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Limited failure censored life test sampling plan in half logistic distribution

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Abstract
The random life time of a product in a submitted lot for acceptance or otherwise is assumed to follow Half Logistic distribution. The decision making process is based on the inspection of a sample of items for their lifetimes taken from the lot. Such a sample is divided into groups to develop a group sampling plan with a mechanism of terminating life testing process as soon as the first failure in each group is noticed. The criterion for accepting the submitted lot is proposed.

Keywords: Single sampling, lot acceptance, group sampling plan, truncated life tests, reliability test plans, order statistics

1. Introduction
Acceptance sampling is concerned with inspection and decision making regarding products. If the quality of a product is measured through the life time, sampling plans to determine acceptability of a product with respect to life time are called Reliability Sampling Plans. When the life time random variable is assumed to follow a specific continuous probability distribution, sampling plans are developed by various researchers covering a wide spectrum of probability models. Most of these works are referred in a recent article by Kantam and Ravikumar (2016) [2].

In all these works, given the termination time of a life test, the construction of the sampling plan consists of determining the minimum number of sample items that are to be life-tested and the acceptance number beyond which the observed failures out of the life-tested items of the sample lead to rejection of the submitted lot, conditioned on pre specified producer's and consumer's risks. Deviating from the methodology of these works, Kantam and Ravikumar (2016) [2] proposed a new sampling plan called Limited Failure Censored Life Test Sampling Plan (LFCLTSP). Similar work is done by Srinivasarao *et al.* (2018) [5] for Dagum distribution. In fact LFCLTSP is an alternative criterion to what is proposed by Jun *et al.* (2006) [1]. Also the probability model considered by Jun *et al.* (2006) [1] is the Weibull distribution whereas the probability model considered in Kantam and Ravikumar (2016) [2] is Burr Type X distribution – both are popular life testing models. This scheme of life testing and termination process is named by some researchers as *Sudden Death Testing* (for example Pascual and Meeker – 1998; Jun *et al.* (2006)) [4, 1]. 'Limited failure censored life tests' is the name proposed by Wu *et al.* (2001) [6].

In the present paper, LFCLTSP is developed for the probability model Half Logistic distribution on lines of Kantam and Ravikumar (2016) [2]. Construction of LFCLTSP for Half Logistic distribution is presented in Section – 2. The results are explained by an example in Section – 3.

2. The proposed sampling plan
Let the limited failure censored sample -$Y_1, Y_2, \dots, Y_m$ which are *m* first order statistics in *m* independent random samples of size *n* each. If *Z* denotes the maximum of $Y_1, Y_2, \dots, Y_m$ it may also be viewed as the total test time/experimental time as opined by Kantam and Srinivasa Rao (2004) [3].

Hence, larger realised value of Z can be considered as an indication that the products in the submitted lot have longer life prompting one to consider the lot as a good lot for acceptability. In other words "$Z > cL$" can be taken as a criterion of acceptance of the lot. Thus Kantam and Ravikumar (2016) [2] proposed following decision rule.

i. Draw a random sample of size $N = m \times n$ and allocate n items to each of the m groups.
ii. Observe Y_i the time to the first failure in the i^{th} group (i=1, 2,…., m).
iii. Identify the quantity $Z = Max(Y_1, Y_2, \ldots, Y_m)$.
iv. Accept the lot if $Z \geq cL$ and reject the lot otherwise (c may be called acceptability constant – a concept similar to the acceptance number in time truncated reliability test plans).

Using the theory of order statistics we can get the cumulative distribution function (cdf) of Z in a closed form as long as the cdf of the base line distribution is in a closed form. Hence the percentiles of Z can be used to get the design parameters m, c analytically.
For our focal distribution namely Half Logistic distribution, the following is the analytical procedure of calculating design parameters of LFCLTSP.
The probability density function (pdf) of Half Logistic distribution (HLD) is given by

$$f(x) = \frac{2e^{-x}}{\left(1+e^{-x}\right)^2}, 0 < x < \infty \quad for \quad x \geq 0. \tag{2.1}$$

The cumulative distribution function (cdf) of Half Logistic distribution is

$$F(x) = \frac{1-e^{-x}}{1+e^{-x}}, 0 < x < \infty \quad for \quad x \geq 0. \tag{2.2}$$

The fraction non-conforming or unreliability is expressed by

$$p = \Pr\{X < L\} = F(L) \tag{2.3}$$

If p is given, the corresponding L is obtained from

$$w = L = -\ln\left[\frac{1-p}{1+p}\right]. \tag{2.4}$$

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from (2.2)
The cdf of least of $X_1, X_2, \ldots, X_n$ is given by

$$F_{(1)}(x) = 1 - \left[1 - F(x)\right]^n \tag{2.5}$$

That is

$$F_{(1)}(x) = 1 - \left[1 - \left(\frac{1-e^{-x}}{1+e^{-x}}\right)\right]^n \tag{2.6}$$

$Y_1, Y_2, \ldots, Y_m$ of the limited failure censored test are now a random sample of size m from $F_{(1)}(x)$. Hence, the cdf of Z – the largest of $Y_1, Y_2, \ldots, Y_m$ is given by

$$G_{(m)}(z) = \left[F_1(z)\right]^m \tag{2.7}$$

$$i.e., G_{(m)}(z) = \left[1 - \left[1 - \left(\frac{1-e^{-x}}{1+e^{-x}}\right)\right]^n\right]^m \tag{2.8}$$

The design parameters m and c of LFCLTSP are obtained with the help of percentiles of $G_{(m)}(z)$ given in (2.8). If α and β are respectively the producer's and consumer's risks for desirable/acceptable lot quality level p_0, undesirable/lot tolerance quality level p_1 then m and c are the solutions of the following two inequalities.

$$G_m(cw_0) \leq \alpha \tag{2.9}$$

$$G_m(cw_1) \geq 1 - \beta \tag{2.10}$$

Where w_0 and w_1 are the solution of (2.4).
The inequalities (2.7), (2.8) respectively imply

$$cw_0 \leq G_m^{-1}(1-\alpha) \tag{2.11}$$

$$cw_1 \geq G_m^{-1}(\beta) \tag{2.12}$$

which jointly lead to
$$\frac{w_0}{w_1} \leq \frac{G_m^{-1}(1-\alpha)}{G_m^{-1}(\beta)} \tag{2.13}$$

Therefore, m can be obtained by the smallest integer satisfying (2.13). The acceptability constant c can be obtained from the equality case in either of the expressions (2.11), (2.12). We have tabulated the values of m and c analytically determined for the selected combinations of p_0, p_1 and is presented in Table 1. The values of m obtained by LFCLTSP can be seen to be consistently smaller, thus this sampling plan indicating less number of items to be put to life test.

Table 1: Design Parameters of LFCLTSP for HLD at α=0.05 and β=0.1

(Min-Max) Approach for HLD at α=0.05 and β=0.1					
p_0	p_1	***m***		***c***	
		$n = 5$	$n = 10$	$n = 5$	$n = 10$
0.005	0.02	6	6	34.40831	17.87877
	0.04	3	3	17.60606	8.9881
	0.06	2	2	9.87983	4.9994
	0.08	2	2	9.87983	4.9994
	0.10	2	2	9.87983	4.9994
	0.14	2	2	9.87983	4.9994
	0.20	2	2	9.87983	4.9994
0.01	0.04	6	6	17.20372	8.93916
	0.06	4	4	12.0781	6.21017
	0.08	3	3	8.80281	4.49394
	0.10	2	2	4.93979	2.49964
	0.14	2	2	4.93979	2.49964
	0.20	2	2	4.93979	2.49964
0.02	0.06	9	11	11.34131	13.43727
	0.08	6	6	8.601	8.93916
	0.10	4	5	6.03845	7.67461
	0.14	3	3	4.40097	4.49394
	0.20	2	2	2.46965	2.49964
0.03	0.08	14	14	9.60959	5.09951
	0.10	8	8	7.02109	3.67539
	0.14	5	5	4.9455	2.55752
	0.20	3	3	2.93349	1.49758
0.04	0.10	15	18	7.44702	4.31488
	0.14	7	8	4.81207	2.7559
	0.20	4	5	3.01802	1.91769
0.05	0.14	11	12	5.08921	2.81917
	0.20	6	6	3.43799	1.7864
0.07	0.20	10	12	3.44329	2.01208

3. Example

The quality assurance in a bearing manufacturing process states that p_0 =0.03, p_1 =0.20, α=0.05, β=0.1 the number of test positions (size of each group, *n*)=10. For this information Table – 1 of suggests *m*=6, *c*=1.91769. Accordingly a random sample of size *N*=50 items are put to test in five groups with 10 items in each group. The observed first failure times in the five groups are Y_1 =120, Y_2 =200, Y_3 =185, Y_4 =55, Y_5 =265. Assuming that the life times follow Half Logistic distribution and a lower specification of L=100 they have at the above p_0, p_1 , α, β, *n*=10, and acceptability constant *c*=1.91769 then cL =191.769. *Z*= The maximum of 55,120,185= 185. Since $Z < cL$. *i.e.,* 185 < 191.769, the lot is to be rejected.

From this example, we see that our approach reached the decision of rejecting the lot by conducting limited failure censored life test for only three groups of 10 items each, resulting in low cost of experimentation and lower number of destructions.

More over it may be recalled that Z are defined as $Z = Max(Y_1, Y_2, \ldots, Y_m)$. If *c* is the acceptability constant and *L* is the lower specification, $Z > cL$. That is acceptance decision of LFCLTSP is considered and gives a stronger conclusion with this illustration.

4. References

1. Jun Ch-H, Balamurali S, Lee SH. Variables Sampling Plans for Weibull Distributed Lifetimes under Sudden Death Testing. IEEE Transactions on Reliability. 2006; 55(1):53-58.
2. Kantam RRL, Ravikumar MS. Limited Failure Censored Life Test Sampling Plan in Burr Type X Distribution. Journal of Modern Applied Statistical Methods. 2016; 15(2):428-454.
3. Kantam RRL, Srinivasa Rao G. A Note on Savings in Experimental Time under Type II Censoring. Economic Quality Control. 2004; 19(1):91-95.
4. Pascual FG, Meeker WQ. The Modified Sudden Death Test: Planning life tests with a limited number of test positions. Journal of Testing and Evaluation. 1998; 26(5):434-443.
5. Srinivasa Rao B, Sricharani P, Ravikumar MS. Limited Failure Censored Life Test Sampling Plan in Dagum Distribution. American Journal of Applied Mathematics and Statistics. 2018; 6(5):181-185.
6. Wu J-W, Tsai T-R, Ouyang L-Y. Limited Failure-Censored Life Test for the Weibull Distribution. IEEE Transactions on Reliability. 2001; 50(1):107-111.