International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2019; 4(3): 47-53

www.mathsjournal.com
Received: 22-03-2019
Accepted: 24-04-2019

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Three dimensional convective MHD flow and heat transfer in a porous medium

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Abstract
The effect of variation of suction velocity on the three dimensional convective flow and heat transfer through a porous medium under the influence of a uniform magnetic field perpendicular to the wall is investigated. A series expansion method is used for solving the governing equations and expressions for velocity and temperature distributions are obtained. The skin friction and the rate of heat transfer at the wall are discussed with the help of various graphs.

Keywords: MHD flow, porous medium, thermal convection, suction velocity, buoyance force

Introduction
It is clear from the experimental and theoretical studies that the problem of laminar flow has gained considerable importance in the field of Aeronautical engineering in view of its applications that the 'laminarization' of the boundary layer over a profile reduces the drag and hence the vehicle power requirement is reduced by a very substantial amount. The development on this subject has been compiled by Lachman [2]. Theoretical & experimental investigations have shown that the transition from laminar to turbulent flow which causes the drag coefficient to increase, may be prevented by the suction of fluid and heat transfer from the boundary of the wall. Gersten and Gross [1] have studied the effect of transverse sinusoidal suction velocity on flow and heat transfer along an infinite vertical porous wall. Singh, Mishra and Narayan [3] discussed the effect of variation of suction velocity on the three dimensional free convective flow and heat transfer through a porous medium. P.K. Path *et al*. [4] discussed the effects of heat and mass transfer on three dimensional MHD free convective flow of a viscous incompressible fluid through a highly porosity medium with periodic permeability. T.S. Reddy *et al*. [5] discussed the effects of hall current, chemical reaction and radiation on a free convection flow bounded by a vertical surface embedded in porous medium under the influence of uniform magnetic field. The object of the present paper is to study the effect of variation of suction velocity on the three dimensional convective flow and heat transfer through a porous medium under the influence of a uniform magnetic field perpendicular to the wall. Because of its vast applications in engineering problems, a large number of researchers have contributed to the investigation in this area. The velocity field and temperature distribution are obtained. The skin friction and the rate of heat transfer at the wall are discussed in detail.

Nomenclature
v (u, v, w)	=	velocity of the fluid
ρ	=	density of fluid
T	=	temperature
L	=	wave length
ε	=	amplitude of oscillations in suction velocity
g	=	acceleration due to gravity
G	=	Grashof number
K	=	Porous permeability
P	=	pressure

Nu	=	Nusselt number
C_p	=	specific heat at constant pressure
Cf_x	=	shear stress along x-axis
F (P, R)	=	skin friction factor
K_1, K_2	=	permeability parameters
B_0	=	magnitude of the uniform magnetic field
σ	=	conductivity
μ	=	coefficient of viscosity
υ	=	coefficient of kinamatic viscosity
M	=	non-dimensional Hartmann number

Formulation of the problem

Let us consider three dimensional convective flow of an incompressible viscous fluid through a porous medium bounded by an infinite vertical porous wall under the influence of a uniform magnetic field perpendicular to the wall. Let the wall be on x-z plane and y-axis be taken normal to the wall and directed into the fluid. The suction velocity distribution is taken in the form:

$$v_w(z) = V_0\left[1 + \varepsilon\, cos\frac{\pi z}{L}\right] \quad \text{... (1)}$$

which consist of a basic steady distribution $v_0 < 0$ with a superposed weak transversally varying distribution ε v_0 cos $(\pi z/L)$ of wave length L. Every physical quantity is independent of x because the wall is infinite in x-direction. The flow always remains three dimensional due to the variation of suction velocity. Let u, v and w be the velocity components in x, y and z directions respectively, and T the temperature distribution. Since the physical variables are the function of y and z only, therefore, the flow is governed by the system of equations:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{... (2)}$$

$$v \quad \frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = g\beta(T - T_\infty) + \upsilon\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - mu - \frac{\upsilon}{K}\mu \quad \text{... (3)}$$

$$v \quad \frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{\upsilon v}{K} \quad \text{... (4)}$$

$$v \quad \frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{\upsilon}{K}w \quad \text{... (5)}$$

$$v \quad \frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right), \quad \text{... (6)}$$

where $m = B_0^2\,\sigma/\rho,\ \ B_0 = \left|B\right|$ and σ is the electrical conductivity of the fluid.

Here the variation in density are neglected every where except in its association with the buoyancy force.

The last term in equation (3) to (5) account for the pressure drop across the porous material. The term mu in equation (3) accounts for uniform magnetic field. The basic flow in the medium is entirely due to the buoyancy force, caused by temperature difference between the wall and the medium. The appropriate boundary conditions are given as:

$$\left.\begin{aligned} &y = 0: u = 0,\ v = v_w(z),\ w = 0,\ T = T_w \\ &y \to \infty: u = 0,\ w = 0,\ P = P\infty,\ T = T\infty \end{aligned}\right\} \quad \text{... (7)}$$

Solution of the problem

If in the suction velocity the amplitude ε of oscillations is small then expanding the physical variables in the powers of ε, we have:

$u = u_0 + \varepsilon\, u_1 + \varepsilon^2\, u_2 + ...$

$v = v_0 + \varepsilon\, v_1 + \varepsilon^2\, v_2 + ...$

$w = w_0 + \varepsilon\, w_1 + \varepsilon^2\, w_2 + ...$

and $Q = Q_0 + \varepsilon\, Q_1 + \varepsilon^2\, Q_2 + ...$ ... (8)

where

$Q = (T - T\infty) / (T_w - T\infty)$. ... (9)

When $\varepsilon = 0$, then the problem reduces to the two dimensional MHD convection flow in an infinite porous medium with constant suction velocity at the wall. In this case the governing equations take the form:

$$\frac{\partial V_c}{\partial y} = 0 \quad \text{... (10)}$$

$$v_0^{\partial} \frac{\partial u_0}{\partial y} = g\beta(T_w - T_\infty)Q_0 + v\frac{d^2u_0}{dy^2} - \frac{v}{K}\left(1 + m\frac{K}{v}\right)u_0 \quad \text{... (11)}$$

$$\frac{dp_0}{dy} = -\frac{\mu}{K}v_0 \quad \text{... (12)}$$

$$v_0 \frac{dQ_0}{dy} = \alpha\frac{d^2Q_0}{dy^2} \quad \text{... (13)}$$

with the boundary conditions

$$\left.\begin{aligned} &y = 0: u_0 = 0, v_0 = v, w_0 = 0, Q_0 = 1 \\ &y \to \infty: u_0 = 0, w_0 = 0, P = P\infty, Q_0 = 0 \end{aligned}\right\} \quad \text{... (14)}$$

Equation (11) shows that v_0 does not depend upon y and thus we take $v_0 = v_0$ (constant).
The following non dimensional quantities are introduced to put the governing equations and boundary conditions in the dimensionless form:

$$\overline{y} = \frac{y}{L}, \overline{z} = \frac{z}{L}, R = -\frac{L}{v}v_0$$

$$\overline{u} = -\frac{Lu}{v}, \overline{v} = -\frac{L}{v}v, \overline{w} = -\frac{Lw}{v}, M = B_0L\sqrt{\sigma/\mu}$$

$$\overline{p} = -\frac{L^2}{Pv^2}P, K_1 = \frac{L^2}{K}, K_2 = K_1 + M^2,$$

$$G = g\beta(T_w - T_\infty)\frac{L^3}{v^2}, P = \frac{v}{\alpha}.$$

Using the above non dimensional quantities, and removing the bars over the physical quantities for convenience the equations (12) to (14) reduce to:

$$R \frac{du_0}{dy} = GQ_0 - \frac{d^2u_0}{dy^2} + K_2u_0 \quad \text{... (16)}$$

$$\frac{dp_0}{dy} = -K_1R \quad \text{... (17)}$$

$$\frac{d^2Q_0}{dy^2} + PR\frac{dQ_0}{d\overline{y}} = 0. \quad \text{... (18)}$$

The boundary condition (15) reduces to

$$\left.\begin{aligned} &\overline{y} = 0: u_0 = 0, v_0 = R, w_0 = 0, Q_0 = 1 \\ &\overline{y} \to \infty: u_0 = 0, w_0 = 0, P_0 = P\infty, Q_0 = 0. \end{aligned}\right\} \quad \text{... (19)}$$

Solving equations (16) to (18) with the help of the boundary conditions (19), we get

$$u_0 = \frac{\partial G}{Lp_1}\left[e^{-\lambda\overline{y}} - e^{-PR\overline{y}}\right]$$

$$v_0 = v_0$$

$$w_0 = 0 \quad \text{... (20)}$$

$$Q = e \quad \text{... (21)}$$

and $P_0 = P\infty$, ... (22)

where

$$\lambda = \frac{R}{2} + \left(\frac{R^2}{4} + K_2\right)^{½}.$$

Now, when $\varepsilon \neq 0$, the series expansion (8) are substituted in equations (2) to (6) and like powers of ε are equated to get perturbation (4) equations of various order of ε. For the small values of ε, we consider the perturbation equations only of o (ε). The governing equations in this case reduce to.

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \qquad \text{... (23)}$$

$$v_0 \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} = \frac{G v^2}{L^3} Q_1 + v\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \frac{v}{K}\left(1 + m\frac{K}{v}\right) u_1 \qquad \text{... (24)}$$

$$v_0 \frac{\partial v_1}{\partial y} = -\frac{1}{\rho}\frac{\partial p_1}{\partial y} + v\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{v}{K} v_1 \qquad \text{... (25)}$$

$$v_0 \frac{\partial w_1}{\partial y} = -\frac{1}{\rho}\frac{\partial p_1}{\partial z} + v\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{v}{K} w_1 \qquad \text{... (26)}$$

$$v_0 \frac{\partial Q_1}{\partial y} + v_1 \frac{\partial Q_0}{\partial y} = \alpha\left(\frac{\partial^2 Q_1}{\partial y^2} + \frac{\partial^2 Q}{\partial z^2}\right) \qquad \text{... (27)}$$

with boundary conditions:

$y = 0$: $u_1 = 0$, $v_1 = v$, cos $\left(\frac{\pi z}{L}\right)$, $w_1 = 0$, $Q_1 = 0$

$y \to \infty$: $u_1 = 0$, $w_1 = 0$, $P_1 = 0$, $Q_1 = 0$. ... (28)

Using the scheme of non dimensionalisation as already introduced and removing the bars over the physical quantities for convenience, the equations (23) to (27) reduce to:

$$\frac{\partial v_1}{\partial \bar{y}} + \frac{\partial w_1}{\partial \bar{z}} = 0 \qquad \text{... (29)}$$

$$R\frac{\partial u_1}{\partial \bar{y}} + v_1 \frac{\partial u_0}{\partial \bar{y}} = G Q_1 - \frac{\partial^2 u_1}{\partial \bar{y}^2} - \frac{\partial^2 u_1}{\partial \bar{z}^2} + K_2 u_1 \qquad \text{... (30)}$$

$$R\frac{\partial v_1}{\partial \bar{y}} = \frac{\partial p_1}{\partial \bar{y}} - \frac{\partial^2 v_1}{\partial \bar{z}^2} - \frac{\partial^2 v_1}{\partial \bar{z}^2} + K_1 v_1 \qquad \text{... (31)}$$

$$R\frac{\partial w_1}{\partial \bar{y}} = \frac{\partial p_1}{\partial \bar{z}} - \frac{\partial^2 w_1}{\partial \bar{y}^2} - \frac{\partial^2 w_1}{\partial \bar{z}^2} + K_1 w_1 \qquad \text{... (32)}$$

$$\frac{\partial^2 Q_1}{\partial \bar{y}^2} + \frac{\partial^2 Q_1}{\partial \bar{z}^2} + PR\frac{\partial Q_1}{\partial \bar{y}} + P v_1 \frac{\partial Q_0}{\partial \bar{y}} = 0. \qquad \text{... (33)}$$

The boundary condition reduces to

$\bar{y} = 0$: $u_1 = 0$, $v_1 = R \cos(\pi\bar{z})$, $w_1 = 0$, $Q = 0$

$\bar{y} \to \infty$: $u_1 = 0$, $w_1 = 0$, $P_1 = 0$, $Q = 0$ ... (34)

Now we are required to find the solutions of the above equations. Clearly the solutions for v_1 $(\bar{y},\bar{z})$ and p_1 $(\bar{y},\bar{z})$ are independent of the main flow component u_1 $(\bar{y},\bar{z})$ and the temperature field, Q_1. Hence we may assume that:

$u_1 \ (\bar{y},\bar{z}) = u_{11}(\bar{y}) \cos(\pi\bar{z})$ ... (35)

$v_1 \ (\bar{y},\bar{z}) = v_0^{\pi} v_{11}(\bar{y}) \cos(\pi\bar{z})$ ... (36)

$w_1 \ (\bar{y},\bar{z}) = - v_0 v_{11}(\bar{y}) \sin(\pi\bar{z})$ ... (37)

$p_1 \ (\bar{y},\bar{z}) = v_0^2 p_{11}(\bar{y}) \cos(\pi\bar{z})$ ... (38)

and $Q_1 \ (\bar{y},\bar{z}) = Q_{11}(\bar{y}) \cos(\pi\bar{z})$, ... (39)

where, prime denotes differentiation with respect to $\bar{y}$.
From these relations in dimensionless form and the governing equations (29) to (33), with the help of boundary conditions (34) we obtain:

$$u_{11} = -\frac{GR}{P_1(\pi - n)}\left[-\left(\frac{\pi}{nB_1} - \frac{n}{\pi B_3} - C_1 + C_2 + C_3\right)e^{-8\overline{\delta y}} + \frac{\pi}{nB_1}e^{-(\lambda+n)\bar{y}}\right]$$

- $\frac{n}{\pi B_3} e^{-(\pi+\lambda)\overline{y}} - C_1 e^{-(n+PR)\overline{y}} + C_2 e^{-(\pi P+PR)\overline{y}} + C_3 e^{-n\overline{y}}$]

$v_{11} = \frac{B_2}{(\pi-n)} e^{-\pi\overline{y}} - n e^{\pi\overline{y}}$

$p_{11} = \frac{nB_2}{(\pi-n)} e^{-\pi\overline{y}}$

and $Q_{11} = \frac{P^2 R}{\pi-n}\left[\frac{\pi}{P_2} e^{-(n+PR)\overline{y}} - \frac{n}{\pi P} e^{-(\pi+PR)\overline{y}} - \frac{C_3}{P_3} e^{-n\overline{y}}\right]$

where

$n = \frac{R}{2} + \left(\frac{R^2}{4} + \pi^2 + K_1\right)^{1/2}$

$\delta = \frac{R}{2} + \left(\frac{R^2}{4} + \pi^2 + K_2\right)^{1/2}$

$\eta = \frac{PR}{2} + \left(\frac{P^2 R^2}{4} + \pi^2\right)^{1/2}$

$B_1 = 2\left(1 + \frac{K_1}{2n\lambda}\right)$

$B_2 = 1 + \frac{K_1}{\pi R}$

$B_3 = \left[2\left(\frac{R^2}{4} + K_2\right)^{1/2}\right]/\lambda$

$P_1 = R^2 (p_2 - p) - K_2$

$p = n (1 + p) + K_1/R$

$P_3 = P_1 P^2/ [n R(P - 1) - K_2]$

$C_1 = \pi [1 + P_1 P/P_2 R] / [(P - 1) R - 2 n - L^2 m / P R \upsilon]$

$C_2 = n P (R + P_1/\pi) / (P^2 R^2 - P R^2 - \pi R + 2\pi P R - K_2)$

and $C_3 = P_3 (\pi/P_2 - n/\pi P)$

Thus

$u_1 (\overline{y},\overline{z}) = (\upsilon/L) (G R/P_1 (\pi - n) \left[-\left(\frac{\pi}{nB_1} - \frac{n}{\pi B_3} - C_1 + C_2 + C_3\right) e^{-\delta\overline{y}} + \right.$

$+ \frac{\pi}{nB_1} e^{-(\lambda+n)\overline{y}} - \frac{n}{\pi B_3} e^{-(\pi+\lambda)\overline{y}} - C_1 e^{-(n+PR)\overline{y}} + C_2 e^{-(\pi+PR)\overline{y}}$

$\left. e^{-n\overline{y}} \right] \cos(\pi\overline{z})$ ... (40)

$v_1 (\overline{y},\overline{z}) = \frac{V_0}{\pi-n}\left[\pi\, e^{-n\overline{y}} - n\, e^{-\pi\overline{y}}\right]\cos(\pi\overline{z})$ ... (41)

$w_1 (\overline{y},\overline{z}) = \frac{nV_0}{\pi-n}\left[e^{-n\overline{y}} - e^{-\pi\overline{y}}\right]\sin(\pi\overline{z})$ ... (42)

$p_1 (\overline{y},\overline{z}) = B_2\, \rho v_0^2 \left(\frac{n}{\pi-n}\right) e^{-\pi\overline{y}} \cos(\pi\overline{z})$ ... (43)

and $Q (\overline{y},\overline{z}) = (p^2 R/(\pi - n)) \left[\frac{\pi}{p^2} e^{-(n+PR)\overline{y}} - \frac{n}{\pi p} e^{-(\pi+PR)\overline{y}} - (C_3/P_3), e^{-n\overline{y}}\right] \cos(\pi\overline{z})$ ... (44)

Result and discussions

Now we shall discuss some flow characteristics of the problem. The shear stress along x-axis is given as:

C f_x $= \frac{t_x}{\rho v/L^2} = (\mu/p)(\partial u/\partial y)_{y=c}/v^2/L^2$

i.e. C f_x = G/P_1 (PR-λ) + ε G F (P, R) cos $(\pi\bar{z})$, ... (45)

where

F (P, R) = $\frac{R}{P_1}(\pi - n)[(\delta - \lambda - n)(\pi/nB_1) - (\delta - \lambda - \pi)(n/\pi B_3)]$

$-(\delta - PR - n)c_1 + (\delta - PR - \pi)c_2 + (\delta - n)c_3]$. ... (46)

The skin friction factor F(P, R) vanishes in the limiting case as R→ 0 and R→∞. This factor is numerically evaluated for different values of R and permeability parameters K_1 and K_2. The results are shown graphically. From figure 1, it is clear that F tends to its limiting values as R→ 0 and R→∞, for each K_2, also F increases with R and attains the maximum values, after which it decreases and approaches to zero. Also as the values of K_2 increases, F decreases significantly.

The heat flux at the wall in terms of Nusselt numbers Nu is given as:

Nu = - $q_w/\rho v_o c_p (T_w - T_\infty) = (k/\rho v_o c_p)(\partial Q/\partial y)_{y=0}$

i.e. Nu = $[1 + \varepsilon(1 - H(P,R)]\, cos(\pi\bar{z})]k/\rho v_0 c_p$, ... (47)

where

H(P,R) = $(1/(-\pi + n))[n\{(n/\pi) - (\pi p/p_2)\} + (\pi P/P_2)(n + PR) - (n/\pi)PR \rightleftarrows -\pi]$ ... (48)

The heat transfer factor H(P, R) takes constant values for limiting values of R for all values of permeability parameter K_1, H→ 1 and R→0 which shows that there is no oscillatory flow, and when R→∞, then H→0 which shows that the heat transfer approaches to quasi-steady value. These results are shown in Fig. 2. It is also observed that the value of H increases as the value of K_1 increases for each value of M.

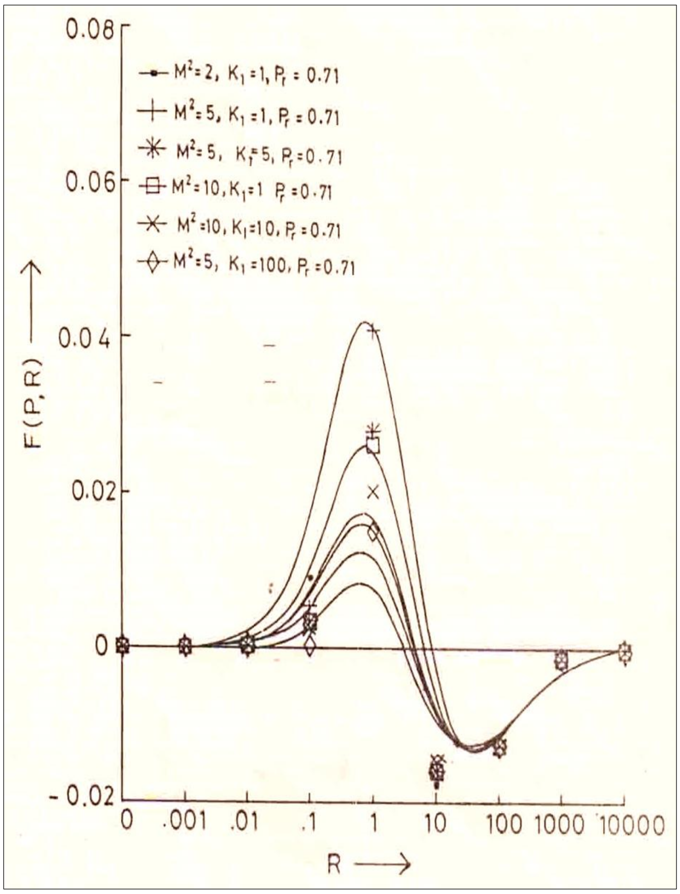

Fig 1: Variation of F (P, R) with

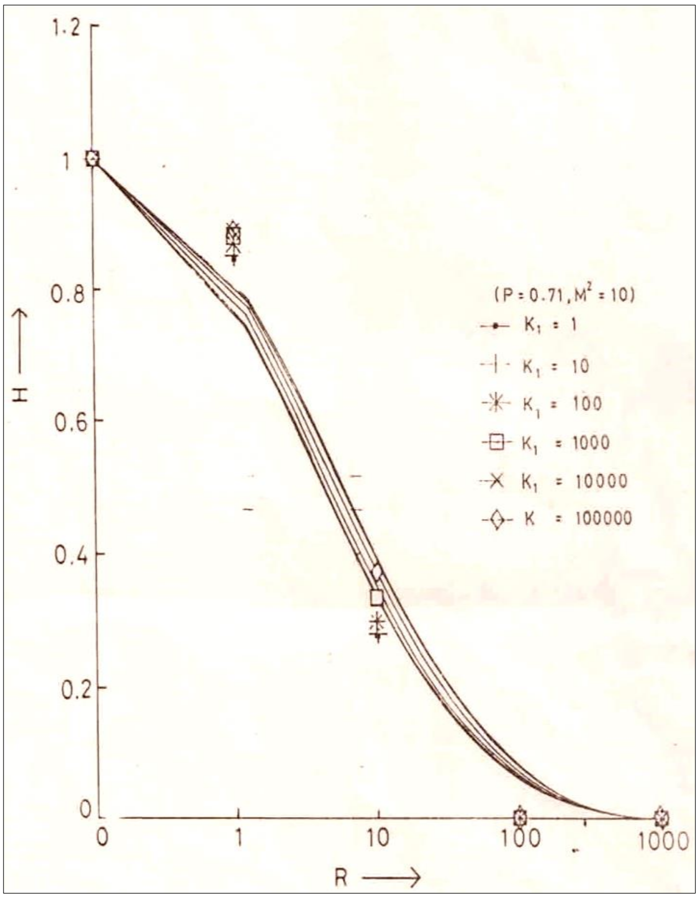

Fig. 2: Variation of H (P, R) with R"

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