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Intuitionistic fuzzy technique to find the critical path

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Abstract
In this paper, intuitionistic fuzzy number has been applied to find the critical path for the project scheduling problems with the aid of interval valid intuitionistic fuzzy numbers (IVIFN). Decision maker's risk attitude index and decision maker's risk index ranking value has also been utilized to find the critical path. In order to explain intuitionistic fuzzy technique to find critical path, numerical example has been illustrated in this paper.

Keywords: Trapezoidal intuitionistic fuzzy number, ranking of intuitionistic fuzzy number, critical path

Introduction
A constructed network is an imperative tool in the development, and it organizes a definite project execution. Network diagram plays a vital role in formative project-completion time. In real life situation, vagueness may arise from a number of possible sources like: due date may be distorted, capital may unavailable, weather situation may root several impediments. Therefore, the fuzzy set theory can play a significant role in this kind of problems to handle the ambiguity about the time duration of deeds in a project network.
Fuzzy set theory as developed by Zadeh [11] and the concept of fuzzy numbers presented by Dubois and Prade [4] are applied to present a fuzzily defined system. Chanas and Zielinski [12] proposed a method to make critical path analysis in the network with fuzzy activity times (interval activity times, fuzzy numbers of L-R type) by directly applying the extension principle [11] to the classical criticality notion treated as a function of activity duration time in the network. And two methods of calculation of the path degree of criticality are presented.
Slyeptsov and Tyshchuk [13] presented an efficient method of computation of fuzzy time windows for late start and finish times of operations in the problem of fuzzy network. G. Liang and T.C. Han [6] proposed a fuzzy critical path for project networks. Elizabeth and L. Sujatha [5] discussed a critical path problem under fuzzy Environment. C.T Chen and S.F Huang [2] proposed a new model that combines fuzzy set theory with the PERT technique to determine the critical degrees of activities and paths, latest and earliest starting time and floats. S.H. Nasution [8] proposed a fuzzy critical path method by considering interactive fuzzy subtraction and by observing that only the non-negative part of the fuzzy numbers can have physical work. Basic definitions of intuitionistic fuzzy set theory have been reviewed. K.Atanassov [1] proposed a procedure to find out the intuitionistic fuzzy critical path using an illustrative example. The results are discussed. Finally, some conclusions are drawn.

2. Preliminaries
2.1 Definition: Fuzzy Set
Let X be the universal set A fuzzy set A in X represented by A=$\{(x,\mu_A(x))/x\varepsilon X\}$,where the function$\mu_A(x)$:X→ [0,1] is the membership degree of element x in the fuzzy set A.

2.2 Definition: Intuitionistic Fuzzy Set
Let X be an Universe of discourse, then an Intuitionistic Fuzzy Set (IFS) A in X is given by A=$\{(x,\mu_A(x),\upsilon_A(x))/x\varepsilon X\}$,where the function $\mu_A(x)$:X→ [0,1] and $\upsilon_A(x)$:X→ [0,1] determine the degree of membership and non-membership of the element$x\varepsilon X$, respectively and for every $x\varepsilon X, 0 \le \mu_A(x) + \upsilon_A(x) \le 1$.

2.3 Definition: Intuitionistic Fuzzy Number
An IFN A is
- An IF subset of the real line.
- Normal ie, there is an $x_0 \varepsilon R$such that $\mu_A(x_0)$ =1 (so $\upsilon_A(x_0)$ =0).
- Convex for the Membership function $\mu_A(x)$.

ie, $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_A(x_1), \mu_A(x_2))$ for every $x_1, x_2 \varepsilon R$, $\lambda \varepsilon [0,1]$.
- Concave for the non-membership function

ie $\upsilon_A(\lambda x_1 + (1-\lambda)x_2) \leq max(\upsilon_A(x_1), \upsilon_A(x_2))$ for every $x_1, x_2 \varepsilon R$, $\lambda \varepsilon [0,1]$.

2.4 Definition: Trapezoidal Intuitionistic Fuzzy Number
An Intuitionistic Fuzzy Number $A = \{< a, b, c, d >< a', b', c', d' >\}$is said to be a trapezoidal intuitionistic fuzzy number if its membership function and non-membership function are given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{(x-d)}{(c-d)} & c \leq x \leq d \\ 0 & otherwise \end{cases} \qquad \upsilon_A(x) = \begin{cases} \frac{(b'-x)}{(b'-a')} & a' \leq x \leq b' \\ 0 & b' \leq x \leq c' \\ \frac{(x-c')}{(d'-c')} & c' \leq x \leq d' \\ 1 & otherwise \end{cases}$$

2.5 Definition: Algebraic Operations of any Two Trapezoidal Intuitionistic Fuzzy Number
Addition ⊕

$$A_1 \oplus A_2 =< a_1, b_1, c_1, d_1 >< a_1', b_1', c_1', d_1' > \oplus < a_2, b_2, c_2, d_2 >< a_2', b_2', c_2', d_2' >$$

$$=< a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2 >< a_1' + a_2', b_1' + b_2', c_1' + c_2', d_1' + d_2' >$$

Subtraction

$$A_1\text{-}A_2 =< a_1, b_1, c_1, d_1 >< a_1', b_1', c_1', d_1' > - < a_2, b_2, c_2, d_2 >< a_2', b_2', c_2', d_2' >$$

$$=< a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2 >< a_1' - d_2', b_1' - c_2', c_1' - b_2', d_1' - a_2' >$$

2.6 Definition: Decision Maker's Risk Attitude Index
For an Intuitionistic fuzzy critical path analysis problem [14], using the Trapezoidal Intuitionistic fuzzy numbers such as $T_{ij} =< a_{ij}, b_{ij} c_{ij}, d_{ij} >$ to denote the intuitionistic fuzzy activity time of activity $A_{ij}$the decision maker's risk attitude index βcan be obtained by

$$\beta = \left[\sum_{A_{ij}\varepsilon}{}_{i} \sum_{ACT}{}_{j} \frac{b_{ij} - a_{ij}}{(a_{ij} - c_{ij}) + (d_{ij} - b_{ij})} \right] / t \text{ ....(1)}$$

where ACT and t denote the set of all actives and the number of actives in a project network, respectively.

2.7 Definition: Decision Maker's Risk Index Ranking Value
The ranking value $R(A_i)$ of the trapezoidal intuitionistic fuzzy number $A_i$can be obtained as follows:

$$R(A_i) = \beta[(d_i - x_1)/(x_2 - x_1 - c_i + d_i)] + (1-\beta)\{1 - [(x_2 - a_i)/(x_2 - x_1 + b_i - a_i)]\} \text{...........(2)}$$

Where $\beta$is the decision maker's risk attitude index, $x_1 = min\{a_1, a_{2,} \ldots\ldots, a_n\}$ and $x_2 = max\{d_1, d_{2,} \ldots\ldots, d_n\}$

2.8 Definition: Rules for Ranking Trapezoidal Intuitionistic Fuzzy Number
Now, we rank the intuitionistic fuzzy numbers A_i and $A_j$according to the following rules [9]:

$A_i > A_j \Leftrightarrow R(A_i) > R(A_j)$
$A_i < A_j \Leftrightarrow R(A_i) < R(A_j)$
$A_i = A_j \Leftrightarrow R(A_i) = R(A_j)$

2.9 Definition: An Intuitionistic Fuzzy Completion Time
Assume that there exists a path P_c in a project network such that
$IFCPM(P_c)=min\{IFCPM(P_i)/P_i \varepsilon P\}$ then the path P_c is an intuitionistic fuzzy critical path.

3. Intuitionistic fuzzy critical path analysis
3.1 ***Notation***

N	: The set of all nodes in a project network.
A_{ij}	: The activity between nodes i and j.
$T_{\mu ij}, T_{\upsilon ij}$	: The intuitionistic fuzzy activity time for membership function and non-membership function.

$EST_{\mu j}$, $EST_{\nu j}$: The Earliest stating time for membership function and non-membership function.
$LFT_{\mu j}$, $LFT_{\nu j}$: The Latest Finishing time for membership function and non-membership function.
$TS_{\mu ij}$, $TS_{\nu ij}$: The Total Slack for membership function and non-membership function.
S(j) : The set of all successor activities of node j.
NS(j) : The set of all nodes connected to all successor activities of node j.
ie, $NS(j) = \{k/A_{jk}\varepsilon S(j), k\varepsilon N\}$
F(j) : The set of all predecessor activities of node j.
NP(j) : The set of all nodes connected to all predecessor activities of node j.
ie,$NP(j) = \{i/A_{ij}\varepsilon F(j), i\varepsilon N\}$
P_i : The ith Path.
P : The set of all paths in a project network.
IFCP($P_{\mu k}$), IFCP($P_{\nu k}$): An intuitionistic fuzzy completion of path $P_{\mu k}$, $P_{\nu k}$ in a project network.

3.2 Properties In The Proposed Intuitionistic Fuzzy Critical Path Analysis

Set the initial node to be zero for starting [7],
i.e. $EST_{\mu 1}$=<0,0,0,0> and $EST_{\nu 1}$=<0,0,0,0>. Then, the following properties are true.

Property 1: $EST_{\mu j} = max\{ EST_{\mu i} \oplus T_{\mu ij}/i\varepsilon NP(j), i \neq 1, j\varepsilon N\}$

$EST_{\nu j} = min\{ EST_{\nu i} \oplus T_{\nu ij}/i\varepsilon NP(j), i \neq 1, j\varepsilon N\}$

Property 2: $LFT_{\mu j} = min\{ LFT_{\mu k} \odot T_{\mu ik}/k\varepsilon NS(j), j \neq n, j\varepsilon N\}$

$LFT_{\nu j} = max\{ LFT_{\nu k} \odot T_{\nu ik}/k\varepsilon NS(j), j \neq n, j\varepsilon N\}$

Property 3:
$TS_{\mu ij} = LET_{\mu j} \odot (EST_{\mu i} \oplus T_{\mu ij}), 1 \leq i < j \leq n; i, j\varepsilon N$
$TS_{\nu ij} = LET_{\nu j} \odot (EST_{\nu i} \oplus T_{\nu ij}), 1 \leq i < j \leq n; i, j\varepsilon N$

Property 4: $IFCPM(P_{\mu k}) = \sum_{\substack{1\leq i<j\leq n \\ i,j\varepsilon P_{\mu k}}} TS_{\mu ij}, P_{\mu k}\varepsilon P$

$$IFCPM(P_{\nu k}) = \sum_{\substack{1\leq i<j\leq n \\ i,j\varepsilon P_{\nu k}}} TS_{\nu ij}, P_{\nu k}\varepsilon P$$

3.3 Intuitionistic Fuzzy Critical Path Analysis Algorithm

In this section [3], an intuitionistic fuzzy critical path analysis algorithm is developed to find a critical path of a project network in an intuitionistic fuzzy environment. The description of the algorithm is presented in the following.

1. Identify activities in a project.
2. Establish precedence relationships of all activities.
3. Estimate the Intuitionistic fuzzy activity time with respect to each activity.
4. Construct the project network.
5. Let $EST_{\mu 1}$=<0,0,0,0>, $EST_{\nu 1}$=<0,0,0,0> and EST_j j=2, 3,…..,n by using property 1.
6. Let $LFT_{\mu n}$=$EST\mu_n$, $LFT_{\nu n}$=$EST_{\nu n}$ and calculate LFT_j, j=n-1, n-2 ,…….2,1 by using property 2.
7. Calculate $TS_{\mu ij} and TS_{\nu ij}$ with respect to each activity in a project network by using property 3.
8. Find all the possible paths and calculate IFCP($P_{\mu k}$), IFCP($P_{\nu k}$) by using proberty 4.
9. Find the intuitionistic fuzzy critical path by using definition 2.9.
10. Find the grade of membership and non-membership that the project can be completed at scheduled time.

4. Numerical Example

In this section [10], a hypothetical project problem is presented to demonstrate the computational process of intuitionistic fuzzy critical path analysis proposed above.
Suppose there is a project network, as Figure, with the set of node N={1,2,3,4}, the intuitionistic fuzzy activity time for each activity as shown in Table 1. All of the durations are in hours.

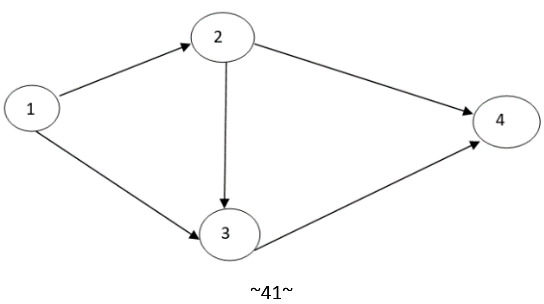

Table 1: The fuzzy activity time for each activity in the project network shown as above figure

Activity A_{ij}	Intutionistic Fuzzy Activity time T_{ij}
1-2	<3, 5, 5, 7> <4, 5, 5, 8>
1-3	<5, 10, 10, 15> <6, 10, 10, 16>
2-3	<1, 3, 4, 5> <2, 3, 4, 5>
2-4	<2, 4, 5, 6> <3, 4, 5, 7>
3-4	<6, 8, 10, 11> <7, 8, 10, 12>

Table 2: Total slack intuitionistic fuzzy time for each activity

Activity A_{ij}	Intutionistic FuzzyActivity time T_{ij}	$EST_{\mu i}$ $EST_{\nu i}$	$LFT_{\mu j}$ $LFT_{\nu j}$	$TS_{\mu ij}$ $TS_{\nu ij}$
1-2	<3,5,5,7> <4,5,5,8>	<0,0,0,0> <0,0,0,0>	<-5,4,9,19> <4,5,5,11>	<-12,-1,4,16> <-4,0,0,7>
1-3	<5,10,10,15> <6,10,10,16>	<0,0,0,0> <0,0,0,0>	<0,8,12,20> <-5,-1,2,8>	<-15,-2,2,15> <-21,-11,-8,2>
2-3	<1,3,4,5> <2,3,4,5>	<3,5,5,7> <4,5,5,8>	<0,8,12,20> <-5,-1,2,8>	<-12,-1,4,16> <-19,-10,-6,2>
2-4	<2,4,5,6> <3,4,5,7>	<3,5,5,7> <4,5,5,8>	<11,18,20,26> <7,9,10,15>	<-2,8,11,21> <-8,-1,1,8>
3-4	<6,8,10,11> <7,8,10,12>	<5,10,10,15> <6,8,9,14>	<11,18,20,26> <7,9,10,15>	<-15,-2,2,15> <-22,-10,-6,2>

Table 3: Rank value of total slack intuitionistic fuzzy time of all possible paths

Paths	IFCP($P_{\mu k}$) K=1–m	Rank value Using equation 1 β=0.602 x_1=-39, x_2=47	Rank	IFCP($P_{\nu k}$) K=1-m	Rank value Using equation 1 β=0.35 x_1=-45, x_2=11	Rank
1 → 2 → 4	<-14,7,15,37>	0.5606	III	<-26,-10,-6,9>	0.5827	III
1 → 2 → 3 → 4	<-39,-4,10,47>	0.5360	II	<-45,-20,-12,11>	0.4487	II
1 → 3 → 4	<-30,-4,4,30>	0.4953	I	<-43,-21,-14,4>	0.4318	I

The critical path for intuitionistic fuzzy network for both membership and non-membership function are 1 → 3 → 4.

5. Conclusion
In this paper, intuitionistic fuzzy number has been applied to find the critical path for the project scheduling problems. Decision maker's risk attitude index and decision maker's risk index ranking value has been applied to find the shortest path. Further, total path can also be calculated by using these above two methods.

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