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A study on interactive fuzzy multi-objective nonlinear programming

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Abstract
This paper presents the method for solving nonlinear programming problems with multiple objectives. We transform these problems to goal multi-objective nonlinear programming problem to solve the corresponding minimax problem to obtain the M-Pareto optimal solution and the membership function value together with the trade-off rate information between the membership functions.

Keywords: M-Pareto optimal solution, goal multi-objective nonlinear programming problem (GMONLP), interactive multi-objective nonlinear programming

Introduction

In most real-world situations, the model coefficients are not exactly known because relevant data is in existent or scarce, difficult to find, the system is subject to change etc. Therefore, mathematical programming models for decision support must take explicitly into account, besides multiple and conflicting objective functions, the treatment of the intrinsic uncertainty associated with the model coefficients [2].

In the fuzzy approaches to multi-objective nonlinear programming problems proposed by Zimmermann [12] and his successors; however, it has been assumed that the fuzzy decision of Bellman and Zadeh (1970) [1] is the proper representation of the fuzzy preferences of the decision maker (DM). Therefore, these approaches are preferable only when the DM feels that the fuzzy decision is appropriate when combining the fuzzy goals and/or constraints. However, such situations seem to occur rarely in practice and consequently it becomes evident that an interaction with the DM is necessary. In the interactive multi-objective nonlinear programming method proposed by Sakawa, (1987) [6], the reference membership values can be viewed as natural extensions of the reference point of Wierzbicki (1979a) [9] in objective function spaces.

In this section, assuming that the DM has a fuzzy goal for each of the objective functions in multi-objective nonlinear programming problems [9], we present an interactive fuzzy multi-objective nonlinear programming method incorporating the desirable features of the interactive approaches into the fuzzy approaches.

Multi-Objective Nonlinear Programming with Fuzzy Coefficients

The multi-objective nonlinear programming with fuzzy coefficient can be formulated as follows [4, 5]:

Maximize $\sum_j \tilde{C}_{rj} x_j^{\alpha j}$ r = 1, 2, 3,……, q (1)

s.t:

$\sum_j \tilde{a}_{ij} x_j \leq \tilde{b}_i$ i = 1, 2, 3, ……, m

$x_j \geq 0$, Where's $\bar{c}_{rj}$, $\bar{a}_{ij}$ and $\bar{b}_i$ are fuzzy numbers.

Multi-Objective Nonlinear Programming with Vector-Minimization Problem [3, 8]

The multi-objective nonlinear programming (MONLP) problem is represented as the following vector-minimization problem:

$$min\,i\,mize z(x) = (z_1(x), z_2(x), \ldots, z_k(x))^T \qquad (2)$$
$$subject to\, x\varepsilon X = \{x\varepsilon R^n / Ax \le b, x \ge 0\}$$

Where x is an n-dimensional vector of decision variables, $z_1(x) = c_1x, \ldots, z_k(x) = c_kx$

are k conflicting nonlinear objective functions, and X is the feasible set of linearly constrained decisions. Fundamental to the MONLP is the concept of Pareto optimal solutions, also known as a non-inferior solution.

Considering the imprecise nature inherent in human judgments in multi-objective nonlinear programming problems, the DM may have a fuzzy goal expressed as "$z_i(x)$ should be substantially less than or equal to some value p_i." In a minimization problem, a fuzzy goal stated by the DM may be to achieve "substantially less than or equal to p_i." This type of statement can be quantified by eliciting a corresponding membership function representing the fuzzy goal to achieve substantially less than or equal to p_i.

To elicit a membership function $\mu_i(z_i(x))$ from the DM to each of objective functions $z_i(x)$, i=1,…k, we first calculate the individual minimum $z_i^{min\,min_{x\varepsilon X} z_i}$ and maximum $z_i^{max\,max_{x\varepsilon X} z_i}$ of each objective function $z_i(x)$ under the given constraints.

Taking into account the calculated individual minimum and maximum of each objective function together with the rate of increase of membership of satisfaction, the DM must determine the subjective membership function $\mu_i(z_i(x))$, which is a strictly monotone decreasing function with respect to $z_i(x)$. Here, it is assumed that $\mu_i(z_i(x))$=0 or →0 if $z_i(x) \ge z_i^0$ and $\mu_i(z_i(x))$=1 or →1 if $z_i(x) \le z_i^1$, where z_i^a represents the value of $z_i(x)$ such that the value of membership function $\mu_i(z_i(x))$ is $a\varepsilon[0,1]$ Within z_i^{min} and z_i^{max}.

We have restricted ourselves to a minimization problem and consequently assumed that the DM has a fuzzy goal such as "$z_i(x)$ should be substantially less than or equal to p_i."

Goal Fuzzy Multi-Objective Nonlinear Programming Problem (Gmonlp)

Multi-objective nonlinear programming problem (GMONLP) may now be expressed as [7, 10]

Fuzzy min $z_i(x) i\varepsilon I_1$

Fuzzy max $z_i(x) i\varepsilon I_2$

Fuzzy equal $z_i(x) i\varepsilon I_3$ (3)

Subject to $x\varepsilon X$

Where $I_1 \cup I_2 \cup I_3 = \{1,2,\ldots k\}, I_i \cap I_j = \varphi, i,j = 1,2,3, i \ne j.$

Here "fuzzy min $z_i(x)$"or "fuzzy max $z_i(x)$" represents the fuzzy goal of the DM such as

"$z_i(x)$ *should* be substantially less than or equal to

p_i or greater than or equal to

q_i, "and" fuzzy equal

$z_i(x)$ "represents the fuzzy goal such as"

$z_i(x)$ should be in the vicinity of

r_i."

Concerning the membership function for the fuzzy goal of the DM such as"

$z_i(x)$ should be in the vicinity of

r_i," it is obvious that a strictly monotone increasing function

$d_{iL}(z_i)(i\varepsilon I_{3L})$ and a strictly monotone decreasing function

$d_{iR}(z_i)(i\varepsilon I_{3R}), (I_3 = I_{3L} \cup I_{3R})$ corresponding to the left and right sides of

r_i must be determined through interaction with the DM. The possible shape of the fuzzy equal membership functions where the left function is nonlinear and the right function is exponential. When the fuzzy equal is included in the fuzzy goals of the DM, it is desirable that zi(x) should be as close to as possible. Consequently, the notion of Pareto optimal solutions defined in terms of objective functions cannot be applied. For this reason, we introduce the concept of M-Pareto optimal solutions which is defined in terms of membership functions instead of objective functions. M refers to membership.

Definition (Pareto Optimal Solution) [8]
x* ε X is said to be a Pareto optimal solution to the MONLP if and only if there does not exist another x ε X such that f_i (x) ≤ $f_i(x^*)$, i = 1,..., k, with strict inequality holding for at least one i.

Definition (M-Pareto Optimal Solution) [11]
$x^* \varepsilon X$ is said to be an M-Pareto optimal solutions to the GMONLP if and only if there does not exist another $x \varepsilon X$ such that $\mu_i(z_i(x)) \geq \mu_i(z_i(x^*)) \forall i$ and $\mu_i(z_i(x) \neq \mu_i(z_i(x^*))$ for at least one j

Having elicited the membership functions $\mu_i(z_i(x)), i = 1, \ldots, k,$ from the DM for each of the objective functions $z_i(x), i = 1, \ldots, k$ the MONLP and /or the GMONLP can be converted into the fuzzy multi-objective optimization problem (FMOP) defined by

$$\max_{x \varepsilon X} imize\ (\mu_1(z_1(x)), \mu_2(z_2(x)), \ldots.. \mu_k(z_k(x)))). \tag{4}$$

By introducing a general aggregation function

$$\mu_D(\mu(z(x))) = \mu_D(\mu_1(z_1(x)), \mu_2(z_2(x)), \ldots., \mu_k(z_k(x))), \tag{5}$$

A general fuzzy multi-objective decision making problem (FMDMP) can be defined by

$$\max_{x \varepsilon X} imize\ \mu_D(\mu(z(x))). \tag{6}$$

Observe that the value of $\mu_D(\mu(z(x)))$ can be interpreted as representing an overall degree of satisfaction with the DM's multiple fuzzy goals.
The fuzzy decision or the minimum operator of Bellman and Zadeh (1970)

$$\min_{i=1,\ldots,k} (\mu_1(z_1(x)), \mu_2(z_2(x)), \ldots., \mu_k(z_k(x))), \tag{7}$$

Can be viewed only as one special example of $\mu_D(\mu(z(x)))$
In the conventional fuzzy approaches discussed thus far, it has been implicitly assumed that the minimum operator is the proper representation of the DM's fuzzy preferences and hence, the FMDMP has been interpreted as

$$\max_{x \varepsilon X} imize \min_{i=1,\ldots,k} (\mu_1(z_1(x)), \mu_2(z_2(x)), \ldots., \mu_k(z_k(x))) \tag{8}$$

Or equivalently

$$\left.\begin{array}{l} \max imize\ v \\ subject to\ v \leq \mu_i(z_i(x), i = 1, \ldots, k \\ x \varepsilon X \end{array}\right\} \tag{9}$$

It should be emphasized here that this approach is preferable only when the DM feels that the minimum operator is appropriate.
DM does not always use the minimum operator when combining the fuzzy goals and/or constraints. Problem the most crucial problem in the FMDMP is the identification of an appropriate aggregation function which will represent the DM's fuzzy preferences.
If $\mu_D(.)$ can be explicitly identified, then the FMDMP reduces to a standard mathematical programming problem. However, this rarely happens, and as an alternative, an interaction with the DM is necessary for finding the satisfying solution of the FMDMP.

Interactive Fuzzy Multi-Objective Nonlinear Programming [7]
In the interactive multi-objective nonlinear programming, the membership functions are

$\mu(z(x)) = (\mu_1(z_1(x)), \mu_2(z_2(x)), \ldots., \mu_k(z_k(x)))^T$ For each of the objective functions $z(x) = (z_1(x), \ldots, z_k(x))^T$ are determined first.

To generate a candidate for the satisfying solution which is also (M-) Pareto optimal, the DM is then asked to specify the aspiration levels of achievement for the membership values of all membership functions, called the reference membership levels.
For the DM's reference membership levels $\overline{\mu} = (\overline{\mu_1}, \ldots. \overline{\mu_k})^T$ the corresponding (M-) Pareto optimal solution, which is nearest to the requirements in the minimax sense or better than that if the reference membership levels are attainable, is obtained by solving the following minimax problem

$$\min_{x \in X} imize \max_{i=1,\dots,k} \{\overline{\mu_i} - \mu_i(z_i(x))\} \quad (10)$$

Or equivalently

$$\left.\begin{array}{l} min\, i\, mize\ \upsilon \\ subject\ to\ \overline{\mu_i} - \mu_i(z_i(x)) \leq \upsilon, i = 1,\dots,k \\ x \in X \end{array}\right\} \quad (11)$$

If all of the membership functions$\mu_i(z_i(x))$,i=1,…k, are linear the mimimax problem becomes a nonlinear programming problem, and hence, we can obtain an optimal solution by directly applying the simplex method of linear programming.

For notational convenience, denote the strictly monotone decreasing functions for the fuzzy min and the right function of the fuzzy equal by $d_{iR}(z_i)(i \in I_1 \cup I_{3R})$and the strictly monotone increasing function for the fuzzy max and the left function of the fuzzy equal by $d_{iL}(z_i)(i \in I_2 \cup I_{3L})$.Then in order to solve the formulated problem on the basis of the linear programming method, convert each constraint $\overline{\mu_i} - \mu_i(z_i(x)) \leq \upsilon, i = 1,\dots.k$,of the minimax problem (11) into the following form the strictly monotone property of $d_{iL}(.)$and $d_{iR}(.)$:

$$\left.\begin{array}{l} min\, i\, mize\ \upsilon \\ z_i(x) \geq {d_{iL}}^{-1}(\overline{\mu_i} - \upsilon), i \in I_2 \cup I_{3L} \\ subject\ to\ z_i(x) \leq {d_{iR}}^{-1}(\overline{\mu_i} - \upsilon), i \in I_1 \cup I_{3R} \\ x \in X \end{array}\right\} \quad (12)$$

If the value of υ is fixed, it can be reduced to a set of linear inequalities. Obtaining the optimal solution $\upsilon^*$to the above problem is equivalent ot determining the minimum value of υ so that there exists an admissible set satisfying the constraints of (12).Since υsatisfies$\overline{\mu}\mu_{max_{max}}$where $\overline{\mu}_{max}$ denotes the maximum value of$\overline{\mu_i}$,i=1,..,k we have the following method solving this problem by combined use of the bisection method and the simplex method of linear programming.

Here, when $\overline{\mu_i} - \upsilon \leq 0$,set $\overline{\mu_i} - \upsilon = 0$,in view of the constraints $\overline{\mu_i} - \upsilon \leq \mu_i(z_i(x))for0 \leq \mu_i(z_i(x) \leq 1, i = 1,\dots,k$. [13]

Step 1: Set $\upsilon = \overline{\mu}_{max}$and test whether an admissible set satisfying the constaints of (3.136) exists or not using phase one of the simplex method. If an admissible set exists, set $\upsilon^* = \overline{\mu}_{max}$Otherwise, go to the next step since the minimum υwhich satisfies the constraints of (12) exists between $\overline{\mu}_{max}$and $\overline{\mu}_{max}$.

Step 2: For the initial value of $\upsilon = \overline{\mu}_{max}$, update the value of υ using the bisection method as follows:

$$\left\{\begin{array}{l} \upsilon_{n+1} = \upsilon_n - 1/2^{n+1} \text{ if an admissible set exists for } \upsilon_n \\ \upsilon_{n+1} = \upsilon_n + 1/2^{n+1} \text{ if no admissible set exists for } \upsilon_n \end{array}\right.$$

For each $\upsilon_n, n = 1,2,\dots$, test whether an admissible set of (12) exists or not using the sensitivity analysis technique for changes in the right hand side of the simplex method and determine the minimum value of υsatisfying the constraints of (12).

We can determine the optimal solution υ^*.Then the DM selects an appropriate standling objective from among the objectives $z_i(x), i = 1,\dots k$.For notational convenience in the following without loss f generality, let it be $z_1(x) and 1 \in I_1$.Then the following linear programming problem is solved for $\upsilon = \upsilon^*$:

$$\begin{array}{l} \min imize\ z_1(x) \\ z_i(x) \geq {d_{iL}}^{-1}(\overline{\mu_i} - \upsilon^*), i(\neq 1) \in I_2 \cup I_3 \\ subject\ to\ z_i(x) \leq {d_{iR}}^{-1}(\overline{\mu_i} - \upsilon^*), i(\neq 1) \in I_1 \cup I_3 \\ x \in X \end{array} \quad (13)$$

The relationships between the optimal solutions of the minimax problem and the (M-) Pareto optimal concept of the MOLP can be characterized by the following theorem.

Theorem [7]

(1) If $x^* \in X$is a unique optimal solution to the minimax problem for some $\overline{\mu_i}, i = 1,\dots k$,then $x^*$is a (M-) Pareto optimal solution to the (G) MONLP.
(2) If $x^*$is a (M-) Pareto optimal solution to the (G) MONLP with $0 < \mu_i(z_i(x^*))$<1 holding for all I, them there exists $\overline{\mu_i}, i = 1,\dots k$, such that $x^*$is an optimal solution to the minimax problem.

The proof of this theorem follows directly from the definitions of optimality and (M-) Pareto optimality by making use of contradiction arguments. It must be noted here that, for generating (M-) Pareto optimal solutions using this theorem, uniqueness of solution must be verified.

The (M-) Pareto optimality of current optimal solution x^*, we solve the following Pareto optimality test problem.

$$\begin{aligned} &\max imize \sum_{i=1}^{k} \varepsilon_i \\ &subject\ to\ \mu_i(z_i(x)) - \varepsilon_i = \mu_i(z_i(x^*)), i = 1, .., k \\ &x \in X, \varepsilon = (\varepsilon_1, \dots \varepsilon_k)^T \geq 0. \end{aligned} \tag{14}$$

For the optimal solution

$\overline{x}$ and

$\overline{\varepsilon}$ to this nonlinear programming problem, if all

$\overline{\varepsilon_i} = 0$, then

x^* is a (M-) Pareto optimal solution ti the (G) MONLP, and (2) if at least one $\overline{\varepsilon_i} > 0$, not x^* but $\overline{x}$ is a (M-) Pareto optimal solution of the (G) MONLP.

The DM must either be satisfied with the current (M-) Pareto optimal solution or act on this solution by updating the reference membership levels. In order to help the DM express a degree of preference, as was discussed in the previous subsection, trade-off information between a standing membership function $\mu_1(z_1(x))$and each of the other membership functions is very useful. Such trade-off information is easily obtainable since it is closely related to the simplex multipliers of problem(13).
Let the simplex multipliers corresponding to the constraints
$z_i(x), i = 2, \dots, k$ of the liner problem (13) be denoted by
$\pi_i^* = \pi_i(x^*), i = 2, \dots, k$,where
$x^*$is an optimal solution of (13) and all the constraints of (13) are active, then by using the results in Haimes and Chankong(1979), the trade-off information between the objective functions can be represented by

$$-\frac{\partial z_1(x)}{\partial z_i(x)} = -\pi_i^*, i = 2, \dots, k. \tag{15}$$

Hence, by the chain rule, the trade-off information between the membership functions is given by

$$-\frac{\partial \mu_1(z_1(x))}{\partial \mu_i(z_i(x))} = \frac{\partial \mu_1(z_1(x))}{\partial z_1(x)} \frac{\partial z_1(x)}{\partial z_i(x)} \left\{\frac{\partial \mu_i(z_i(x))}{\partial z_i(x)}\right\}^{-1}, i = 2, \dots, k. \tag{16}$$

Therefore, for each i=2,…,k we have the following expression

$$-\frac{\partial \mu_1(z_1(x))}{\partial \mu_i(z_i(x))} = \pi_i^* \frac{\partial \mu_1(z_1(x))/\partial z_1(x)}{\partial \mu_i(z_i(x))/\partial z_i(x)}, i = 2, \dots, k. \tag{17}$$

It should be stressed here that in order to obtain the trade-off rate information from (17), all the constraints of problem (13), must be active. Therefore, if ther are inactive constraints, it is necessary to replace
$\overline{\mu_i}$ for inactive constraints by
$\overline{\mu_i}(z_i(x^*))$ and solve the corresponding problem to obtain the simplex multipliers.
We can now construct the interactive algorithm in order to derive the satisfying solution for the DM from the (M-) Pareto optimal solution set where the steps marked with an asterisk involve interaction with the DM. This interactive fuzzy multi-objective programming method can also be interpreted as the fuzzy version of the reference point method (RPM) with trade-off information.

Interactive Fuzzy Multi-Objective Nolinear Programming Algorithm
Step 0: Calculate the individual minimum and maximum of each objective function under the given constraints.
Step 1: Elicit a membership function from the DM for each of the objective functions.
Step 2: Set the initial reference membership levels to 1.
Step 3: For the reference membership values, solve the corresponding minimax problem to obtain the (M-) Pareto optimal solution and the membership function value together with the trade-off rate information between the membership functions.
Step 4: If the DM is satisfied with the current levels of the (M-) Pareto optimal solution, stop. Then the current (M-) Pareto optimal solution is the satisfying solution of the DM. Otherwise, ask the DM to update the current reference membership levels by considering the current values of the membership functions together with the trade-off between the membership functions and return to step 3.
It should be stressed to the DM that any improvement of one membership function can be achieved only at the expense of at least one to the other membership functions.

Numerical Example
The corresponding GMONLP can be formulated as follows:

$$\begin{aligned} &fuzzymax \quad x_1^2 + 2x_2 \\ &fuzzymin \quad 3x_1 + 2x_2 \\ &fuzzyequal\ x_1^2 - 2x_2 \\ &subjectto \quad x \in X \end{aligned} \tag{18}$$

Where X is the feasible region.
First, observe that the individual minimums and maximums of each objective function are

$Z_1^{\min x} = 0$, $Z_1^{\max x} = 10.6$, $Z_2^{\min x} = 0$, $Z_2^{\max x} = 15.6$, $Z_3^{\min x} = 0$, $Z_2^{\max x} = -9.6$

Considering these values, assume that the DM determines the following linear membership functions for the fuzzy goals.

$$\begin{cases} fuzzy \ \max \ \mu_1(6) = 0, \ \mu_1(7) = 1, \\ fuzzy \ \min \ \mu_2(16) = 0, \ \mu_2(9) = 1, \\ fuzzy \ equal \ \mu_{3L}(-6) = 0, \ \mu_{3L}(0) = 1, \ \mu_{3R}(0) = 1, \ \mu_{3R}(4) = 0, \end{cases}$$

Where linear functions are assumed from $\mu_i = 0$ to $\mu_i = 1$ for i=1,2,3.
Then, for the initial reference membership value 1, solving the corresponding minimax problem yields the M-Pareto optimal solution.

$z_1 = 6.833, z_2 = 10.94, z_3 = -2.722, \quad (x_1 = 2.056, x_2 = 2.389),$

The corresponding membership values

$\mu_1 = 0.6111, \quad \mu_2 = 0.6111, \quad \mu_3 = 0.6111,$

And the trade-off rates between the membership functions

$-\frac{\partial \mu_2}{\partial \mu_1} = 1.2, -\frac{\partial \mu_3}{\partial \mu_1} = 0.8271,$

On the basis of such information the DM updates the reference membership values to

$\overline{\mu_1} = 0.7, \quad \overline{\mu_2} = 0.8, \quad \overline{\mu_3} = 0.5,$

Improving the satisfaction levels for the profit and the amount of pollution at the expenses of the ratio of two products.
For the updated reference membership values, the corresponding minimax problem yields the M-Pareto optimal solution.

$z_1 = 6.983 \ \ z_2 = 10.19 \ \ z_3 = -3.772 \quad x_1 = 1.606, x_2 = 2.689),$

The membership values

$\mu_1 = 0.6611, \quad \mu_2 = 0.7611, \quad \mu_3 = 0.4611,$

And the trade-off rates $-\frac{\partial \mu_2}{\partial \mu_1} = 1.2, \quad -\frac{\partial \mu_3}{\partial \mu_1} = 0.8271,$

Conclusion

If the DM is satisfied with the current values of the membership functions, the procedure steps; otherwise, a similar procedure continues until the satisfying solution for the DM is obtained.
By using the Interactive fuzzy multi-objective nonlinear programming algorithm, we get a new solution to a nonlinear multi-objective programming problem with fuzzy co-efficient.

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