International Journal of Statistics and Applied Mathematics 2019; 4(5): 134-139

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2019; 4(5): 134-139

www.mathsjournal.com
Received: 27-07-2019
Accepted: 29-08-2019

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Combined effect of surface tension and hydromagnetics on the Kelvin-Helmholtz instability of superposed viscous fluids saturating porous medium

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Abstract
This paper deals with the effects of square of the Alfvén velocity, surface tension and the medium porosity on the instability of superposed viscous fluids in hydromagnetics saturating porous medium. Graphs are plotted by giving permissible numerical value to the non-dimensional parameters. The growth rate of the perturbations decreases slightly with the increase in medium porosity has very slight stabilizing effect on the system. The square of Alfvén velocity in imaginary growth rates of the perturbations the Kelvin-Helmholtz instability has stabilizing effect on the system. Surface tension has large enough stabilizing effect on the system. The effect of surface tension dissipates the energy of any disturbance more than that carried out by the magnetic field. In other words, the role of the square of Alfvén velocity can help the surface tension to find more stability on the Kelvin-Helmholtz instability problem, while the surface tension plays the fundamental role to generate the complete stability.

Keywords: Kelvin helmholtz instability, surface tension, medium permeability, medium porosity

1. Introduction

The instability of the plane interface separating two uniform superposed streaming fluids under varying assumptions of hydrodynamics and hydromagnetics in the presence of surface tension has been discussed in the monograph of Chandrasekhar [1]. There are diverse applications of the Kelvin-Helmholtz instability like to examine the horizontal and temporal variability of the out-of-cloud vertical velocity, the stratospheric gravity wave response to the convection to determine the vertical and spatial extent of turbulence due to gravity wave breaking, to provide a more realistic evolving background flow and convective initiation. Alterman [2] has studied the effect of surface tension to the Kelvin-Helmholtz instability of two rotating fluids. Reid [3] studied the effect of surface tension and viscosity on the stability of two superposed fluids. Bellman and Pennington [4] further investigated in detail illustrating the combined effects of viscosity and surface tension.

Sharma and Kumari [5] have studied hydromagnetic instability of streaming fluids in porous medium theoretically including surface tension and have found that the magnetic field and surface tension have stabilizing effect and completely suppress the Kelvin-Helmholtz instability for small wavelengths. The medium porosity reduces the stability range given in terms of a difference in streaming velocities and the Alfvén velocity. Some solar activities in the solar atmosphere are created by Kelvin-Helmholtz instability in the presence of magnetic field and subsequent reconnection processes and Kelvin-Helmholtz instability plays an important role in energy transfer mechanism in the solar atmosphere. The effect of the Kelvin-Helmholtz instability is shown to convert shear flow in compression flow that derives reconnection. Khalil Elcoot [6] has studied the new analytical approximation forms for non-linear instability of electric porous media. Asthana *et al*. [7] have been studied Kelvin-Helmholtz instability of two viscous fluids in porous medium for two dimensional flow. Rudraiah *et al*. [8] have studied the study of surface instability of Kelvin-Helmholtz type in a fluid layer bounded above by a porous layer and below by a rigid surface. The effect of porosity in astrophysical context and the plasma outflow occur in regions which are created by the Kelvin-Helmholtz vortices.

We believe that the mechanism presented here opens promising possibilities of further investigation. However a clear understanding of the role of the Kelvin-Helmholtz instability in reconnection requires fully three-dimensional flows.

Keeping in mind the fact that effect of surface tension in streaming and superposed fluids is of scientific importance mentioned above, the present paper is therefore to investigate the effects of the surface tension on the instability of electrically conducting, streaming superposed viscous three dimensional fluids saturating porous medium numerically using the software Mathematica version-5.2.

2. Materials and Methods

The initial state whose stability we wish to examine is that of an incompressible, electrically infinitely conducting viscous fluid in which there is a horizontal streaming in the x - direction with a velocity q through a homogeneous and isotropic porous medium of medium porosity ε and medium permeability k_1. A uniform horizontal magnetic field H and acceleration due to gravity $g(0,0,g)$ pervade the system. Suppose that at some prescribed level z_s, the density may change discontinuously and bring into play effect due to effective interfacial tension T_s, n_s denote normal to the interface and a subscript s distinguishes the value of the quantity at $z = z_s$.

Then the equations of motion, continuity, incompressibility for the viscous fluid and the Maxwell's equations saturating porous media relevant to the problem are

$$\frac{\rho}{\varepsilon}\left(\frac{\partial q}{\partial t} + \frac{1}{\varepsilon}(q.\nabla)q\right) = -\nabla p - \rho g + \sum_s \left[T_s\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) z_s\right]\delta(z - z_s)n_s - \frac{\mu}{k_1}q + \frac{1}{4\pi}(\nabla \times H) \times H, \tag{1}$$

$$\nabla.q = 0, \tag{2}$$

$$\varepsilon\frac{\partial \rho}{\partial t} + (q.\nabla)\rho = 0, \tag{3}$$

$$\nabla.H = 0, \tag{4}$$

$$\varepsilon\frac{\partial H}{\partial t} = \nabla \times (U \times H), \tag{5}$$

Where $q\ (U(z),0,0), p, \rho, g, \mu$ and $H(H,0,0)$ denote, respectively, the fluid velocity, fluid pressure, fluid density, acceleration due to gravity, viscosity and magnetic field. $\delta(z - z_s)$ denotes Dirac's delta function and the magnetic permeability is assumed to be unity.

The initial stationary state solution is given by

$$q = (U(z), 0, 0), \rho = \rho(z), p = p(z), H\ (H, 0, 0). \tag{6}$$

This initial state is given a small disturbance. As a consequence of this, let $q\ (u, v, w), h(h_x, h_y, h_z), \delta p, \delta\rho$ and $\delta z_s\ (x, y, t)$, denote, the perturbations in fluid velocity $q\ (U(z), 0, 0)$, magnetic field H, pressure p, density ρ and surface of separation z_s, respectively.

Using the initial stationary state solutions given by (6) and the linear theory (i.e neglecting the product perturbations and higher order perturbations), the equations (1)–(5) in the linearized perturbed form become

$$\frac{\rho}{\varepsilon}\left(\frac{\partial u}{\partial t} + \frac{U}{\varepsilon}\frac{\partial u}{\partial x} + \frac{w}{\varepsilon}\frac{\partial U}{\partial z}\right) = -\frac{\partial}{\partial x}\delta p - \frac{\mu}{k_1}u, \tag{7}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial v}{\partial t} + \frac{U}{\varepsilon}\frac{\partial v}{\partial x}\right) = -\frac{\partial}{\partial y}\delta p - \frac{\mu}{k_1}v + \frac{H}{4\pi}\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right), \tag{8}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial w}{\partial t} + \frac{U}{\varepsilon}\frac{\partial w}{\partial x}\right) = -\frac{\partial}{\partial z}\delta p - \frac{\mu}{k_1}w + \frac{H}{4\pi}\left(\frac{\partial h_z}{\partial x} - \frac{\partial h_x}{\partial z}\right) + \sum_s T_s\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\delta z_s\right]\delta(z - z_s) - g\delta\rho, \tag{9}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{10}$$

$$\left(\varepsilon\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\delta\rho = -w\frac{\mathrm{d\rho}}{\mathrm{dz}}, \tag{11}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0, \tag{12}$$

$$\left(\varepsilon\frac{\partial h_x}{\partial t} + U\frac{\partial h_x}{\partial x}\right) = H\frac{\partial u}{\partial x} + h_z\frac{\partial U}{\partial z}, \tag{13}$$

$$\left(\varepsilon\frac{\partial h_y}{\partial t} + U\frac{\partial h_y}{\partial x}\right) = H\frac{\partial v}{\partial x}, \tag{14}$$

$$\left(\varepsilon\frac{\partial h_z}{\partial t} + U\frac{\partial h_z}{\partial x}\right) = H\frac{\partial w}{\partial x}. \tag{15}$$

In equation (9), δz_s can be expressed in terms of the normal component of the velocity w_s and z_s since

$$\left(\varepsilon\frac{\partial}{\partial t} + U_s\frac{\partial}{\partial x}\right)\delta z_s = w_s, \tag{16}$$

The disturbances are analyzed into normal modes by seeking solutions of the above equations, whose dependence on x, y and t is of the form

$$\exp i\left(k_x x + k_y y + nt\right), \tag{17}$$

Using (17), equations (7)-(16) become

$$\left[\frac{i\rho}{\varepsilon}\left(n + \frac{U}{\varepsilon}k_x\right) + \frac{\mu}{k_1}\right]u + \frac{\rho}{\varepsilon^2}w\frac{dU}{dz} = -ik_x\delta p, \tag{18}$$

$$\left[\frac{i\rho}{\varepsilon}\left(n + \frac{U}{\varepsilon}k_x\right) + \frac{\mu}{k_1}\right]v = -ik_y\delta p + \frac{H}{4\pi}\left(ik_x h_y - ik_y h_x\right), \tag{19}$$

$$\left[\frac{i\rho}{\varepsilon}\left(n + \frac{U}{\varepsilon}k_x\right) + \frac{\mu}{k_1}\right]w = -\frac{\partial}{\partial z}\delta p + \frac{H}{4\pi}\left(ik_x h_z - \frac{\partial}{\partial z}h_x\right) - k^2\sum_s T_s\,\delta z_s\delta(z - z_s) - g\delta\rho, \tag{20}$$

$$ik_x u + ik_y v + Dw = 0, \tag{21}$$

$$i(\varepsilon n + Uk_x)\delta\rho = -w\frac{d\rho}{dz}, \tag{22}$$

$$i(\varepsilon n + k_x U)h_x = ik_x Hu + h_z DU, \tag{23}$$

$$i(\varepsilon n + k_x U)h_y = ik_x Hv,`` \tag{24}$$

$$ik_x h_x + ik_y h_y + Dh_z = 0,` \tag{26}$$

$$i(\varepsilon n + k_x U)h_z = ik_x Hw, \tag{25}$$

$$i(\varepsilon n + k_x U_s)\delta z_s = w_s, \tag{27}$$

where $D = \frac{d}{dz}$.

Multiplying equation (18) by $-ik_x$ and (19) by $-ik_y$ and adding the resulting equations and using (21), we get

$$\left[\frac{i\rho}{\varepsilon}\left(n + \frac{U}{\varepsilon}k_x\right) + \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\right]Dw = -k^2\delta p + \frac{H}{4\pi}\left(k_x k_y h_y - k_y^2 h_x\right), \tag{28}$$

Eliminating u, v, h_x, h_y, h_z and δp from equations (21)-(27) and using (20) and (28), one obtains after simplification,

$$D\left[\left\{\frac{i\rho}{\varepsilon}\left(n + \frac{k_x U}{\varepsilon}\right) + \frac{\rho\upsilon}{k_1}\right\}Dw - \frac{ik_x\rho}{\varepsilon^2}(DU)w\right] + \frac{ik_x^2H^2k^2}{4\pi(\varepsilon n + k_x U)}w - k^2\left\{\frac{i\rho}{\varepsilon}\left(n + \frac{k_x U}{\varepsilon}\right) + \frac{\rho\upsilon}{k_1}\right\}w - igk^2\left[(D\rho) - w\frac{k^2T_s}{g}\delta(z - z_s)\right]\frac{w}{\varepsilon n + k_x U} - ik_x\frac{H^2}{4\pi}D\left[\frac{1}{(\varepsilon n + k_x U)}\left\{k_x Dw - \frac{k^2(DU)}{(\varepsilon n + k_x U)}w - \frac{ik_y^2(DU)\left(\frac{\rho}{\varepsilon^2} - \frac{k_x^2H^2}{4\pi(\varepsilon n + k_x U)^2}\right)w}{\frac{i\rho}{\varepsilon^2}(\varepsilon n + k_x U) + \frac{\mu}{k_1} - \frac{ik_x^2H^2}{4\pi(\varepsilon n + k_x U)}}\right\}\right] - \frac{ik_x k_y^2H^2}{4\pi}D\left[\frac{\frac{\mu}{k_1}(DU)w}{\frac{i\rho}{\varepsilon^2}(\varepsilon n + k_x U) + \frac{\mu}{k_1}\upsilon - \frac{ik_x^2H^2}{4\pi(\varepsilon n + k_x U)}}\right] = 0, \tag{29}$$

where $D = \frac{d}{dz}$.

Two uniform streaming fluids separated by a horizontal boundary

Let two uniform fluids of densities ρ_1 and ρ_2 be separated by a horizontal boundary at $z = 0$ and the density ρ_2 of the upper fluid be less than the density ρ_1 of the lower fluid so that, in the absence of streaming, the configuration is stable one. Let the two fluids be streaming with velocities U_1 and U_2. Then in each region of constant ρ, ν, T' where T' is the surface tension on the plane interface separating two fluids and U, equation (29) reduces to

$$(D^2 - k^2)w = 0. \tag{30}$$

The boundary conditions to be satisfied are

(i) w must be bounded both when $z \to +\infty$ (in the upper fluid) and $z \to -\infty$ (in the lower fluid).

(ii) Since U is discontinuous at $z = z_s$, the uniqueness of the normal displacement of any point on the interface implies, according to (16), that

$\frac{w}{(\varepsilon n+k_x U)}$ must be continuous at the interface. (31)

(iii) Integrating equation (29) between $z_s-\eta_1{}'$ and $z_s+\eta_1{}'$ passing to the limit $\eta_1{}'=0$, we obtain in view of (31), the jump condition

$$\Delta_s\left[\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_x U}{\varepsilon}\right)+\frac{\rho\nu}{k_1}\right\}Dw-\frac{ik_x^2H^2}{4\pi}\Delta_s\frac{Dw}{(\varepsilon n+k_xU)}\right]=igk^2\left[\Delta_s(\rho)-\frac{k^2T'}{g}\right]\left(\frac{w}{\varepsilon n+k_xU}\right),\text{for } z=z_s. \quad (32)$$

while the equation valid everywhere else $z\neq z_s$ is

$$D\left[\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_xU}{\varepsilon}\right)+\frac{\rho\nu}{k_1}\right\}Dw\right]k^2\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_xU}{\varepsilon}\right)+\frac{\rho\nu}{k_1}\right\}w-\frac{ik_x^2H^2}{4\pi(\varepsilon n+k_xU)}(D^2-k^2)w=igk^2(D\rho)\left(\frac{w}{(\varepsilon n+k_xU)}\right), \quad (33)$$

where $\Delta_s(f)=f(z_s+0)-f(z_s-0)$is the jump which a quantity experiences at the interface $z=z_s$ and the subscript $s$distinguishes the value, a quantity known to be continuous at an interface, takes at $z=z_s$.
Since $\frac{w}{(\varepsilon n+k_xU)}$ must be continuous on the surface $z=0$ and w cannot increase exponentially on either side of the surface, the solutions appropriate for the two regions are

$$w_1=A_1(\varepsilon n+k_xU_1)\,e^{+kz},(z<0) \quad (34)$$

$$w_2=A_1(\varepsilon n+k_xU_1)\,e^{-kz},(z>0) \quad (35)$$

where A is a constant.
Applying the boundary condition (32) to the solutions (34) and (35), we obtain the characteristic equation

$$n^2+\left[\frac{2k_x}{\varepsilon}(\alpha_1U_1+\alpha_2U_2)-\frac{i\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\right]n+\left[\frac{k_x^2}{\varepsilon^2}(\alpha_1U_1^2+\alpha_2U_2^2)-\frac{ik_x}{k_1}(\alpha_1\nu_1U_1+\alpha_2\nu_2U_2)-2k_x^2V_A^2-gk\left\{(\alpha_2-\alpha_1)+\frac{k^2T'}{g(\rho_1+\rho_2)}\right\}\right]=0, \quad (36)$$

where $\nu_1(=\mu_1/\rho_1)$ and $\nu_2(=\mu_2/\rho_2)$ are the kinematic viscosities of fluids 1 and 2, respectively.
$\alpha_1=\frac{\rho_1}{\rho_1+\rho_2},\alpha_2=\frac{\rho_2}{\rho_1+\rho_2}$ and $V_A{}^2=\frac{H^2}{4\pi(\rho_1+\rho_2)}$ is the square of Alfvén velocity.
Now the special case in which the lower and upper fluids are streaming with velocities $U(=U_1)$ and $-U\ (=U_2)$, respectively is considered for the sake of convenience.
Then the equation (36) reduces to

$$n^2+\left[\frac{2k_x}{\varepsilon}(\alpha_1-\alpha_2)U-\frac{i\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\right]n+\left[\frac{k_x{}^2U^2(\alpha_1+\alpha_2)}{\varepsilon}-\frac{ik_x}{k_1}(\alpha_1\nu_1-\alpha_2\nu_2)U-2k_x{}^2V_A{}^2-\mathrm{g}k\left\{(\alpha_1-\alpha_2)+\frac{k^2T'}{\mathrm{g}(\rho_1+\rho_2)}\right\}\right]=0. \quad (37)$$

which is in the good agreement with the earlier results by Sharma and Kumari [5]. In the absence of surface tension i.e. $T'=0$, equation (37) reduces to the equation (36) of
In order to solve equation (37) numerically by putting $n=n_r+in_i$ and equating the real and imaginary we obtain

$$(n_r{}^2-n_i{}^2)+\left[\frac{2k_x}{\varepsilon}(\alpha_1-\alpha_2)U\right]n_r+\left[\frac{\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\right]n_i+\left[\frac{k_x{}^2U^2}{\varepsilon}(\alpha_1+\alpha_2)-2k_x{}^2V_A{}^2-gk\left\{(\alpha_1-\alpha_2)+\frac{k^2T'}{g((\rho_1+\rho_2))}\right\}\right]=0, \quad (38)$$

And

$$\left[\frac{2k_x}{\varepsilon}(\alpha_1-\alpha_2)U\right]n_i-\left[\frac{\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\right]n_r+2n_rn_i-\left[\frac{k_x}{k_1}(\alpha_1\nu_1-\alpha_2\nu_2)U\right]=0. \quad (39)$$

On solving equations (38) and (39), we get

$$4\,n_r{}^4\ +8\,An_r{}^3\ +(5\,A^2+B^2+4\,C)n_r{}^2+(A^3+AB^2+4\,AC)n_r+(-D^2+ABD+A^2C)\ =0 \quad (40)$$

And

$$4\,n_i{}^4-8\,Bn_i{}^3\ +(5\,B^2+A^2-4\,C)n_i{}^2+(B^3+\ A^2B+\ 4\,BC)n_i-(D^2-ABD+B^2C)\ =0 \quad (41)$$

where

$$\left.\begin{aligned} A &= \frac{2k_x}{\varepsilon}(\alpha_1-\alpha_2)U, \\ B &= \frac{\varepsilon}{k_1}(\alpha_1\nu_1+\alpha_2\nu_2), \\ C &= \frac{{k_x}^2U^2}{\varepsilon}(\alpha_1+\alpha_2)-2{k_x}^2{V_A}^2-gk\left((\alpha_1-\alpha_2)+\frac{k^2T'}{g((\rho_1+\rho_2))}\right), \\ D &= \frac{k_x}{k_1}(\alpha_1\nu_1-\alpha_2\nu_2)U, \end{aligned}\right\}. \tag{42}$$

3. Results and Discussions

The real growth rates of the unstable mode have been examined numerically satisfying equations (41). In figure 1, the growth rate n_i has been plotted versus wavenumber k for fixed permissible values of the parameters $k_1 = 2, g = 980\ cm/sec^2, \rho_1 = 0.95,\ \rho_2 = 1.8, \varepsilon = 0.9, \nu_2 = 2, U = 1\ km/sec,\ \alpha_1 = 0.35,\ \alpha_2 = 0.65,\ k_x = k/\sqrt{2}$ for three different values of the medium porosity $\varepsilon = 0.2, 0.6, 0.9$, respectively. It is clear from the graph that growth rate n_i decreases slightly as with the increase in ε, showing thereby very little stabilizing effect of medium porosity on the system for a fixed wavenumber.

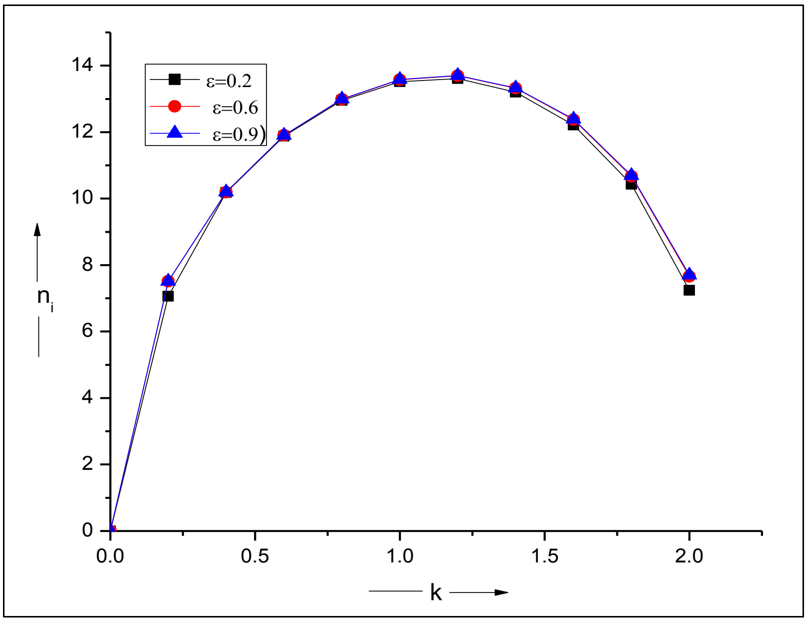

Fig 1: Variation of n_i with wave number k for three different values of the medium porosity $\varepsilon = 0.2, 0.4, 0.9$.

Figure 2 show that the four different values of the square of Alfvén velocity (accounting for magnetic field) $V_A^2 = 10, 50, 100$ respectively. It is clear from the graph that k_c takes different values of the critical wavenumber $k_c \to 3.0, 2.2$ at $V_A^2 = 50, 100$, respectively, while the value of k_{max} at $V_A^2 = 100$ is 1.2 approaching complete stability (or neutral stability). The graphs show that the square of Alfvén velocity has complete stabilizing effect on the system.

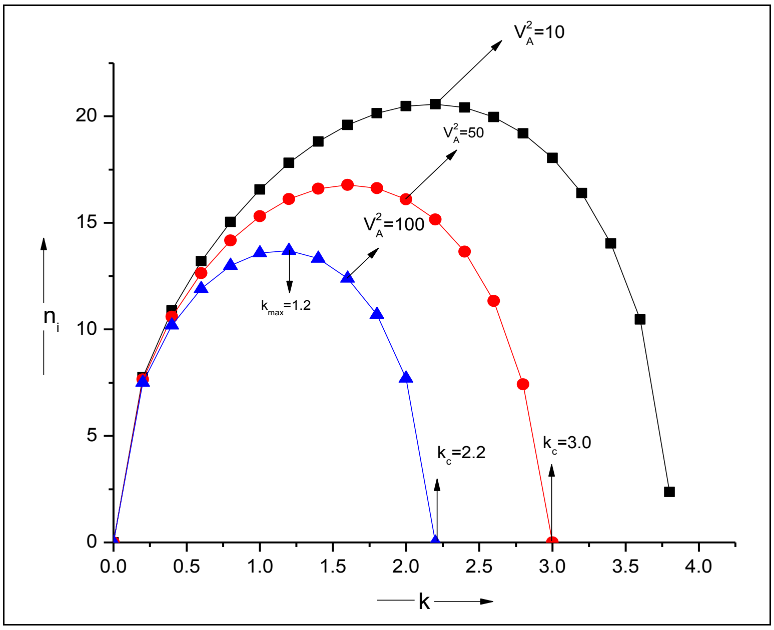

Fig 2: Variation of n_i with wave number k for three different values of the square of Alfvén velocity $V_A^2 = 10, 50, 100$.

Figure 3 has been plotted for the real growth rate n_i versus wavenumber k for four different values of the surface tension $T' = 0, 50, 75, 100$, respectively. It is clear from the graphs that k_c takes different values $k_c \rightarrow 2.2, 2.0$ at $T' = 75, 100$ respectively, while the value of k_{max} at $T' = 75, 100$ is 1.0 approaching to more stability. This means that the simultaneous presence of surface tension and magnetic field has a crucial capability to suppress the instability, showing thereby the large enough stabilizing effect of surface tension on the system.

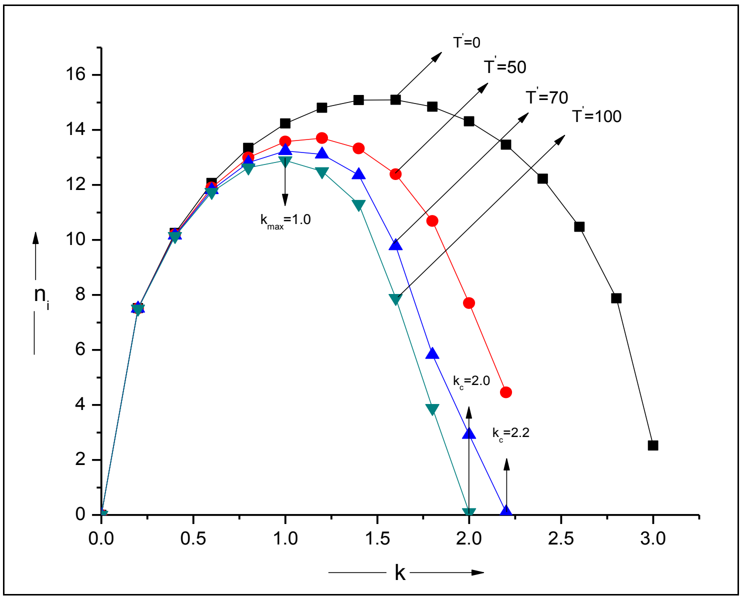

Fig 3: Variation of n_i with wave number k for different values of the surface tension $T' = 0, 50, 75, 100$.

4. Conclusions

A study has been made to investigate numerically the simultaneous effect of the square of the Alfv$\acute{e}$n velocity, surface tension and the medium porosity on the instability of superposed viscous fluids in hydromagnetics saturating porous medium. The principal conclusions drawn are as follows:

i) The growth rate of the perturbations decreases slightly with the increase in medium porosity has very slight stabilizing effect on the system.

ii) The square of Alfv$\acute{e}$n velocity in imaginary growth rates of the perturbations the Kelvin-Helmholtz instability has stabilizing effect on the system.

iii) Surface tension has large enough stabilizing effect on the system. The effect of surface tension dissipates the energy of any disturbance more than that carried out by the magnetic field. In other words, the role of the square of Alfv$\acute{e}$n velocity can help the surface tension to find more stability on the Kelvin-Helmholtz instability problem, while the surface tension plays the fundamental role to generate the complete stability.

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