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MHD free convection heat transfer having vertical fin in a square wavy cavity

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Abstract
MHD free convection heat transfer in a square wavy cavity with single vertical fin attached to its lower heated wall has been numerically simulated in this paper. The bottom wall is kept at a constant heat temperature T_n and the upper wall is kept at a constant cold temperature T_c, while the wavy vertical walls are kept at adiabatic. The magnetic field of strength B_0 is applied parallel to x-axis. Finite element method based Galerkin weighted residual technique is used to solve the governing equation. The Prandlt number for the flow inside the enclosure is 0.71. A parametric study has been carried out to investigate the effect of Rayleigh number and Hartmann number on the fluid flow and heat transfer characteristic inside the cavity. The obtained results indicated that the heat transfer rate is enhanced with the growth of Rayleigh number when Hartmann number is kept constant. A set of graphical results are presented in terms of streamlines, isotherms, velocity profiles, temperature profiles, local Nusselt number and average Nusselt number. The results are validated comparing with the previous published works.

Keywords: Free convection, MHD, Square wavy cavity and fin

1. Introduction

Free convection is one of the most important phenomena in thermal systems because of its wide range of applications. The relevant research output has numerous applications such as solar energy collectors, cooling of nuclear reactors, ventilation of rooms, fire prevention, electronic cooling devices, biological and geothermal devices etc.

A large number of research works have been done on different shapes of cavities. Bilgen [1] studied numerically natural convection heat transfer inside a differentially heated cavity with a horizontal fin attached to the hot wall. The result showed that the heat transfer rate was minimum with the fin attached to the middle or near the middle of the heated wall. Shi and Khodadadi [2] investigated the steady laminar natural convection heat transfer in a differentially heated square cavity due to a thin fin on the hot wall. They found that for higher value Rayleigh number, heat transfer rate was enhanced irrespective of the fin position or length. Ben-Nakhi *et al.* [3] studied numerically conjugate natural convection in a square enclosure with inclined thin fin of arbitrary length. Tasnim *et al.* [4] investigated numerically natural convection heat transfer in a square cavity with a baffle on hot wall. They observed that the effect of fin position on the heat transfer rate was depended strongly affected by Rayleigh number and the fin length. Jani *et al.* [5] carried out the effect of magneto hydrodynamic free convection fluid flow in a square cavity heated from below and cooled from other walls. They showed that free convection parameter and Hartmann number have notable effect on flow structure and temperature field. Xu *et al.* [6] reported the effect of fins and their height on natural convection flow transition cavity. They found that the flow near the finned wall changes from a steady to periodic unsteady flow at a critical Rayleigh number that is sensitive to the fin length. Sun *et al.* [7] reported the effect of triangular fins on mixed convection in a lid-driven cavity. They observed that the triangular fin is a good control parameter for flow structure, temperature field and rate of heat transfer. Pirmohammadi *et al.* [8]. Studied steady laminar free convection flow in presence of a magnetic field in an enclosure heated from left and cold from right wall.

The result showed that with increasing Hartmann number the rate of convective transfer and the average Nusselt number increase. Mahmud and Faster [9] investigated the magneto hydrodynamic free convection and entropy generation in a square porous cavity. They found that the fluid velocity is reduced with increasing the value of Hartmann number. Xu and Saha [10] studied the transition to an unsteady flow due to an adiabatic fin on the sidewall of a square cavity. Gdhaidh *et al.* [11] investigated analytically the enhancement of natural convection heat transfer within closed enclosure using parallel fins. They observed that as the fin number increases the maximum heat source temperature decreases. Elatar *et al.* [12] performed a numerical study on laminar natural convection inside square enclosure with adiabatic horizontal wall with a single horizontal fin at different length and positions attached to the hot wall. They investigated the effect of Rayleigh number, fin lengths and fin positions of the enclosure on fluid flow structure and heat transfer characteristics. Ali *et al.* [13] studied the magneto hydrodynamic mixed convection flow in a hexagonal enclosure. They found that Hartmann number and Richardson number have considerable effect on the flow field and temperature field. Rostami [14] numerically simulated the unsteady fluid flow and heat transfer behavior in a cavity with vertical wavy walls and horizontal straight walls. Abu-Nada and Chamkha [15] examined the mixed convection flow of nanofluid in lid-driven cavity with a wavy wall. Sheremet *et al.* [16] studied the flow and convective heat transfer in a partially heated wavy porous cavity filled with a nanofluid. Alsabery *et al.* [17] reported the effect of rotating solid cylinder on entropy generation and convective heat transfer in a wavy porous cavity heated from below.

Based on the above literature survey and to the best of author’s knowledge, it appears that very little work reported on the wavy enclosure having vertical fin. There is no present study on MHD free convection heat transfer having vertical fin in a square wavy cavity will be numerically analyzed in this present study. The objective of this present study is to analyze the increase of the heat transfer rate in fluid through wavy cavity having vertical fin.

2. Physical Configuration

The physical model and boundary conditions of the present problem under consideration are shown in Fig. 1. Which represents of two dimensional square wavy enclosure with side length W. The bottom wall is kept at a constant heat temperature T_h and the upper wall is kept at a constant cold temperature T_c ($T_h > T_c$), while the wavy vertical walls are kept at adiabatic. A heated fin of length (l) and thickness (b) is attached to the bottom wall at a position (h) from the left surface. The gravitational force acts in the vertically downward direction and a uniform external magnetic field of strength *(B_0)* is applied along the horizontal direction.

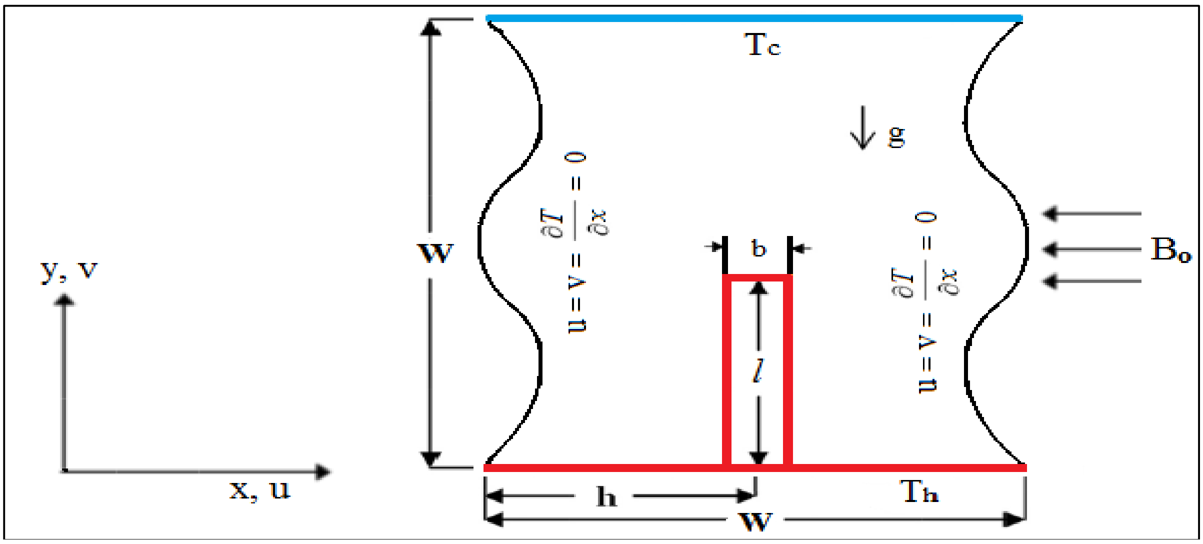


Fig 1: Schematic description of the physical model and boundary conditions

3. Mathematical Formulations

The physical domain is shown in Fig. 1. The flow is considered steady, laminar, incompressible and two-dimensional, which is filled with electrically conducting fluids $Pr = 0.71$ interact.The governing differential equations for free convection flow using conservation of mass, momentum and energy can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\sigma {B_0}^2 v+\rho g\beta(T-T_c) \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

where u and v are the velocity components along x and y directions respectively, p is the pressure, ρ is the density, μ is the dynamic viscosity, β is the coefficient thermal expansion, σ is the electrical conductivity, B_0 is the magnetic field, T is the temperature, g is the gravitational force, $v = \frac{\mu}{\rho}$ is the kinematic viscosity and α is the thermal diffusivity.

Boundary conditions

The boundary conditions for the present problem are specified as follows:
On the top wall: $u = 0, v = 0, T = T_c$
On the left vertical wall: $u = 0, v = 0, \frac{\partial T}{\partial x} = 0, A(1 - \cos(2\pi\lambda x))$
On the right vertical wall: $u = 0, v = 0, \frac{\partial T}{\partial x} = 0, 1 - A(1 - \cos(2\pi\lambda x))$
On the bottom wall: $u = 0, v = 0, T = T_h$

For the Fin surface: $0 \le y \le l;\ x = h + \frac{b}{2}$ and $x = h - \frac{b}{2};\ u = v = 0$

Using the following dimensionless parameters, the governing equations can be converted to the dimensionless forms:

$$X = \frac{x}{W};\ Y = \frac{y}{W};\ U = \frac{uW}{\alpha};\ V = \frac{vW}{\alpha};\ P = \frac{pW^2}{\rho\alpha^2};\ \theta = \frac{T - T_c}{T_h - T_c};\ H = \frac{h}{W};\ L = \frac{l}{W} \text{ and } B = \frac{b}{W} \tag{5}$$

where, X and Y are the coordinates varying along horizontal and vertical directions, U and V are the velocity components in the X and Y directions respectively, θ is the dimensionless temperature and P is the dimensionless pressure. After substitution of the dimensionless variables into equations (1) - (4), we get the following dimensionless equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{6}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{7}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - Ha^2 PrV + RaPr\theta \tag{8}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{9}$$

In the above equations, Pr is the Prandtl number, Ha is the Hartmann number or magnetic parameter and Ra is the Rayleigh number defined by:

$$Pr = \frac{v}{\alpha};\ Ha = B_0 W\sqrt{\frac{\sigma}{\rho v}} \text{ and } Ra = \frac{g\beta(T_h - T_c)W^3}{\alpha v} \tag{10}$$

The transformed non-dimensional boundary conditions are as follows:
On the top wall: $U = 0, V = 0, \theta = 0$
On the left vertical wall: $U = 0, V = 0, \frac{\partial\theta}{\partial x} = 0, 1 - A(1 - \cos(2\lambda\pi X))$
On the right vertical wall: $U = 0, V = 0, \frac{\partial\theta}{\partial x} = 0, A(1 - \cos(2\lambda\pi X))$
On the bottom wall: $U = 0, V = 0, \theta = 1$

For the fin surface: $0 \le Y \le L;\ X = H + \frac{B}{2}$ and $X = H - \frac{B}{2};\ U = V = 0$

Nusselt Number

First, the heat transfer by conduction was equated to the heat transfer by convection

$$h^* \Delta T = -k\frac{\partial T}{\partial n} \tag{11}$$

where n is the normal distance to coordinate surface. By introducing the dimensionless variables, defined in equation (5) into equation (11), local Nusselt number is defined as:

$$Nu_L = -\frac{\partial\theta}{\partial N}\bigg|_{Surface} \tag{12}$$

The average Nusselt number on the cold wall is obtained as follows:

$$Nu_{av} = -\int_{2+2W}^{3+2W} \frac{\partial\theta}{\partial N}\bigg|_{Surface} dS \tag{13}$$

4. Numerical Technique

The nonlinear governing partial differential equations, i.e., mass, momentum and energy conservation equations are transformed into a system of integral equations by using the Galerkin weighted residual method of finite-element formulation. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations with the help of Newton's method. Lastly, these linear equations are solved by applying Triangular factorization. For numerical computation and post processing, the software COMSOL Multiphysics is used.

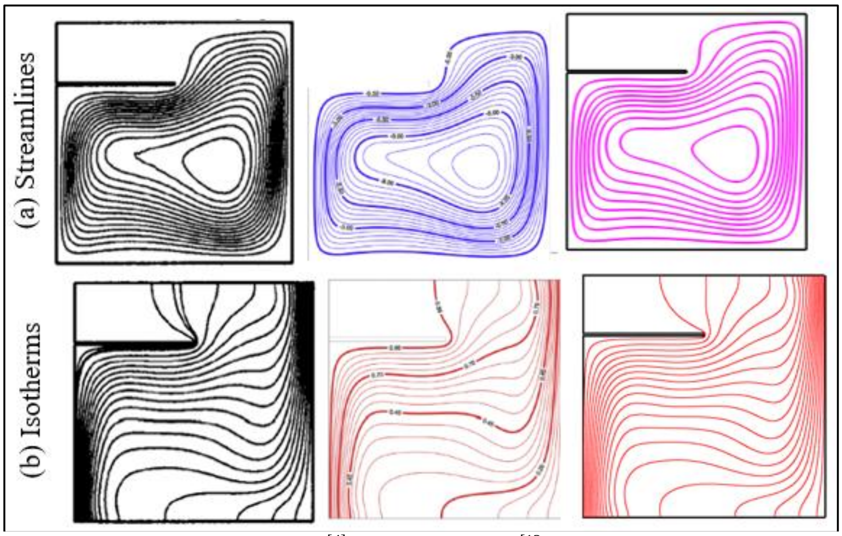
5. Program Validation and Comparison with Previous Work

In order to check the accuracy of the numerical results obtained in this problem, a comparison is made between results of the average Nusselt number obtained from this study with other published results for enclosure without a fin as shown in Table 1. The maximum derivation found was 1.55% for the case of $Ra = 10^6$.

Table 1: Comparison of average Nusselt number obtained from the present study and those reported in the literature.

Ra	10^4	10^5	10^6
Shi and Khodadadi [2]	2.247	4.532	8.893
Tasnim and Collins [4]	2.244	4.524	8.855
Elatar *et al.* [12]	2.234	4.517	8.948
Present result	2.245	4.522	8.835

In addition, a comparison of the streamlines and isotherms is made for the present results with those of Tasnim and Collins [4] and Elatar *et al.* [12] at ($Ra = 10^5, B = 0.01, L = 0.5$ and $H = 0.75$) as shown in Fig. 2. The shapes of streamline contours are almost identical. For the isotherms, one can see the strong agreement of the present results with those by Tasnim and Collins [4] and Elatar *et al.* [12] as seen in the figure.



Tasnim and collins [4] Elatar *et al.* [12 Present results

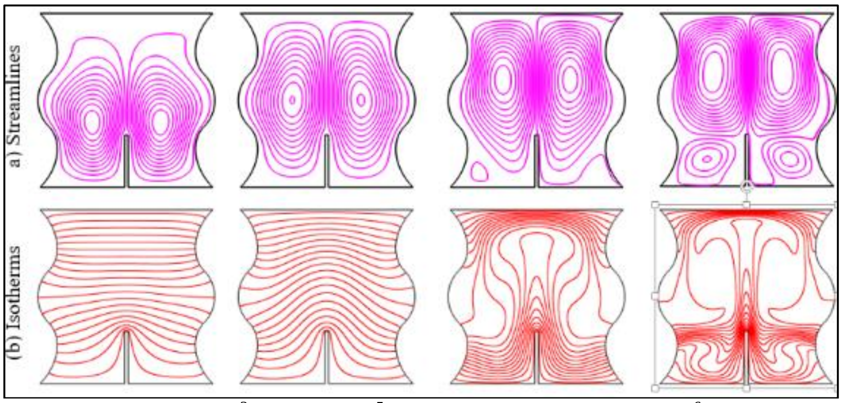
Fig 2: Comparison of Streamlines and isotherms with $Ra = 10^5$, $B = 0.1$, $L = 0.50$ and $H = 0.75$

6. Result and Discussion

In this numerical analyze, the MHD free convection heat transfer in a square wavy cavity with single vertical fin attached to its heated wall has been carried out. The effect of the heat transfer inside the square wavy cavity is studied using the following range of values are given Rayleigh number($10^3 \leq Ra \leq 10^6$), Hartmann number ($0 \leq Ha \leq 50$), Fin length ($L = 0.30$), Position $H = 0.5$ and Thickness ($B = 0.02$) while $Pr = 0.71$ and then illustrated graphically.

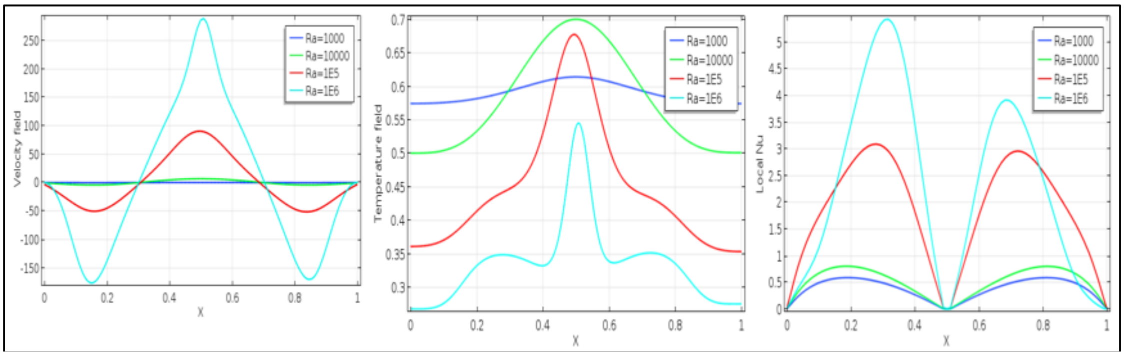
Effect of Rayleigh number

The results are presented in terms of streamlines, isotherms in Fig. 3 and in terms of velocity profiles, temperature profiles and local Nusselt number along the bottom heated wall are shown in Fig. 4. In Fig. 3(a) shows that when Ra between 10^3 and 10^5 the strength of buoyancy inside the cavity is significant and two elliptic-shaped eddies inside the cavity, Again when $Ra = 10^6$ the strength of buoyancy inside the cavity is more significant and four elliptic-shaped eddies appear inside the cavity. Conduction dominant heat transfer is observed from the isotherms in Fig. 3(b) for Rayleigh number ($10^3 \leq Ra \leq 10^6$) with $Pr = 0.71$ and $Ha = 10$. It can be seen from the figure that the isotherms appear almost parallel to the horizontal walls for the lower value of Ra. With increase in Rayleigh number, isotherms are found to concentrate the top and both sides of the fin respectively and isotherm lines are bending more which means increasing heat transfer through convection. Variation of vertical velocity profiles along the horizontal centre line for different Rayleigh number with $Pr = 0.71$ and $Ha = 10$ of cavity is shown in Fig. 4(a). It can be seen from this figure that for lower values of Rayleigh number vertical velocity profiles has smaller change but higher value of Rayleigh number velocity profiles has larger change. Variation of vertical dimensionless temperature profiles along the horizontal centre line for different Rayleigh number with $Pr = 0.71$ and $Ha = 10$ of cavity is shown in Fig. 4(b). It can be seen that, the temperature increases with the increasing of Rayleigh number. The local Nusselt number along the bottom hot wall including both fin surfaces and the fin tip for different Rayleigh number with $Pr = 0.71$ and $Ha = 10$ of the cavity are shown in Fig. 4(c). It can be seen from this figure that, lower value of Rayleigh number local Nusselt number has smaller change but higher value of Rayleigh number local Nusselt number has larger change.



Ra=10^3 Ra=10^5 Ra=10^6

Fig 3: Streamlines and isotherms for different values of Rayleigh number $(10^3 \le Ra \le 10^6)$ at $\lambda = 2, Pr = 0.71$ and $Ha = 10$.



A B C

Fig 4: Variation of Rayleigh number$(10^3 \le Ra \le 10^6)$ on (a) Velocity profiles (b) Dimensionless temperature and (c) Local Nusselt number along the horizontal wall while $Pr = 0.71, Ha = 10$ and $\lambda = 2$.

Effect of Hartmann number

The results are presented in terms of streamlines, isotherms in Fig. 5 and in terms of velocity profiles, temperature profiles and local Nusselt number along the bottom hot wall are shown in Fig. 6. Fig. 5(a) shows that when $Ha = 0$ the strength of buoyancy inside the cavity is significant and two elliptic-shaped eddies appear inside the cavity. Again when Hartmann number increases, the strength of the buoyancy inside the cavity is more significant and two elliptic-shaped eddies move downward and close to the bottom wall appear. Conduction dominant heat transfer is observed from the isotherms in Fig. 5(b) for Hartmann number $(0 \le Ha \le 50)$ with $Pr = 0.71$ and $Ra = 10^4$. When lower value Hartmann number isotherms lines concentrate near the upper wall and top side of the fin respectively and isotherm lines are bending more which means increasing heat transfer through convection. When Hartmann number increases the isotherms appear almost parallel to the horizontal walls. Variation of vertical velocity profiles along the horizontal centre line for different Hartmann number with $Pr = 0.71$ and $Ra = 10^4$ of cavity is shown in Fig. 6(a). It can be seen from this figure that absolute value of maximum and minimum value of velocity increases with decreasing Hartmann number but velocity decreases with increases for Hartmann number. Variation of vertical dimensionless temperature profiles along the horizontal centre line for different Hartmann number with $Pr = 0.71$ and $Ra = 10^4$ of cavity is shown in Fig. 6(b). It can be claimed that, the temperature increases with the decreasing of Hartmann number. The local Nusselt number along the bottom hot wall including both fin surfaces and the fin tip for different Hartmann number with $Pr = 0.71$ and $Ra = 10^4$ of the cavity are shown in Fig. 6(c). It can be seen from this figure that, the highest value of Hartmann number bring smaller change but lower value of Hartmann number causes larger change.

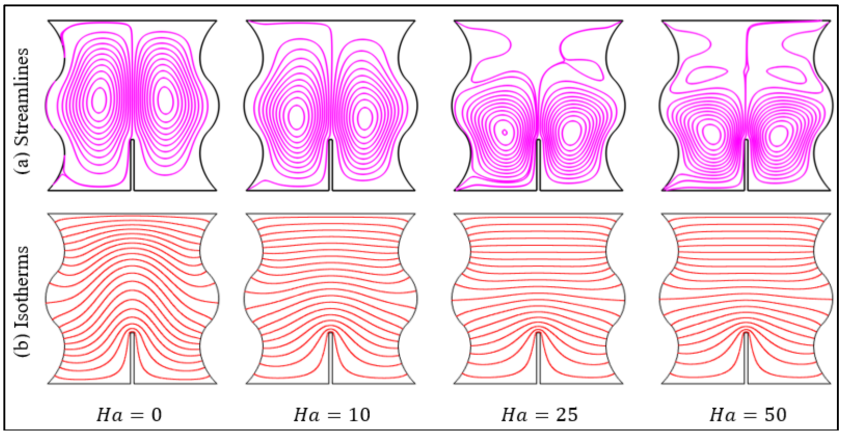
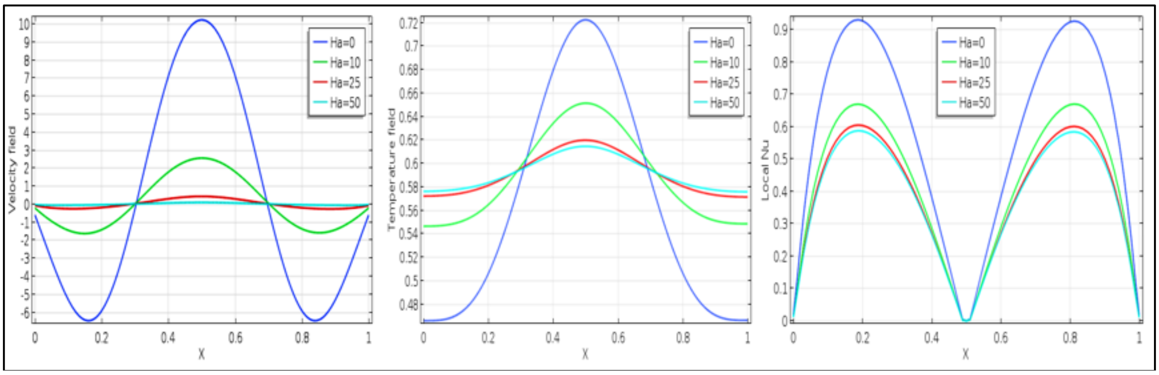


Fig 5: Streamlines and isotherms for different values of Hartmann number$(0 \leq Ha \leq 50)$ at $\lambda = 2, Ra = 10^4$ and $Pr = 0.71$.



A b c

Fig 6: Variation of Hartmann number$(0 \leq Ha \leq 50)$ on (a) Velocity profiles (b) Dimensionless temperature and (c) Local Nusselt number with $Pr = 0.71, Ra = 10^4$ and $\lambda = 2$.

Plot of the average Nusselt number of the fin surface as a function of Hartmann number $(0 \leq Ha \leq 50)$ at different Rayleigh numbers $(10^3 \leq Ra \leq 10^6)$ is shown in Fig. 7. It can be seen from this figure that, average Nusselt number increases when the value of Ra increases for a particular value of Ha. Moreover, the increasing rate of heat transfer is more enhanced for higher Ra with lower Ha. At a constant Hartmann number, with increase in Rayleigh number the buoyancy force increases and the heat transfer is enhanced. Therefore at high Rayleigh numbers, a relatively stronger magnetic field is needed to decrease the rate of heat transfer.

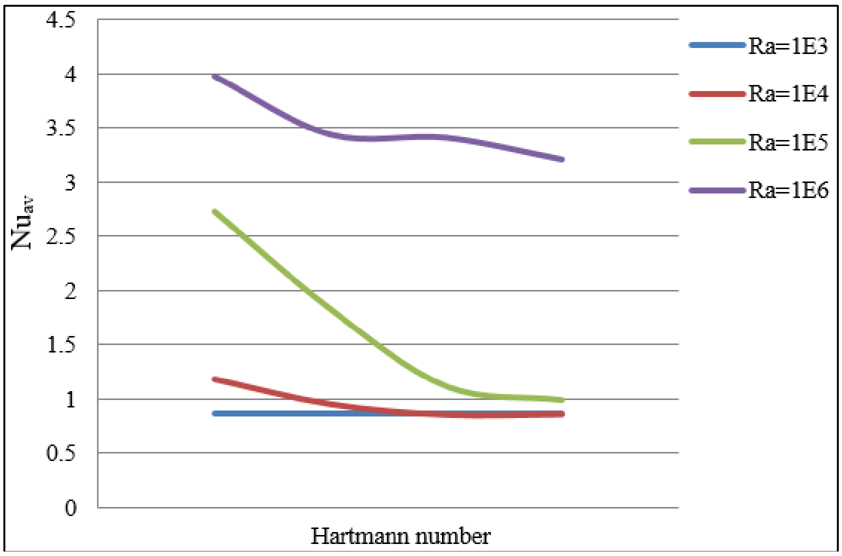


Fig 7: Variation of the average Nusselt number of the fin surface versus Hartmann number $(0 \leq Ha \leq 50)$ for different values of Rayleigh numbers $(10^3 \leq Ra \leq 10^6)$ while $Pr = 0.71$.

7. Conclusion

The effect of magnetic field on the flow structure and heat transfer behaviors for free convection flow and heat transfer within a square wavy cavity with single vertical fin has been studied numerically by using finite element method. The numerical procedure was validated by comparing the average Nusselt number for a square enclosure obtained by the code with the existing results in the literature. Very good agreements were observed between them. Based on the findings, it can be concluded that:

- The flow field inside the cavity has appeared to be noticeably effected by the variation of Rayleigh number.
- Velocity, temperature as well as heat transfer rate increased with the increasing Rayleigh number.
- Velocity, temperature as well as heat transfer rate increased with the decreasing Hartmann number.
- No significant change occurred in isotherms for higher values of Hartmann number. It is also found that, the heat transfer rate is enhanced for greater Rayleigh number with lower Hartmann number.

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