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Problem of opening of two interior griffith cracks by body forces in an infinite orthotropic strip

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Abstract
By using Fourier Transform, the stress and displacement field in the vicinity of two Griffith crack opened by symmetrical system of body forces in an orthotropic strip has been obtained. It is obvious extension and the quest for finding the effects of other crack upon single Griffith crack.

Keywords: Composite material, Stress Intensity Factor, Point body Force, Interior griffith crack, Boundary conditions

Introduction

After discussing about problem of an Interior Griffith crack. It is obvious to solve the problem of two Griffith cracks, so as to know the effects on physical Quantities of a Griffith crack because of another crack. In present paper we shall extend the problem of cracks occupying the region

$y = 0, b < |\varkappa| < c$,The boundary conditions.

are $\sigma_{xy}(x, 0) = 0,$

$\sigma_{xy}(x, \pm\delta) = 0$ for $0 \leq |x| < \infty$ 1.1

$u_y(x, \pm\delta)$=0 (1.2)

& Mixed boundary conditions

$\sigma_{yy}(x, 0) = 0, b < |x| < c$ (1.3)

$u_y(\varkappa, 0) = 0, \quad 0 \leq |x| \leq b, |x| \geq c$ (1.4)

where 2 (c-b) and 2δ are the length and width of the cracks and strip respectively.

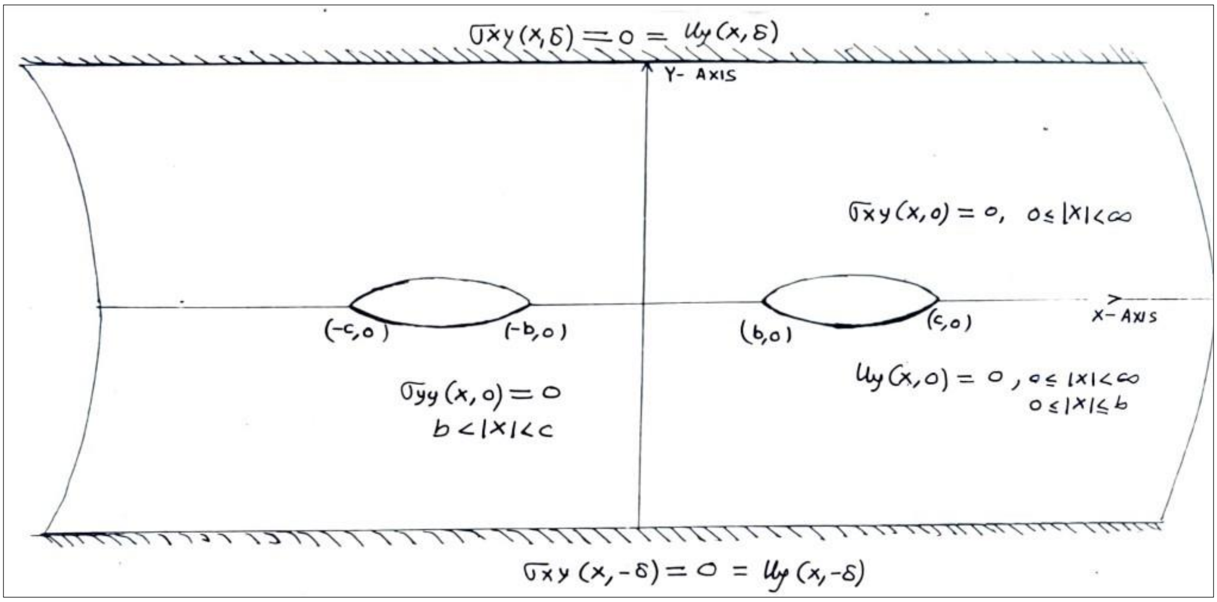

Fig 1: Geometry of the problem

Here we define the Fourier Transform as

Fcs $(\xi, \alpha_n) = \int_0^\infty \int_0^d cos\ (\xi x) \sin(\alpha_n y)\, F(x, y) dxdy$

$$\alpha_n = \frac{n\Pi}{\delta} = nq$$

Now we shall formulate the problem, and the reduction to triple integral equation will be done there in

Formulation and Reduction to Triple Integral Equation

Problem can be divided into two sections as body force problem & crack opening problem. Boundary Conditions over body force problems can be expressed as

$$\sigma\varkappa y^{(1)}(x, \pm\delta) = 0, 0 \leq |x| < \infty \quad (2.1)$$

$$u_y^{(1)}(x, \pm\delta) = 0, 0 \leq |x| < \infty \quad (2.2)$$

$$\sigma_{xy}^{(1)}(x, 0) = 0, 0 \leq |x| < \infty \quad (2.3)$$

$$u_y^{(1)}(x, 0) = 0, 0 \leq |x| < \infty \quad (2.4)$$

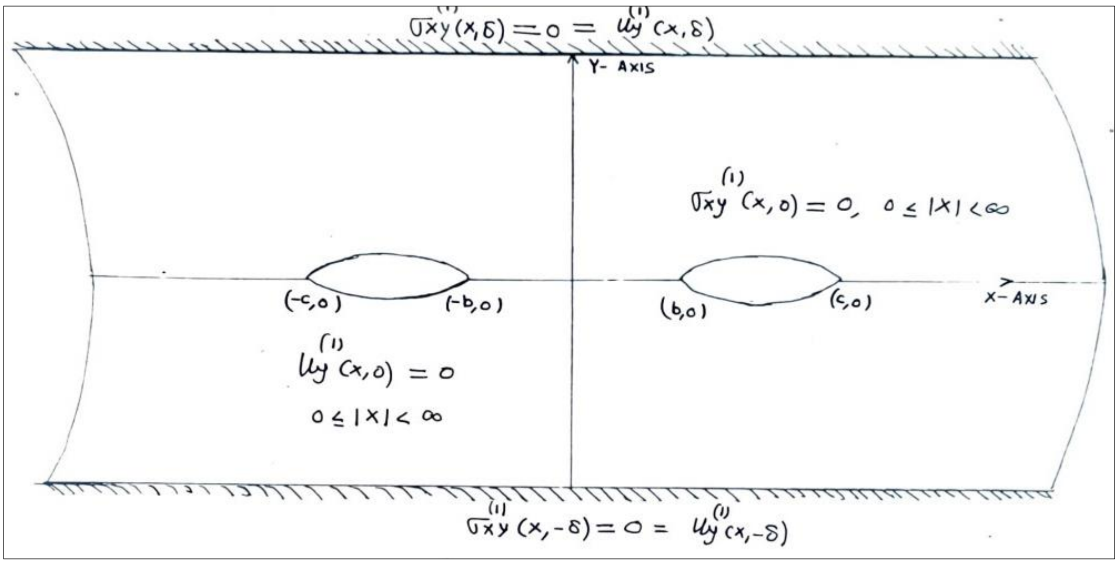

Fig 2: Body Force Problem

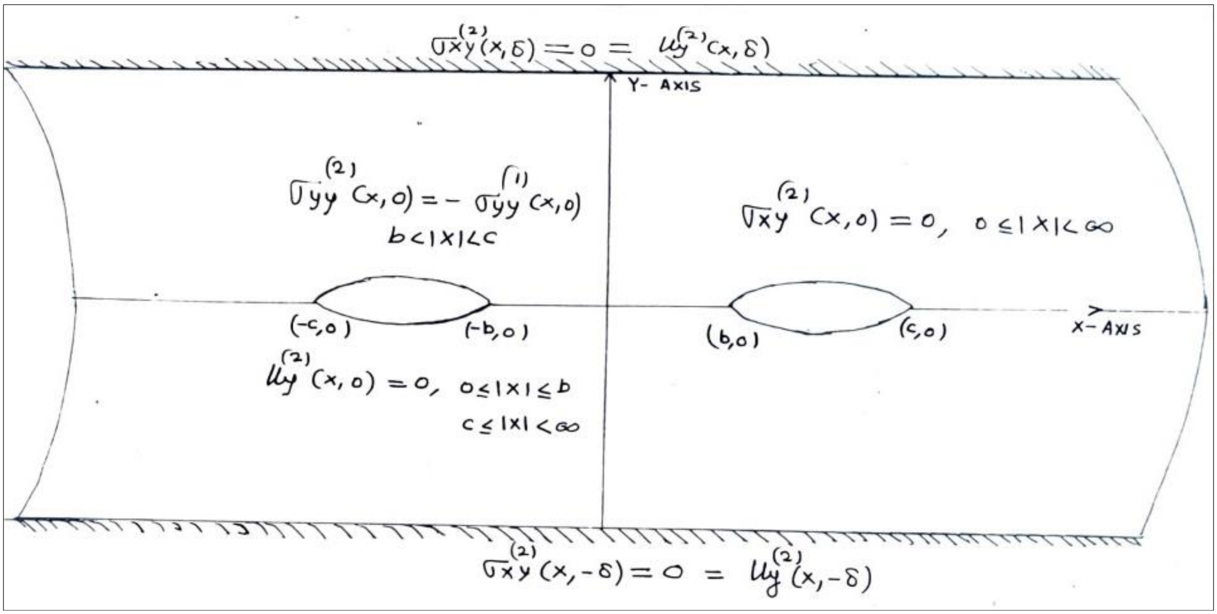

Fig 3: Crack Opening Problem

Reduced boundary Conditions from $(1.1 - 1.4)$ after using $(2.1 - 2.4)$ & using previous solutions

$$r_1 A(\xi) + r_2 c(\xi) = 0 \tag{2.5}$$

$$r_1 \cos h\,(r_1\xi d)A(\xi) + r_1 \sin h\,(r_1\xi d)B(\xi) +$$

$$r_2 \cos h\,(r_2\xi d)C(\xi) + r_2 \sin h\,(r_2\xi d)D(\xi) = 0 \tag{2.6}$$

The Boundary conditions (1.2) gives after using (2.1)

$$H_{12}A(\xi) + \alpha_1 \sin\mathrm{h}\;(r_1\xi d)B(\xi) + \alpha_2 \sin\mathrm{h}\;(r_2\xi d)D(\xi) = 0 \tag{2.7}$$

Where

$$H_{12} = \alpha_1 \cos\mathrm{h}\;(r_1\xi d) + \alpha_2 \cos\mathrm{h}\;(r_2\xi d)$$

$$\alpha_i = r_1(a_{11}r_i^2 - \alpha_0), i = 1,2 \tag{2.8}$$

After three constants solved by previous equation, we can find fourth constant is determine by mixed boundary conditions We set as

$$\frac{\alpha_3}{r_1}\int_0^\infty \xi A(\xi)\cos(\xi\varkappa)\,d\xi = 0, x \in \mathrm{I}_1 \mathrm{U}\, \mathrm{I}_3 \tag{2.9}$$

$$\int_0^\infty \xi^2 A(\xi)[1 + H(\xi d)]\cos(\xi\varkappa)\,d\xi = \alpha_3\sigma_{yy}(x,0)\; x \in \mathrm{I}_2 \tag{2.10}$$

where,

$$H(y) = \alpha_3 \tan h\,(r_1, y) + \alpha_4 \sin h\,(r_2\, y) cosec\; h\,(r_1, y) + r_1\alpha_5 \cot h\,(r_2, y) - 1 \tag{2.11}$$

$$\alpha_3 = r_1 r_2 a_{11}(r_1^2 - r_2^2), \alpha_4 = (r_1 + r_2)\alpha_2$$

$$\alpha_5 = r_1 r_2[a_{11}(r_1 + r_2)^2 - 3r_1 r_2 - \alpha_0]$$

$$I_1 = [0, b), I_2 = (b, c), I_3 = [c, \infty)$$

Solution of integral equation
Assuming the trial solution

$$\pi\xi^2 A(\xi) = 2\int_b^c g(t)\sin(\xi t)\,dt \tag{3.1}$$

It can be satisfied identically if

$$\int_b^c g(t)dt = 0 \tag{3.2}$$

Putting substitution of (3.1) into (2.10) we can find Fredholm Integral Equation of second kind as

$$g(t) = \frac{\alpha_3}{\Pi^2 \delta(t)} \left(\int_b^c \frac{x\ \delta(x)\, \sigma_{yy}^{(1)}\ (x, 0)}{(x^2 - t^2)} dx + \int_b^c g(y) k\ (y, t)\ \ dy + D \right)$$

where,

$$k(y, t) = \int_b^c \frac{x\delta(\varkappa) L(x,t)\ dk}{x^2 - y^2} \tag{3.3}$$

$$L(x, t) = \int_0^\infty H(\xi d) \sin(\xi t) \cos(\xi x)\ d\xi$$

Expanding H(y) in powers of $e^{-n\xi d}$ we get

$$H(\xi d) = 2 \sum_{n=0}^\infty e^{-2nr_1\xi d}\ \left[< (-1)^n \alpha_3 + r_1\alpha_5 + \alpha_4 < e^{(r_2 - r_1)\xi d} - e^{(r_1 - r_2)\xi d} > -1 + e^{-2r_1\xi d} \right] \tag{3.4}$$

Now substituting above values & evaluating the integrals we get

$$L(x_2, t) = \frac{2t}{t^2 - x^2}(\alpha_3 + r_1\alpha_5^{-1}) + \alpha_4 \left\{ \frac{(x+t)}{4\, r_1^2 d^2 + (x+t)^2} + \frac{(t-x)}{4\, r_1^2 d^2 + (t-x)^2} - \frac{(x+t)}{(r_1 + r_2)^2 d^2 + (x+t)^2} - \frac{(t-x)}{(r_1 + r_2)^2 d^2 + (t-x)^2} \right\} + \left[(\alpha_3 + r_1\alpha_1 + 1) + \frac{(x+t)}{a_1^2 + (x+t)^2} + \frac{(t-x)}{a_1^2 + (t-x)^2} \right] + \alpha_4 \left[\frac{(x+t)}{a_2^2 + (t+x)^2} + \frac{(t-x)}{a_2^2 + (t-x)^2} - \frac{(x+t)}{a_3^2 + (t+x)^2} - \frac{(t-x)}{a_3^2 + (t-x)^2} \right] \tag{3.5}$$

where

$a_1 = 2nr_1 d, a_2 = [(2n-1)r_1 - r_2]d = a_{21}d\ a_3 = [r_2(2n+1)r_1]d = a_{31}d$

Now by Expanding L(x,t) in powers of d^{-n}

$$L(x, t) = \frac{2t}{t^2 - x^2} L_1(x, t)$$

$$L_1(x, t) = \left[\alpha_6 + 2t \left(\frac{\propto_7}{d^2} - \frac{\propto_8}{d^4} \right) x^2 \right) + \Sigma \frac{2t}{d^2} \frac{\propto_6}{n^2 r_1^2} + \propto_4 \left[\frac{1}{a_{21}^2} + \frac{1}{a_{31}^2} \right] - \frac{63x^2}{d^2} \left(\frac{\propto_6}{n^4\, r_1^4} \right) + \propto_4 \left(\frac{1}{a_{21}^4} + \frac{1}{a_{31}^4} \right) \tag{3.6}$$

Where

$$\propto_6 = \propto_3 + r_1 \propto_5 - 1, \propto_7 = \frac{\propto_4}{4} \left(\frac{1}{r_1^2} + \frac{1}{(r_1 + r_2)^2} \right), \propto_8 = \frac{\propto_4}{16} \left(\frac{1}{r_1^4} + \frac{1}{(r_1 + r_2)^4} \right)$$

Now putting value of L(x,t) in (3.3) & evaluating the integrals we get

$$k(y, t) = \frac{2t^2 I_1}{(y^2 - t^2) d^4} (b^2 - 2t^2) + \propto_4 \sum_{n=1}^\infty \left(-3 \propto_9 + F_n{}^d(y, t) \right) \tag{3.7}$$

Where

$$I_m = mb^{2m-2} \frac{\Gamma\left(\frac{2m-1}{2}\right)}{2\Gamma m} + (2t^2 - b^2 - c^2) I_{m-1} \tag{3.8}$$

$$I_o = \begin{cases} \frac{\pi}{2}\left(\frac{1}{\delta(t)}\right) & 0 \le t \le b \\ 0 & b < t < c \\ \frac{\pi}{2}\left(-\frac{1}{\delta(t)}\right) & t > c \end{cases}$$

$$\alpha_9 = \frac{1}{a_{21}^4} + \frac{1}{a_{31}^4} \;\&\; \sum \propto_9 = \frac{\pi^2}{48}$$

F_n^d will be of O ($\frac{1}{d^2}$) type

Solution of Fredholm integral equation

For solving Fredholm Equation of second type given by (3.2) – (3.6),we use the method of Sneddon and we use the Lowengrub, let we assume

$$g_1(t) = \sum_{n=0}^{\infty} g_n(t)\, d^{-n} \tag{4.1}$$

Where d is half width of strip. Now putting from (4.1) to (3.3) and comparing the coefficients of different powers of d^{-n},we get

$$g_0(t) = \frac{\alpha_3}{\Pi^2 \delta(t)}\left[\int_b^c \frac{\delta(x)\sigma_{yy}^1(x,0)}{x^{2-}t^2} dx + D\right]$$

$$g_1(t) = g_2(t) = g_3(t) = g_5(t) = g_6(t) = 0 \tag{4.2}$$

$$g_4(t) = G_1(t)\int_b^c \frac{y g_0(y) dy}{y^2 - t^2} \tag{4.3}$$

$$G_1(t) = \frac{t^3[b_1^2 - 2t^2]}{\pi^2(t)}$$

&

$$b_1^2 = c^2 + b^2 - \frac{\pi^2}{16}, g_{2n+1}(t) = 0 \tag{4.4}$$

Physical Quantities

The Quantities of interest in fracture mechanics are crack opening displacement i.e., $uy(x,0), x \in I_2$ and stress components for

$x \in I_1 \cup I_3$

By evaluating crack shape series for interval b < x < c.

$$u_y{}^{(2)}(x,0) = \frac{r_1}{\propto_3}\int_x^c g(t)dt, b < x < c \tag{5.1}$$

where g(t) is solution of Fredholm Integral equation discussed earlier.

Stress component

The Shear stress $\sigma_{xy}^{(2)}(x,0)$ is zero throughout for $0 \le |x| < \infty$, the normal stress component $\sigma_{yy}^{(2)}(x,0)$ is given by the value of series given by (2.10) for $x \in I_1 \cup I_3$

$$\sigma_{xy}^{(2)}(x,0) = \frac{1}{\propto_3}\left[\int_b^c \frac{t}{x^2 - t^2} g(t)dt + \int_b^c g(t)\, L(x,t)dt\right] - \propto_3 \sigma_{yy}^{(1)}(x,0), x \in I_1 \cup I_3 \tag{6.1}$$

Where L(x,t) and g (t) are discussed before

Stress intensity factor

The Stress intensity factors at crack tips $(b \pm ,0)$ are defined as

$$k_b = \lim_{x \to b^-} (\sqrt{b-x})\sigma_{yy}^{(2)}(x,0) \; k_c = \lim_{x \to c^+} (\sqrt{x-c})\sigma_{yy}^{(2)}(x,0) \tag{7.1}$$

Making use of above definition and relation given by (6.1).

$$k_b = \frac{1}{\propto_3} \lim_{x \to 5} \sqrt{(b-x)} \int_b^c \frac{tg(t)}{x^2-t^2} dt$$

$$k_c = \frac{1}{\propto_3} \lim_{x \to c} \sqrt{x-c} \int_b^c \frac{tg(t)}{x^2-t^2} dt$$

Parts which are not in singular (6.1), will not contribute in stress intensity factor.

Conclusion

Using above calculation and taking special case by taking body force as point force acting at $(\pm d, 0)$ in negative and positive directions of x, respectively we can find square root singularity is observed in normal stress component at crack tips $(\pm b, 0)$ and $(\pm c, 0)$ while normal stress component due to body force does not contain singularity. By computing and evaluating the integral seems infinitively correct & also that stress intensity factors $k_c > k_b$ when $c < d$ & $k_c < k_b$ when $d < b < c$.

Symbols used in discussion

$\sigma_{xx}, \sigma_{xy}, \sigma_{yy}$: Components of Stress Tensor

u_x, u_y: Components of displacement vector

2δ: Widht of strip

2b: Length of Crack

q: $\frac{\pi}{\delta}$

$\delta(x)$: $|(x^2 - b^2)(c^2 - x^2)|^{\frac{1}{2}}$

$u_y^{(2)}(x, 0)$: $Crack\ Opening\ Displacement$

I_1, I_2, I_3: $Intervals$

$K_\propto$: $Stress\ Intensity\ Factor$

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