighthood of $\pi(x, t)$. Thus every point of the trajectory is an interior point.

Theorem 4: the omega limit set Ω_x is minimal if for any two points y and z Ω_x and index $a \epsilon A$ there exists a $\tau \in R$ such that $\pi(y, \tau) \epsilon V_a(z)$.

Proof: Let us suppose on the contrary, i.e., Ω_x is not minimal. This implies there is a minimal set $M \subset \Omega_x$ Thus M and Ω_x M are two disjoint, empty, invariant sets.

for $\in M, \pi(y, t) \notin \Omega_x - M$ for all $t \in R$.

Compact set M and any point $z \in \Omega_{x-}$ M of a Housedroff space X can be by two disjoint neighbourhoods Hence there exists a and b in A that $V_a(z) \cap V_b(M) = \emptyset$. This implies $\pi(y, t) \notin V_a(z)$forall. $t \in R$.

References

1. Bhaitha NP, SZEGO GP. Dynamical System: Stability Theory and. Application L.N.M.-35 springer Varlag New York, 1967.
2. Birkhoff GD. Dynamical system Amer, Math Soc, Collq. Pub. New York 1927, 9.
3. Nemytskii VVI, Sepanove VV. Qualitative theory of differential equations Priceton University Press New Jersy, 1960.