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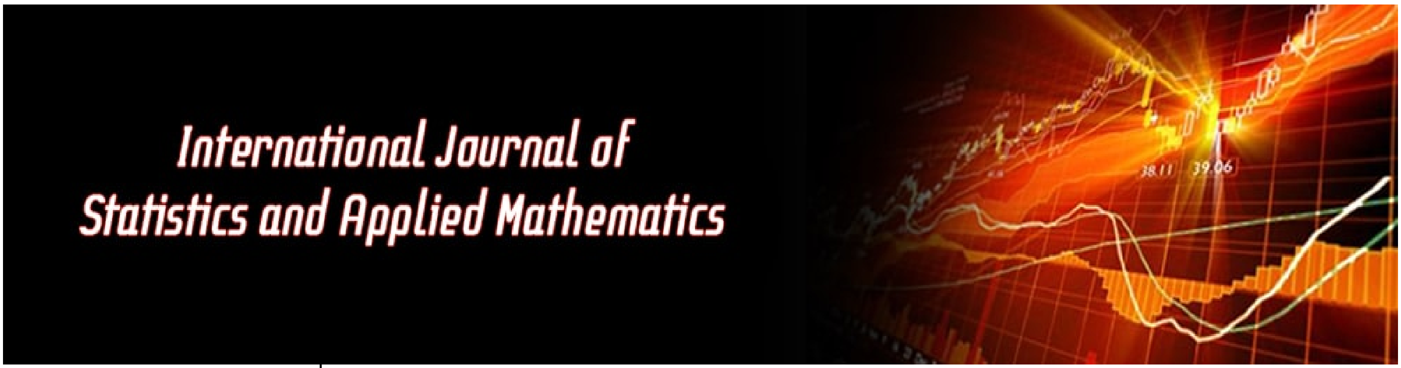

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Assessment on investigation of buckling performance of composite plates

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Abstract
The material of composite is one of the most widely used material when strength to weight ratio is considered, this make it more ideal material in the space-air craft industries where strength to weight ratio is more important. The in-plane and out - plane i.e. bending and twisting constants have been derived in terms of properties of elasticity of individual laminate and its thickness. The staking sequence and its orientation also play a role. Thus knowing the elastic property, the thickness and orientation of individual ply we can get the elastic properties of composite laminate.

Keywords: Laminates, mechanics of composites, fiber-reinforced, statistical properties/methods, composite plate

1. Introduction
The composite materials are typically a combination of usually light, weak and flexible binding material with a more dense, strong and high strength-to-weight and stiffness-to-weight ratios are reliably obtained. The coupling between in plane and out of plane displacements in common in these materials.

These coupling reduces the effective strength of stiffness of composites. Leissa [1] has shown that the coupling may be reduced in anti-symmetrically laminated plate by increasing the stacking of plies.

Abrams and Scheyhing [2] presented numerous configurations for which the elastic behavior of the laminate is similar to that of a homogeneous orthotropic or an isotropic material. The stiff reinforcing materials in fibrous or whisker from. Their constitutive equations of this multi layered plate or shall element is solved and compared with its counter parts for homogeneous media in order to determine the mathematical relations to ensure composite orthotropic and isotropic elastic behavior.

Bartholomew [3] has shown that selection of a suitable ply stacking order for uncoupled, orthotropic laminates enables orthotropic in bending, as well as in plane, to be achieved.

All the individual ply should be identical in the form of thickness and material properties, and the fibre directions lie in proportion about the orthotropic axes for the laminate. After achieving in-plane orthotropy, the remaining requirements for designing an especially orthotropic laminate are achieved by managing ply stacking sequence.

In the following lines are going to give mathematical analysis in section 2, section 3 will give relation between stiffness coefficients.

2. Analytical analysis
Following the notation of Bartholomew the matrix of elastic stiffness coefficients, *Qij*, for a ply is given as

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \quad (2.1)$$

With

$$Q_{11} = \frac{E_1}{\eta_0}, \; Q_{12} = \frac{\eta_{12} E_2}{\eta_0} = \frac{\eta_{21} E_1}{\eta_0}, \tag{2.2}$$

$$Q_{22} = \frac{E_2}{\eta_0}, \;\; Q_{66} = G_{12}, \eta_0 = 1 - \eta_{12}\eta_{21}, \tag{2.3}$$

Where E1, E2, η_{12}, η_{21} are Young's modulus and Poisson ratio. These elastic stiffness coefficients we referred to orthotropic axes.

If the fibre of such ply are taken to be inclined at an angle θ respective to x- axis of the reference plane, then stiffness $\bar{Q}$ is given as

$$\bar{Q} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \tag{2.4}$$

$$2\bar{Q}_{11} = [Q_{11}\cos^2\theta + (Q_{12} + Q_{66})\sin^2\theta](1 + \cos 2\theta) \;\; + [(Q_{12} + Q_{66})\cos^2\theta + Q_{22}\sin^2\theta](1 - \cos 2\theta) \tag{2.5}$$

$$8\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})(1 - \cos 4\theta) + 2Q_{12}(3 - \cos 4\theta) \tag{2.6}$$

$$2\bar{Q}_{22} = [Q_{11}\sin^2\theta + (Q_{12} + Q_{66})\cos^2\theta](1 - \cos\theta) \;\; +[(Q_{12} + Q_{66})\sin^2\theta + Q_{22}\cos^2\theta](1 + \cos 2\theta) \tag{2.7}$$

$$2\bar{Q}_{16} = \sin^2\theta[(Q_{11} - Q_{12} - 2Q_{66})\cos^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^2\theta] \tag{2.8}$$

$$2\bar{Q}_{26} = \sin^2\theta[(Q_{11} - Q_{12} - 2Q_{66})\cos^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos^2\theta] \tag{2.9}$$

$$8\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})(1 - \cos 4\theta) \\ +2Q_{66}\langle 3 - \cos 4\theta \rangle \tag{2.10}$$

3. Relation Between in Bending Moments, Twisting Couples with Strains and Plane Stresses

We assume that small deformation occurs for this we take thin plate theory. The assumption gives that reaction of laminates of applied membrane stresses and bending moments is linear. Elastic behavior is given by

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \in^0 \\ k \end{bmatrix} \tag{3.1}$$

In above N & M are membrane and bending stress resultant and twisting couple.

$$N = \left[N_x, N_y, N_{xy} \right] \tag{3.2}$$

$$M = \left[M_x, M_y, M_{xy} \right] \tag{3.3}$$

Where $\in^0, k$ are strains of middle plane and curvatures, respectively and given as

$$\in^0 = \left[\in_x^0, \in_y^0, \in_{xy}^0 \right] \tag{3.4}$$

$$k = \left[k_x, k_y, k_{xy} \right] \tag{3.5}$$

Now we consider a laminate of 4 plies as shown below and measured *zk* from top to bottom.

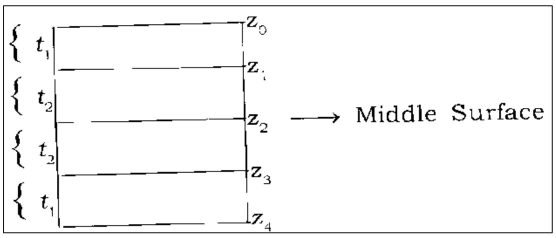

Fig 1: Geometry of 4-Layered Composite

The matrices A, B and D are defined below. k^{th} ply of the laminate lie between *Zk* and *Zk*–1. *t*1, *t*2 etc. are ply-thickness. We consider 4-layered composite as shown in above figure.

4. Relation between reduced stiffness constants of four layered composite and matrices of coefficients

The components of membrane stiffness matrices A, coupling matrices B and bending stiffness matrices D are given in terms of reduced-stiffness coefficients as

$$Q_{ij} = \sum_{k-1}^{4} \left(\bar{Q}_{ij}\right)_k (Z_k - Z_{k-1})$$

$$= \left(\bar{Q}_{ij}(\theta_1)\right)_1 (Z_1 - Z_0) + \left(\bar{Q}_{ij}(\theta_2)\right)_2 (Z_2 - Z_1)$$

$$+ \left(\bar{Q}_{ij}(\theta_2)\right)(Z_3 - Z_2) + \left(\bar{Q}_{ij}(\theta_2)\right)_4 (Z_2 - Z_3) \tag{4.1}$$

We consider fibre directions aligned at $+\theta_1$ and $+\theta_2$ of the co-ordinate system in the direction of *x*-axis.

$$A_{ij} = 2\bar{Q}_{ij}(\theta_1)\left\langle t_1 + \alpha_{ij} t_2 \right\rangle \tag{4.2}$$

$$\alpha_{ij} = \left(\bar{Q}_{ij}(\theta_2) / \left(\bar{Q}_{ij}(\theta_1)\right)\right) \tag{4.3}$$

Where $\bar{Q}_{ij}(\theta)$ is reduced stiffness evaluated at an angle of orientation of fibre, θ. For an antisymmetric angle ply plate

$$\alpha_{ij} = 1 \Rightarrow \theta_1 = \theta_2$$

Then extensional stiffness, A_{ij}, is given as $A_{ij} = 2\,\bar{Q}_{ij}(\theta_1).$
Thickness of plate is *t*.

Then

$$A_{ij} = \bar{Q}_{ij}(\theta)t \tag{4.4}$$

Coupling matrix is defined as

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{4}\bar{Q}_{ij}(\theta_1)\left(z_k^2 - z_{k-1}^2\right)$$

$$= \bar{Q}_{ij}(\theta_1)\left[t_2^2(1 + \alpha_{ij}) - (t_1 + t_2)^2\right] \tag{4.5}$$

Where $\alpha_{ij} = \bar{Q}_{ij}(\theta_2) / \bar{Q}_{ij}(\theta_1).$ It we want that there should not be coupling, then we take

$$t_2^2(1+\alpha_{ij}) - (t_1 + t_2)^2 = 0,$$

Or

$$1 + \alpha_{ij} - (1 + R_1)^2 = 0, \quad R_1 = t_1 / t_2, \tag{4.6}$$

Or

$$R_1^2 + 2R_1 - \alpha_{ij} = 0, \; R_1 = -2 \pm \sqrt{1 + \alpha_{ij}}, \tag{4.7}$$

If we take $\alpha_{ij} = 1$ i.e., the reduced stiffness coefficients of outer and inner ply are same than, $R_1 = \cdot 4$ approximately

5. Conclusion

We can say that accordingly above analysis gives that inner ply should Abe thicker than the outer one. If so coupling will not occur. Thus, $B_{ij} = 0$

Similarly for D_{ij} with $\alpha_{ij} = 1$
Weget

$$D_{ij} = \begin{cases} 8/3\,\bar{Q}_{ij}(\theta_1)\,t^3, & j \neq 4 \\ 0 & , \; j = 4 \end{cases}$$

Thus we see that bending stiffness coefficients depend upon total thickness of plate and not on the thickness of independent ply

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