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Fitting of modified exponential model between rainfall and ground water levels: A case study

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Abstract
Present paper deals with the application of Time Series model to analyze and predict Rainfall (RF) and Ground water levels (GWLs) in Anantapuramu district based on the data collected from January 2007 to December 2016. Through Modified Exponential model for the purpose of analysis the district is divided into five zones namely, Anantapur, Penukonda, Kadiri, Kalyandurg, and Dharmavaram Revenue Divisions. Forecasts are obtained after fitting the models for 1. RF and 2. GWLs in the above five zones and conclusions are drawn based on the results obtained.

Keywords: Rainfall, ground water level, modified exponential, validation of the model, prediction

1. Introduction

A Time Series is an arrangement of statistical data in chronological order i.e. in accordance with occurrence of time, is known as 'Time Series' [8]. Such series have a unique important place in the fields of Economic and Business statistics since the series relating to prices, consumption and production of various commodities; money in circulation; bank deposits and bank clearings; sales and profits in a departmental store, agricultural and industrial production, national income and foreign exchange reserves, prices and dividends of shares in a stock exchange market etc. are all time series spread over a long period of time [5, 6, 11].

A time series depicts the relationship between two variables, one of them being time. Example: The population (U_t) of a country in different years (t); temperature (U_t) of a place on different days (t) of the week; rainfall (U_t) of a place on different days (t) of the month; ground water levels (U_t) of a place on different months (t) of the year etc. [7, 11, 12]

Thus, if the values of a phenomenon or variable at times $t_1, t_2, \dots\dots\dots, t_n$ are $u_1, u_2, \dots\dots\dots, u_n$ respectively, then the series.

$$T: t_1, t_2, \dots\dots\dots, t_n$$
$$U_t: u_1, u_2, \dots\dots\dots, u_n$$

Constitute a time series. Thus, a time series invariably gives a bivariate distribution, one of the two variables being time (t) and the other being the value (U_t) of the phenomenon at different points of time. The values of t may be given yearly, monthly, weekly, daily or even hourly, usually but not always at equal intervals of time [9, 11].

Anantapur is one of the districts of Andhra Pradesh facing frequent droughts and rain shadow area since many years. Because of these conditions standards of living, financial conditions and filthy conditions of surroundings in the living area of the people are very poor because of very irregular monsoon and decreasing greenery conditions. Many diseases relating to Climatic Conditions water and air pollutions and so on are spreading frequently in many areas of the districts. It is very necessary to concentrate on above mentioned conditions of the people in this area. Main factors for this situation are water, Climatic Conditions and irregular monsoons, surroundings in the living area; less medical facilities and so on these things are to be scientifically analyzed to improve the conditions of living standards of the people areas [1-3].

Because of very flimsy and irregular monsoons water pollutions and air pollution are high and ground water resources are also going down and down from time to time hence time series models are more suitable to analyze these conditions [4]. All these factors through the common man into a gloomy and pessimistic conditions. To improve these conditions of people, one has to analyze different parameters influencing these Conditions [10].

For the present work Average Ground Water Level (GWL) measured in meters (m) from 194 Piezometer points spread throughout the district and Average Rainfall measured in mille meters (m.m) of the district are considered. The data on the above variables are collected from the records of Ground Water and Water Audit Department Anantapuramu on Ground Water Levels (GWLs) and the data on Rainfall is collected from the Chief planning office, Anantapuramu from 2001 Jan to 2017 Oct. Further, Rainfall data is recorded on daily basis and Ground Water Levels are recorded on monthly basis from the respective records maintained by them [13].

From the data collected, the data relating to January to December months from 2007 to 2016 is considered for the purpose of analysis of this paper on both the variables i.e. Ground Water Level and Rainfall. Further Anantapuramu district consisting of 63 mandals is divided into five Revenue Divisions for the administrative convenience and hence for the analysis these five Revenue Divisions are considered as five zones and are given in the following table along with their respective Mandal [13].

Table 1: Zonal-wise (Revenue division) of mandals in Anantapuramu District.

S. No	Zone-I Anantapuramu RD	Zone-II Penukonda RD	Zone-III Kadiri RD	Zone-IV Kalyandurg RD	Zone-V Dharmavaram RD
1	Anantapuramu	Agali	Amadagur	Beluguppa	Bathalapalli
2	Atmakur	Amarapuram	Bukkapatnam	Bommanahal	C.K.Palli
3	B.K.Samudram	Chilamathur	Gandlapenta	Brahmasamudram	Dharmavaram
4	Garladinne	Gorantla	Kadiri	D.Hirehal	Kanaganipalli
5	Gooty	Gudibanda	Kothacheruvu	Gummaghatta	Mudigubba
6	Guntakal	Hindupur	N.P.Kunta	Kalyandurg	Ramagiri
7	Kudair	Lepakshi	Nallacheruvu	Kambadur	Raptadu
8	Narpala	Madakasira	Nallamada	Kanekal	Tadimarri
9	Pamidi	Parigi	O.D.Cheruvu	Kundurpi	
10	Peddapappur	Penukonda	Puttaparthi	Rayadurg	
11	Peddavadugur	Roddam	Talupula	Settur	
12	Putlur	Rolla	Tanakal		
13	Singanamala	Somandepalli			
14	Tadipatri				
15	Uravakonda				
16	Vajrakarur				
17	Vidapanakal				
18	Yadiki				
19	Yellanur				
Total (63)	19	13	12	11	8

Similarly, zonal wise Piezometer Points are also provided in the following table, from which GWLs are measured.

Table 2: Zonal-wise of Piezometer Points in Anantapuramu District.

	Zone-I Anantapuramu RD	Zone-II Penukonda RD	Zone-III Kadiri RD	Zone-IV Kalyandurg RD	Zone-V Dharmavaram RD
Piezometer Points (194)	54	50	31	32	27

The data is collected on Average Rainfall and Average Ground Water Levels are given in the following Table-1.3 for a ready reference.

Table 3: Average Rainfall and Average Ground water levels data from 2007 to 2016

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL
2007	65.60	10.57	58.20	22.58	67.20	14.23	52.00	14.97	60.50	17.03
2008	53.90	9.96	77.90	20.73	65.20	9.27	61.30	10.88	62.70	9.09
2009	45.40	12.17	50.60	17.53	46.30	11.08	57.10	9.58	38.70	10.24
2010	53.90	12.74	71.50	15.02	70.80	12.03	64.60	8.58	56.30	11.79
2011	39.50	12.69	42.30	15.20	48.90	11.48	31.80	8.93	36.60	12.84
2012	43.20	14.98	43.40	20.49	45.30	16.08	40.50	13.76	41.90	13.22
2013	35.00	15.94	52.30	23.03	47.10	18.69	34.80	16.98	38.10	14.30
2014	31.10	15.87	30.30	23.40	27.10	21.16	37.10	18.92	22.80	16.30
2015	44.10	14.90	62.60	26.88	66.30	25.80	46.00	19.26	54.30	17.66
2016	33.50	15.57	33.40	27.27	32.30	15.35	25.70	19.51	30.10	16.15

2. Statistical Analysis

Some of the Preliminary Statistical analysis is done for the data provided in the above table -1.3, such as yearly averages of Rainfall and Ground water levels are calculated and Karl-Pearson's Correlation Coefficient (r) is calculated between Average Rainfall(X) and Average Ground water levels (Y) Zonal wise by using the following formula,

$$r = \frac{cov(x,y)}{\sqrt{v(x).v(y)}} \quad .....(2.1)$$

and are given in the following Table-2.1.

Table 2.1: Correlation coefficient between average rainfall and average ground water level.

Years	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V
2007-2016	-0.84	-0.26	-0.20	-0.58	-0.23

By studying on the above Correlation Coefficients we can observed that all the Correlation Coefficients are negative, that is the relation between Rainfall and Ground Water levels is negative, that is, if Rainfall is increasing the Ground water level is decreasing, it is true, because the depth of the water level will decrease when rainfall is increases. By observing the

Correlation coefficients in the above Table-2.1 in Zone-I and Zone-IV they are strongly negatively related, as the other Zones are weakly negatively related. We observe that in Zone-I and Zone-IV additional to rainfall, other water resources like, High Level Canal (HLC) in these zones that also helps to improve the Ground water level.

To forecast Rainfall and Ground Water Levels through Modified Exponential model for different zones we consider

The Modified Exponential Model = $K + ab^t$ (2.2)

$$\text{Where } b = \left(\frac{y_3-y_2}{y_2-y_1}\right)^{\left(\frac{1}{t_2-t_1}\right)} \quad (2.3)$$

$$a = \frac{(y_2-y_1)^2}{(y_3-2y_2+y_1)} * \left(\frac{y_2-y_1}{y_3-y_2}\right)^{\left(\frac{t_1}{t_2-t_1}\right)} \quad (2.4)$$

$$k = \frac{(y_1y_3-y_2^2)}{(y_3-2y_2+y_1)} \quad (2.5)$$

To fit the above Modified Exponential model and to estimate the values of the parameters ‘a’, ‘b’ and ‘k’ by solving the related normal equations and following trend curve is fitted for the data given in table-1.3 and fitted model is given below.

The fitted Modified Exponential model for Average RF and Average GWLs:

A: For Rainfall

Zone-I: Modified Exponential Curve = (19.34) + (49.50)* $(0.84)^t$

Zone-II: Modified Exponential Curve = (24.20) + (110.88)* $(0.70)^t$

Zone-III: Modified Exponential Curve = (113.51) + (-39.80)* $(1.10)^t$

Zone-IV: Modified Exponential Curve = (36.29) + (78.54)* $(-0.56)^t$

Zone-V: Modified Exponential Curve = (7.32) + (84.70)* $(0.81)^t$

B: For Ground water levels

Zone-I: Modified Exponential Curve = (-6.60) + (14.96)* $(1.05)^t$

Zone-II

Modified Exponential Curve = (18.50) + (1.71)*$(-1.14)^t$

Zone-III

Modified Exponential Curve = (8.62) + (0.24)*$(1.64)^t$

Zone-IV

Modified Exponential Curve = (10.56) + (0.11)*$(-1.72)^t$

Zone-V

Modified Exponential Curve = (57.58) + (-51.16)*$(0.97)^t$

3. Validation of the fitted model

Validation of the fitted model is necessary to check the suitability of the model for the given data this is done by considering X = Years and Y = Average RF or Average GWL given in table-1.3 and estimated the Average RF (Y) or Average GWL (Y) denoted by $\hat{y}$. The estimated Average RF and Average GWLs are given in the following tables.

Table 3.1: Estimated Average RF $\hat{y}$ for Modified Exponential Curve.

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates
2007	65.60	60.92	58.20	101.82	67.20	69.73	52.00	-7.69	60.50	75.93
2008	53.90	54.49	77.90	78.53	65.20	65.35	61.30	60.64	62.70	63.22
2009	45.40	48.55	50.60	61.90	46.30	60.58	57.10	22.15	38.70	52.21
2010	53.90	44.09	71.50	50.81	70.80	55.40	64.60	44.14	56.30	43.74
2011	39.50	40.13	42.30	43.05	48.90	49.43	31.80	31.58	36.60	36.97
2012	43.20	36.67	43.40	37.51	45.30	43.06	40.50	38.65	41.90	31.04
2013	35.00	34.19	52.30	33.07	47.10	35.90	34.80	34.72	38.10	26.80
2014	31.10	31.72	30.30	30.85	27.10	28.34	37.10	37.08	22.80	23.41
2015	44.10	29.74	62.60	28.64	66.30	19.58	46.00	35.50	54.30	20.03
2016	33.50	27.76	33.40	27.53	32.30	10.43	25.70	36.29	30.10	17.48

Table 3.2: Estimated Average GWL $\hat{y}$ for Modified Exponential Curve.

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates
2007	10.57	9.11	22.58	16.55	14.23	9.01	14.97	10.37	17.03	7.95
2008	9.96	9.86	20.73	20.72	9.27	9.27	10.88	10.89	9.09	9.49
2009	12.17	10.75	17.53	15.97	11.08	9.68	9.58	10.00	10.24	11.02
2010	12.74	11.65	15.02	21.39	12.03	10.36	8.58	11.52	11.79	12.05
2011	12.69	12.55	15.20	15.20	11.48	11.47	8.93	8.90	12.84	13.58
2012	14.98	13.45	20.49	22.24	16.08	13.29	13.76	13.41	13.22	15.12
2013	15.94	14.49	23.03	14.22	18.69	16.28	16.98	5.66	14.30	16.14
2014	15.87	15.54	23.40	23.37	21.16	21.18	18.92	18.99	16.30	17.68
2015	14.90	16.59	26.88	12.94	25.80	29.22	19.26	-3.93	17.66	18.70
2016	15.57	17.78	27.27	24.84	15.35	42.40	19.51	35.49	16.15	19.72

In the above tables -3.1 and 3.2 for the validation of the model Mean Square Errors (MSE's) are calculated zone wise by considering

$$\text{MSE} = \sum(y-\hat{y})^2 \quad \ldots..(3.1)$$

Where y represents actual or observed values given in table-1.3 and $\hat{y}$ is the estimated values through fitted Modified Exponential model is given in tables- 3.1 and 3.2. Using fitted Modified Exponential model respectively. MSE's were calculated and are given in the following table.

Table 3.3: MSE's for Average RF- Modified Exponential Model.

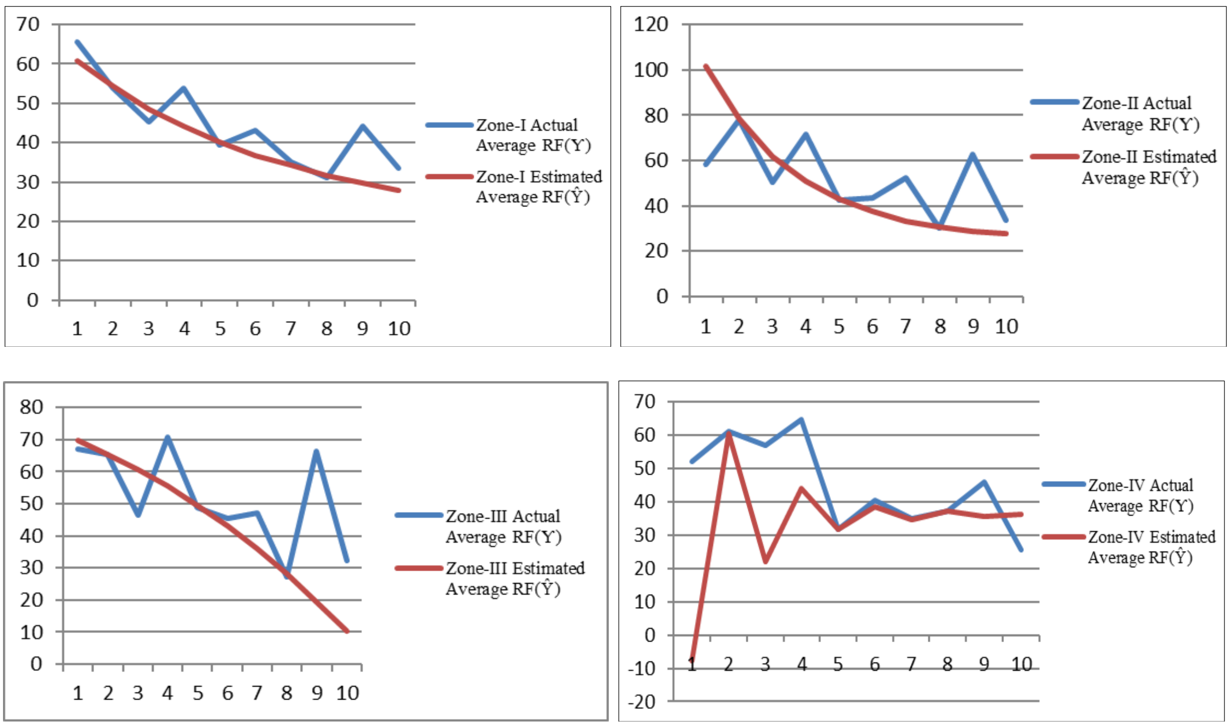
Type of the Model	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V
Modified Exponential	411.64	4051.96	3240.83	5429.32	2158.46

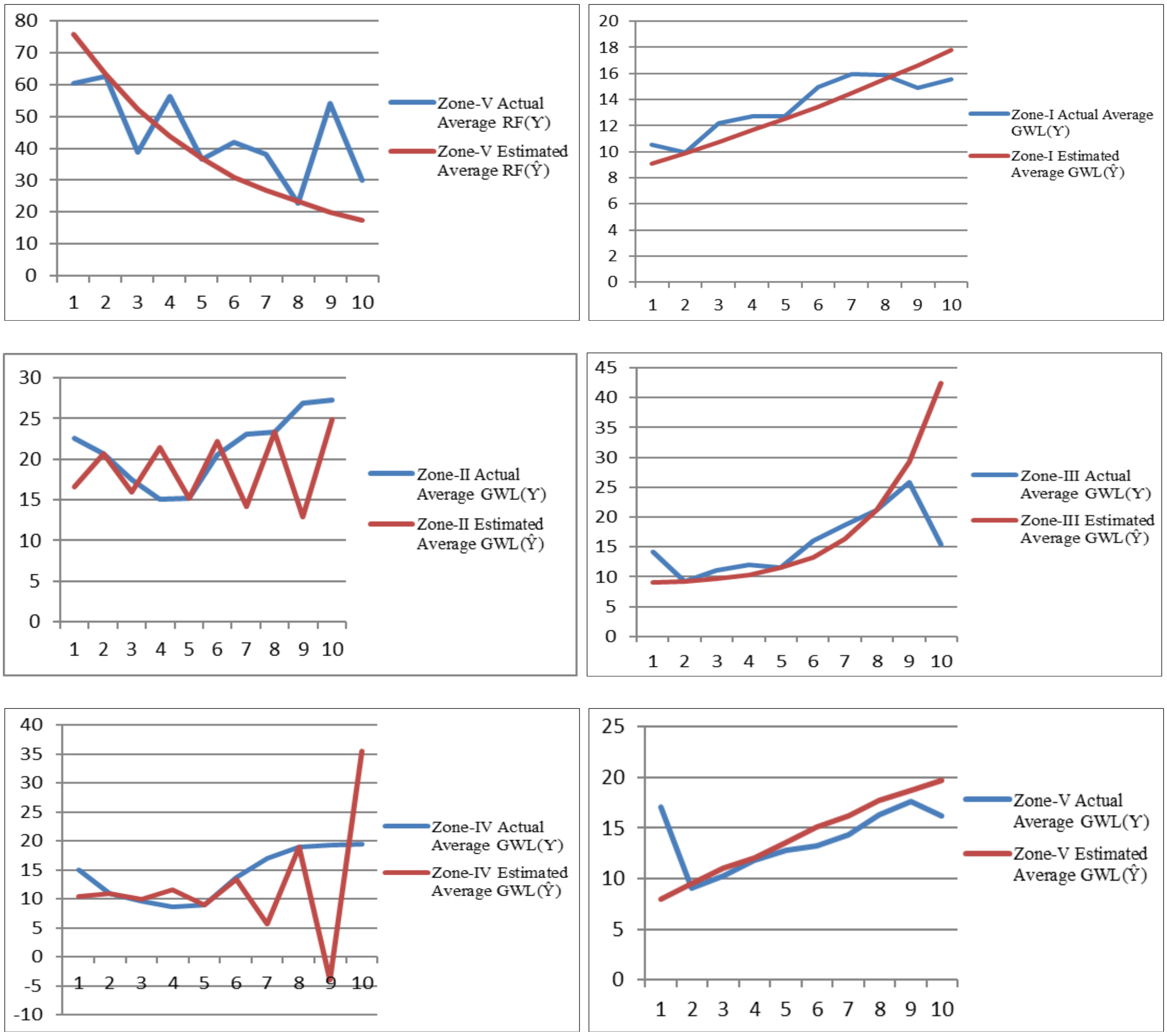
Table 3.4: MSE's for Average GWL – Modified Exponential Model.

Type of the Model	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V
Modified Exponential	17.66	360.28	788.99	951.39	106.56

4. Conclusions

By Comparing MSE's for RF and GWLs through Modified Exponential model under consideration, for RF of zone-I is least and GWLs for zone-I Modified Exponential model is the most suitable model because MSEs for zone-I is least. Next to zone-I, zone-V has least MSEs. Thus next to zone-I for zone-V Modified Exponential model is best suitable model for the RF and GWLs. Further, the behaviors of RF and GWL through this model i.e. Modified Exponential model in different zones are represented in the following Figure-3.1. Similar conclusions can be drawn from the following graphs also.





On y-axis RF measured in Mille Meters or Average GWLs measured in Meters.

Fig 1: Behavior of RF and GWL actual and Modified Exponential Forecasts in Zone –I, II, III, IV and V

5. Other Statistical Analysis

Now we proceed to analyse the given estimates in tables-3.1 and 3.2 using Duncan's Multiple Range (DMR) Test.

Table 4.1: Zonal wise DMR Test Results for Average RF and Average GWL for Modified Exponential Curve.

Zones	RF	GWL
Zone – I	40.82 [a]	13.17 [a]
Zone – II	49.37 [a]	18.74 [a]
Zone – III	43.78 [a]	17.21 [a]
Zone – IV	33.30 [a]	12.13 [a]
Zone – V	39.08 [a]	14.14 [a]

Note: In the above table – 4.1 symbols 'a 'indicates there is no significance variation between zones.

6. Critical Difference (C.D) Test: Average RF for Years

Table 5.1: Year wise Aggregate Average RF for Modified Exponential estimates

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Average	60.14	64.44	49.07	47.63	40.23	37.38	32.93	30.28	26.69	23.89
Ranking	IX	X	VIII	VII	VI	V	IV	III	II	I

Table 5.2: If we can arranged Ascending Order

Year	2016	2015	2014	2013	2012	2011	2010	2009	2007	2008
Average	23.89	26.69	30.28	32.93	37.38	40.23	47.63	49.07	60.14	64.44

S.E = $\sqrt{2 \times Error\ M.S.S/m}$
= 9.26

1% l.o.f C.D = 2.58×9.26
= 23.89

Above notation indicates that 2016, 2015, 2014, 2013, 2012, 2011, 2010 years Average RF come under one category and 2015, 2014, 2013, 2012, 2011, 2010, 2009 years Average RF and 2012, 2011, 2010, 2009, 2007 years Average RF and also 2010, 2009, 2007, 2008 come under another category because there is no Significant Difference in average RF. These years are ranked based on their respective Average RF.

7. Critical Difference (C.D) Test: Average GWL for Years

Table 5.3: Year wise Aggregate Average Ground Water Levels for Modified Exponential estimates

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Average	10.59	12.04	11.48	13.39	12.34	15.50	13.35	19.35	14.70	28.04
Ranking	I	III	II	VI	IV	VIII	V	IX	VII	X

Table 5.4: If we can arranged Ascending Order

Year	2007	2009	2008	2011	2013	2010	2015	2012	2014	2016
Average	10.59	11.48	12.04	12.34	13.35	13.39	14.70	15.50	19.35	28.04

S.E = $\sqrt{2 \times Error\ M.S.S/m}$ = 3.59

1% l.o.f C.D = 2.58×3.59 = 9.26

Above notation indicates that 2007, 2009, 2008, 2011, 2013, 2010, 2015, 2012, 2014 Average GWLs come under one category and 2014, 2016 Average GWLs, come under another category because there is no Significant Difference in average ground water levels. These years are ranked based on their respective average GWLs.

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