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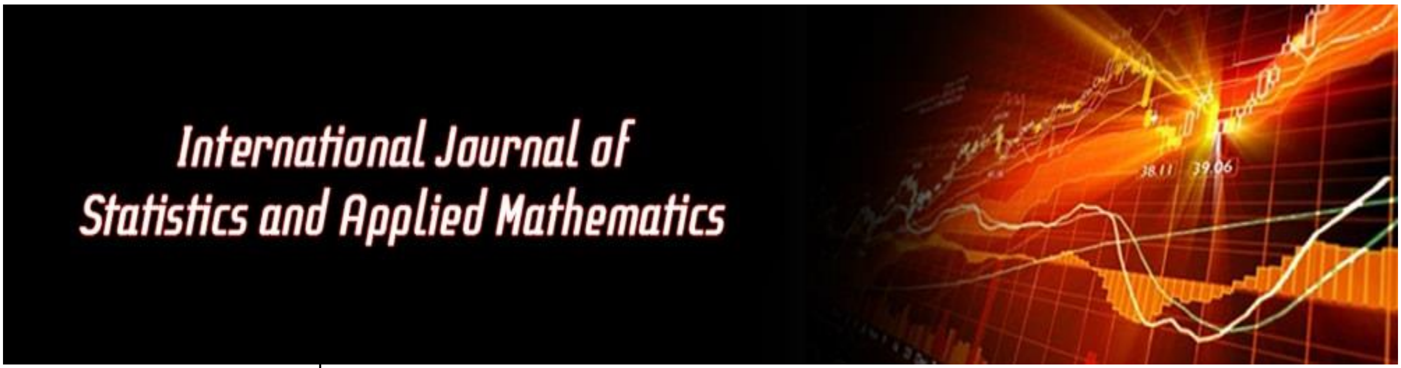

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Stability of stratified elastico-viscous Rivlin-Ericksen fluid in the presence of variable magnetic field and rotation saturating porous medium

Sumit Gupta

Abstract
The influence of viscosity, viscoelasticity, medium permeability and medium porosity on the stability of a stratified elastic-viscous Rivlin –Ericksen fluid is examined for viscoelastic polymeric solutions in the simultaneous presence of a variable horizontal magnetic field $H(H_0(z),0,0)$ and uniform horizontal rotation $\Omega(\Omega,0,0)$ in porous medium. The effects of coriolis force on the stability are chosen along the direction of the magnetic field and transverse to that of the gravitational field $g(0,0,-g)$. Assuming the exponential stratifications in density, viscosity and viscoelasticity, the appropriate solution for the case of free boundaries is obtained using a linearized stability theory and normal mode analysis method. The dispersion relation is obtained and the behaviour of growth rates with respect to kinematic viscosity, kinematic viscoelasticity, medium permeability and medium porosity is examined numerically using Newton-Raphson method through the software Fortran-90 and Mathcad. In contrast to the Newtonian fluids, the system is found to be unstable, for stable stratifications, for small wavelength perturbations. It is found that the magnetic field stabilizes the certain wave number band, for unstable stratification in the presence of rotation and this wave number range increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity implying thereby the stabilizing effect of magnetic field and kinematic viscoelasticity and the kinematic viscosity has a stabilizing effect on the system for the low wave number range. The medium permeability has enhancing effect on the growth rates with its increase for a fixed wave number. These results are shown graphically.

Keywords: Rivlin –Ericksen fluid, magnetic field, rotation, viscosity, viscoelasticity, medium permeability

1. Introduction
The flow through porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamics. When we consider flow in porous medium, some additional complexities arise which are due to the interaction between the fluids and the porous medium. Here we consider those fluid flows for which Darcy's law is applicable. This law is empirical in nature and is usually considered valid for creeping flows where the Reynolds's number as defined for a porous medium is less than one. Darcy's law states that the gross effect, as the fluid slowly percolated through the pores of rock, is that usual viscous term in the equation of elastic-viscous fluid motion will be replaced by the resistance terms $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)q\right]$, where μ and μ' are the coefficients of viscosity and viscoelasticity, of Walters' (model B') fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity (seepage) of the fluid. The stability of flow of a single component fluid through porous medium taking into account the Darcy's resistance has been studied by Lapwood [1] and Wooding [2]. The effect of the Earth's magnetic field on the stability of such a flow is of interest in geophysics particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The physical properties of comets and meteorites strongly suggest importance of porosity in astrophysical context (McDonnell) [3].

The stability derived from the character of the equilibrium of an incompressible heavy fluid of variable density (i.e. of a heterogeneous fluid) was investigated by Rayleigh [4]. He demonstrated that the system is stable or unstable according as the density decreases everywhere or increases everywhere. An experimental demonstration of the development of the Rayleigh–Taylor instability was performed by Taylor [5]. The effect of a vertical magnetic field on the development of Rayleigh–Taylor instability was considered by Hide [6]. Reid [7] studied the effect of surface tension and viscosity on the stability of two superposed fluids. The Rayleigh–Taylor instability of a Newtonian fluid has been studied by several authors accepting varying assumptions of hydrodynamics and hydromagnetics and Chandrasekhar [8] in his celebrated monograph has given a detailed account of these investigations. Bellman and Pennington [9] further investigated in detail illustrating the combined effects of viscosity and surface tension. Gupta [10] again studied the stability of a horizontal layer of a perfectly conducting fluid with continuous density and viscosity stratifications in the presence of a horizontal magnetic field. The Rayleigh–Taylor instability problems arise in oceanography, limnology and engineering. There are many viscoelastic fluids which cannot be characterized either by Maxwell's constitutive relations or by Oldroyd's constitutive relations. One such class of the viscoelastic fluids is the Rivlin–Ericksen fluid. Rivlin and Ericksen [11] have proposed a theoretical model for such viscoelastic fluid. This and other class of polymers is used in the manufacture of parts of space–crafts, aeroplanes, tyres, belt conveyers, ropes cushions, seats, foams, engineering equipments etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical appliances. Joshi [12] has discussed the viscoelastic Rivlin–Ericksen incompressible fluid under time–dependent pressure. The stability of partially ionized superposed plasmas in the presence of variable horizontal magnetic field has been studied by Sharma and Thakur [13]. Srivastava and Singh [14] have studied the unsteady flow of a dusty viscoelastic Rivlin–Ericksen fluid through channel of different cross–sections in the presence of the time–dependent pressure gradient. Sharma and Kumari [15] have studied the stability of stratified fluid in porous medium in the presence of suspended particles and variable magnetic field. In another study, Garg *et al*. [16] have studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivlin–Ericksen fluid in the presence of a uniform magnetic field. Sharma and Gupta [17] also have studied the stability of stratified rotating viscoelastic Rivlin-Ericksen fluid in the presence of variable magnetic field.

Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent [18] has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. In stellar atmospheres and interiors, the magnetic field may be (and quite often is) variable and may altogether alter the nature of the instability. Coriolis force also plays an important role on the stability of the system. In all the above studies the fluid has been assumed to be Newtonian.

With the growing importance of non–Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. Fredricksen [19] has a good review of non–Newtonian fluids whereas Joseph [20] has also considered the stability of viscoelastic fluids. There are many viscoelastic fluids which cannot be characterized either by Maxwell's constitutive relations or by Oldroyd's constitutive relations. This class of fluids is used in the manufacture of parts of space cafts, aeroplane, tyres, beltconveyors, rops, cushions, seats, foams, plastics, engineering equipments etc. The magnetic field stabilizes the system. The viscoelasticity of the medium has damping effects on the growth rates but has enhancing effects for certain ranges of the wave-numbers.

Keeping in mind the importance of non–Newtonian fluids, medium permeability and medium porosity in modern technology and their various applications mentioned above, the present paper is devoted to consider the stability of rotating stratified elastico–viscous Rivlin –Ericksen fluid in the presence of variable magnetic field and rotation in porous medium.

2. Materials and Methods

The initial stationary state whose stability we wish to examine is that of an incompressible, heterogeneous infinitely extending and conducting $(\sigma \to \infty)$ elastico–viscous Rivlin –Ericksen fluid of thickness d bounded by the planes $z = 0, d$ and of variable density, kinematic viscosity and viscoelasticity, arranged in horizontal strata in a porous medium of variable porosity and medium permeability so that the free surface is almost horizontal and the electrical conductivity $\eta = \frac{1}{4\pi\mu_e\sigma}$ is zero. The fluid is acted on by gravity force $g(0,0,-g)$, a uniform horizontal rotation $\Omega(\Omega,0,0)$ and a variable horizontal magnetic field $H(H_0(z),0,0)$.The character of the equilibrium of this stationary state is determined by supposing that the system is slightly disturbed and then, following its further evolution.

The equations expressing conservation of momentum, mass, incompressibility and Maxwell's equations for the elastico–viscous Rivlin –Ericksen fluid are

$$\rho\left[\frac{\partial q}{\partial t} + \frac{1}{\in}(q \cdot \nabla)q\right] = -\nabla p + \rho g - \frac{1}{k_1}\left(\mu + \mu' \frac{\partial}{\partial t}\right)\nabla^2 q + \frac{2\rho}{\in}(q \times \Omega) + \frac{\mu_e}{4\pi}[(\nabla \times H)\times H], \quad (1)$$

$$\nabla \cdot q = 0, \quad (2)$$

$$\in \frac{\partial \rho}{\partial t} + (q \cdot \nabla)\rho = 0, \quad (3)$$

$$\nabla \cdot H = 0, \quad (4)$$

$$\in \frac{\partial H}{\partial t} = \nabla \times (q \times H), \tag{5}$$

Where μ_e, the medium permeability, is assumed to be constant. Equation (3) represents the fact that the density of a particle remains unchanged as we follow it with its motion.
Let $\delta\rho,\ \delta p,\ q(u,v,w)$ and $h(h_x,h_y,h_z)$ denote, respectively, the perturbations in density $\rho(z)$, pressure $p(z)$, velocity V (0,0,0) and horizontal magnetic field $H(H,0,0)$. Then the equations (1)–(5) after perturbations in the cartesian form become

$$\frac{\rho}{\in}\frac{\partial u}{\partial t} + \frac{\rho}{\in}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial}{\partial x}\delta p - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2 u + \frac{\mu_e}{4\pi}\left(h_z\frac{\partial}{\partial z}H_0\right) + \frac{2}{\in}\rho\, v\,\Omega, \tag{6}$$

$$\frac{\rho}{\in}\frac{\partial v}{\partial t} + \frac{\rho}{\in}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial}{\partial y}\delta p - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2 v + \frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_y - \frac{\partial}{\partial y}h_x\right) - \frac{2}{\in}\rho\, u\,\Omega, \tag{7}$$

$$\frac{\rho}{\in}\frac{\partial w}{\partial t} + \frac{\rho}{\in}\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial}{\partial z}\delta p - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2 w + \frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_z - \frac{\partial}{\partial z}h_x - \frac{h_z}{H_0}\frac{\partial}{\partial z}H_0\right) - g\delta\rho, \tag{8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (9) \qquad \in\frac{\partial}{\partial t}(\delta\rho) + w\frac{\partial \rho}{\partial z} = 0, \tag{10}$$

$$\frac{\partial}{\partial x}h_x + \frac{\partial}{\partial y}h_y + \frac{\partial}{\partial z}h_z = 0, \tag{11}$$

$$\in\frac{\partial}{\partial t}h_x = \frac{\partial}{\partial x}\left\{u(H_0 + h_x) - uh_y\right\} - \frac{\partial}{\partial z}\left\{w(H_0 + h_x) - uh_z\right\}, \tag{12}$$

$$\in\frac{\partial}{\partial t}h_y = \frac{\partial}{\partial z}\left\{vh_z - wh_y\right\} - \frac{\partial}{\partial x}\left\{uh_y - v(H_0 + h_x)\right\}, \tag{13}$$

$$\in\frac{\partial}{\partial t}h_z = \frac{\partial}{\partial x}\left\{w(H_0 + h_x) - uh_z\right\} - \frac{\partial}{\partial y}\left\{vh_z - wh_y\right\}, \tag{14}$$

$$\frac{\rho}{\in}\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2 u + \frac{\mu_e}{4\pi}h_z\frac{\partial}{\partial z}H_0 + \frac{2}{\in}\rho\, v\,\Omega, \tag{15}$$

$$\frac{\rho}{\in}\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2 v + \frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_y - \frac{\partial}{\partial y}h_x\right) - \frac{2}{\in}\rho\, u\Omega, \tag{16}$$

$$\frac{\rho}{\in}\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2 w + \frac{\mu_e H_0}{4\pi}\left(\frac{\partial h_z}{\partial x} - \frac{\partial h_x}{\partial z} - \frac{h_x}{H_0}\frac{\partial H_0}{\partial z}\right) - g\delta\rho, \tag{17}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{18}$$

$$\in\frac{\partial}{\partial t}(\delta\rho) + w\frac{\partial \rho}{\partial z} = 0, \tag{19}$$

$$\frac{\partial}{\partial x}h_x + \frac{\partial}{\partial y}h_y + \frac{\partial}{\partial z}h_z = 0, \tag{20}$$

$$\in \frac{\partial}{\partial t} h_x = H_0 \frac{\partial}{\partial x} u - w \frac{\partial}{\partial z} H_0 , \quad (21)$$

$$\in \frac{\partial}{\partial t} h_y = H_0 \frac{\partial}{\partial x} v , \quad (22)$$

$$\in \frac{\partial}{\partial t} h_z = H_0 \frac{\partial}{\partial x} w . \quad (23)$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y, z and time t is given by

$$f(z) \exp\left(ik_x x + ik_y y + nt \right), \quad (24)$$

where $f(z)$ is the some function of z –only; k_x, k_y are the wave-numbers in the x – and y –directions, respectively, $k = \left(k_x^2 + k_y^2\right)^{1/2}$ is the resultant wave-number and n is the growth rate of the disturbance which is, in general, a complex constant.

Equations (15)–(23) using expression (24) become $\frac{\rho}{\in} n u = -ik_x \delta p - \frac{1}{k_1}(\mu + \mu' n)\left(D^2 - k^2\right)u + \frac{\mu_e}{4\pi} h_z DH_0 + \frac{2}{\in} \rho v \Omega$, (25)

$$\frac{\rho}{\in} n v = -ik_y \delta p - \frac{1}{k_1}(\mu + \mu' n)\left(D^2 - k^2\right)v + \frac{\mu_e H_0}{4\pi}\left(ik_x h_y - ik_y h_x \right) + \frac{2}{\in} \rho u \Omega , \quad (26)$$

$$\frac{\rho}{\in} n w = -D \delta p - \frac{1}{k_1}(\mu + \mu' n)\left(D^2 - k^2\right)w + \frac{\mu_e H_0}{4\pi}\left(ik_x h_z - Dh_x - \frac{h_x DH_0}{H_0} \right) - g \delta\rho , \quad (27)$$

$$ik_x u + ik_y v + Dw = 0 , \quad (28)$$

$$\in n\delta\rho + wD\rho = 0 , \quad (29)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0 , \quad (30)$$

$$\in nh_x = ik_x H_0 u - w DH_0 , \quad (31)$$

$$\in nh_y = ik_x H_0 v , \quad (32)$$

$$\in nh_z = ik_x H_0 w , \quad (33)$$

Now substituting the values of h_x, h_y and h_z from equations (31)–(33) in equations (25)–(27), we get

$$\frac{\rho}{\in} n u = -ik_x \delta p - \frac{1}{k_1}(\mu + \mu' n)\left(D^2 - k^2\right) u + \frac{\mu_e}{4\pi}\left(\frac{ik_x H_0 w}{n} \right) DH_0 + \frac{2}{\in} \rho v \Omega , \quad (34)$$

$$\frac{\rho}{\in} n v = -ik_y \delta p - \frac{1}{k_1}(\mu + \mu' n)\left(D^2 - k^2\right) v + \frac{\mu_e}{4\pi} H_0 \left(\frac{ik_x H_0 \zeta_z}{n} + \frac{ik_y w DH_0}{n} \right) - \frac{2}{\in} \rho u \Omega , \quad (35)$$

$$\frac{\rho}{\in} n w = -D \delta p - \frac{1}{k_1}(\mu + \mu' n)\left(D^2 - k^2\right)w + \frac{\mu_e H_0}{4\pi \in}\left[-\frac{k_x^2 H_0 w}{n} - D\left(\frac{ik_x H_0 u}{n} - \frac{w DH_0}{n} \right) - \left(\frac{ik_x H_0 u}{n} - \frac{w DH_0}{n} \right)\frac{DH_0}{H_0} \right] + \frac{g\left(D\rho\right)w}{\in} , \quad (36)$$

Where's $\zeta_z = ik_x v - ik_y u$, is the z –component of vorticity.

Multiplying equations (34) and (35) by $-ik_y$ and ik_x, respectively, and then adding we get

$$\frac{\rho n}{\in}\zeta_z = -\frac{\rho}{k_1}(\upsilon+\upsilon' n)(D^2-k^2)\zeta_z - \frac{\mu_e k_x^2 H_0^2}{\in 4\pi n}\zeta_z + \frac{2}{\in}\Omega Dw \text{, or } \zeta_z = \frac{2n\Omega Dw}{n^2 - \frac{\in n}{k_1}(\upsilon+\upsilon' n)(D^2-k^2)+k_x^2 V_A^2}, \quad (37)$$

Where $\upsilon = \frac{\mu}{\rho}$, $\upsilon' = \frac{\mu'}{\rho}$ and $V_A^2 = \frac{\mu_e H_0^2}{4\pi\rho}$ (square of the Alfvén's velocity).

Substituting the value of ζ_z in equation (35), we get

$$\frac{\rho}{\in}nv = -ik_y\delta p - \frac{1}{k_1}(\mu+\mu' n)(D^2-k^2)v - \frac{\mu_e H_0}{4\pi \in n}\left(\frac{2\Omega n Dw\, ik_x}{n^2 - n(\mu+\mu' n)(D^2-k^2)+k_x^2 V_A^2}\right) + \frac{\mu_e H_0}{4\pi n} ik_y w D(H_0) - 2\frac{\rho}{\in}u\Omega. \quad (38)$$

Multiplying equations (34) and (36) by $-ik_x$ and $-ik_y$, respectively, and then adding and using (28), we obtain

$$\frac{\rho}{\in}nDw = -k^2\delta p - \frac{\rho}{k_1}(\upsilon+\upsilon' n)(D^2-k^2)Dw + \left(\frac{2n\Omega}{n^2 - \frac{\in n}{k_1}(\upsilon+\upsilon' n)(D^2-k^2)+V_A^2 k_x^2}\right)\left(\frac{\mu_e H_0^2}{4\pi \in n}k_x^2 k_y - \frac{2}{\in}\rho\right)Dw \quad (39)$$

Eliminating u, v and δp from equations (35)–(39) using equations (29), after little algebra, we get

$$\in\frac{n\rho}{k_1}(\upsilon+\upsilon' n)(D^2-k^2)^2 w - \frac{n\rho}{k_1}(n^2+k_x^2V_A^2)(D^2-k^2)w - \left[n^2(D\rho)\left(1+\frac{4\Omega^2}{n^2 - \frac{n\in}{k_1}(\upsilon+\upsilon' n)(D^2-k^2)+k_x^2 V_A^2}\right) - \frac{\mu_e k_x^2 D(H_0^2)}{4\pi \in n}\right]Dw + gk^2(D\rho)w = 0. \quad (40)$$

Equation (40) is the general equation formulating the effect of variable magnetic field and uniform rotation on the stability of stratified Rivlin –Ericksen fluid saturating porous medium.

2.1 The case of exponentially varying stratifications

In order to obtain the solution of the stability problem of a layer of Walters' (model B') fluid, we suppose that the density ρ, viscosity μ, viscoelasticity μ' medium porosity $\in$ and medium permeability μ' vary exponentially along the vertical direction i.e.

$$\rho = \rho_0 e^{\beta_1 z}, \quad \mu = \mu_0 e^{\beta_1 z}, \quad \mu' = \mu_0' e^{\beta_1 z}, \quad \in = \in_0 e^{\beta_1 z}, k_1 = k_{10}e^{\beta_1 z} \quad (41)$$

Where ρ_0, μ_0, μ_0', H_1, $\in_0$, k_{10} and β_1 are constants and so the kinematic viscosity $\upsilon\left(=\frac{\mu}{\rho}=\frac{\mu_0}{\rho_0}\right)$, the kinematic viscoelasticity $\upsilon'\left(=\frac{\mu'}{\rho}=\frac{\mu_0'}{\rho_0}\right)$ and the Alfvén velocity $V_A = \left(\frac{\mu_e H_0^2}{4\pi\rho}\right)^{1/2} = \left(\frac{\mu_e H_1^2}{4\pi\rho}\right)^{1/2}$ are constant everywhere.

Using the stratifications of the form (41), equation (40) transforms to

$$(D^2-k^2)^3 w - \frac{2}{\frac{\in_0 n}{k_{10}}(\upsilon_0+\upsilon_0' n)}(n^2+k_x^2V_A^2)(D^2-k^2)^2 w + \frac{1}{n^2\frac{\in_0}{k_{10}}(\upsilon_0+\upsilon_0' n)^2}\left[n^4 + k_x^2V_A^2(2n^2+k_x^2V_A^2) - V_A^2 k_x^2\beta_1 n(\upsilon_0+\upsilon_0' n)\right.$$
$$\left. - gk^2\beta_1 n(\upsilon_0+\upsilon_0' n)\right](D^2-k^2)w - \frac{1}{\frac{\in_0 n}{k_{10}}(\upsilon_0+\upsilon_0' n)^2}\left[4\Omega^2 n^2 + V_A^2k_x^2\beta_1(n^2+k_x^2V_A^2) - gk^2\beta_1(n^2+k_x^2V_A^2)w\right] = 0. \quad (42)$$

Considering the case of two free boundaries, we must have

$$w = D^2 w = 0 \text{ at } z = 0 \text{ and } z = d. \quad (43)$$

The appropriate solution of equation (42) satisfying the above boundary condition is

$$w = A_0 \sin \frac{m\pi z}{d}, \tag{44}$$

Where's m is an integer and A_0 is a constant.
Substituting the value of w from equation (44) in equation (42) we obtain dispersion relation

$$n^4\left[\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)^2\right]+n^3\left[2\upsilon_0 L_3\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)\right]+n^2\left[L_3^2\upsilon_0^2+\left(2k_x^2V_A^2-\frac{g k^2\beta_1}{L_3}\right)(1-\upsilon_0' L_3)+\frac{4\in_0 \Omega^2 k_x^2}{L_3 k_{10}^2}-\frac{1}{L_3}V_A^2 k_x^2\beta_1(1+\upsilon_0' L_3)\right]+n\left[v_0 L_3\left(2k_x^2V_A^2-\frac{\in_0^2 g\beta_1 k^2}{k_{10}^2 L_3}\right)-\frac{1}{L_3}V_A^2 k_x^2\beta_1(1+\upsilon_0' L_3)\right]+k_x^2V_A^2\left[k_x^2V_A^2-\frac{\beta_1}{L_3}\left(g k^2+V_A^2 k_x^2\right)\right]=0, \tag{45}$$

Where's $L_3=\left[k^2+\frac{m_1^2\pi^2}{d^2}\right]$.

Equation (45) is biquadratic in n and is the dispersion relation governing the effects of uniform rotation, variable horizontal magnetic field, viscosity, viscoelasticity medium permeability and medium porosity on the stability of stratified Rivlin –Ericksen fluid.

3. Results and Discussions

3.1. Case of stable stratifications (i.e. $\beta_1<0$) and $(k_{10}>4\in_0 v_o')$, Equation (45) does not admit any positive real root or complex root with positive real part using Routh–Hurwitz criterion; therefore, the system is always stable for disturbances of all wave-number.

3.1.1 Case of unstable stratifications (i.e. $\beta_1>0$) and $(k_{10}<4\in_0 v_o')$, If $\beta_1>0$, $\frac{k_x^2V_A^2}{k^2}\left(1-\frac{\beta_1}{L_3}\right)<\frac{\beta_1}{L_3}g$, the constant term in the equation (45) is negative and therefore has at least one root with positive real part using Routh–Hurwitz criterion; so the system is unstable for all wave-numbers satisfying the inequality

$$k^2<\frac{\beta_1 d^2 g\sec^2\theta-V_A^2\left(m_1^2\pi^2-\beta_1 d^2\right)}{V_A^2 d^2}, \tag{46}$$

Where θ is the angle between k_x and k i.e. $(k_x=k\cos\theta)$.

If $\beta_1>0$, (unstable stratifications) $1>\frac{\beta_1}{L_3}$ and $V_A^2>\frac{\beta_1 g k^2}{L_3 k_x^2\left(1-\frac{\beta_1}{L_3}\right)}$, equation (45) does not admit of any positive real root or complex root with positive real part, therefore, the system is stable. The system is clearly unstable in the absence of magnetic field, rotation and for non–viscoelastic fluid.

$$n^4\left[\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)^2\right]+n^3\left[2\upsilon_0 L_3\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)\right]+n^2\left[L_3^2\upsilon_0^2-\frac{g k^2\beta_1}{L_3}(1+\upsilon_0' L_3)+\frac{1}{L_3}V_A^2 k_x^2\beta_1(1+\upsilon_0' L_3)\right]-n\left[v_0 L_3\frac{\in_0^2 g\beta_1 k^2}{L_3 k_{10}^2}\right]=0. \tag{47}$$

For $\beta_1>0$, the constant term in the equation (45) is negative and therefore has at least one root with positive real part therefore the system is clearly unstable. The magnetic field, therefore, stabilizes potentially unstable stratifications for small wave-length perturbations

$$k^2>\frac{\beta_1 d^2 g\sec^2\theta-V_A^2\left(m_1^2\pi^2-\beta_1 d^2\right)}{V_A^2 d^2}. \tag{48}$$

Also, it is clear that the wave-number range, for which the potentially unstable system gets stabilized, increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity. All long wave-length perturbations satisfying equation (48) remain unstable and are not stabilized by magnetic field.

The behaviour of growth rates with respect to kinematic viscosity υ_0, kinematic viscoelasticity υ_0' and square of the Alfvén velocity V_A^2 satisfying equation (45) has been examined numerically using Newton–Raphson method through the software Mathcad**.** Figure (1) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissiblevalueof $\beta_1=2$, $\in_0=0.5, k_{10}=6, m_1=1$, $d=6\,\text{cm}$, $\Omega=1$ revolution/minute, $\upsilon_0'=1$, $g=980\text{cm/s}^2$, $k_x=k\cos 45^0$, $V_A^2=55$ for three values of $\upsilon_0'=$2, 3 and 4 respectively. These values are the permissible values for the respective parameters and are in good agreement with the corresponding values used by Chandrasekhar [8] while describing various hydrodynamic and hydromagnetic stability problems. The graph shows that for fixed wave-numbers, the growth rate increases for certain wave number with the increase in kinematic viscoelasticity υ_0', which indicates the destabilizing effect of viscoelasticity whereas the growth rate decreases for certain wave numbers implying thereby the stabilizing effect of kinematic viscoelasticity on the system in the presence of medium permeability and medium porosity for low wave numbers range.

Figure (2) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ revolution/minute, $\upsilon_0' = 1$, $g = 980\text{cm/s}^2,\ k_x = k\cos 45^0,\ V_A^2 = 55, \in_0 = 0.5, k_{10} = 6,$ for three values of $\upsilon_0 =$2, 4 and 6 respectively. The graph shows that for fixed wave-numbers, the growth rate increases for certain wave number with the increase in kinematic viscosity υ_0 which indicates the destabilizing influence of kinematic viscosity, whereas the growth rate decreases for certain wave numbers, implying thereby the stabilizing effect of kinematic viscosity on the system in the presence of medium permeability and medium porosity.

Figure (3) shows the variation of growth rate n_r (positive real value of n) with respect to wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ revolution/minute, $\upsilon_0 =$ 4, $\upsilon_0' = 2$, $g = 980\,\text{cm/s}^2,\ k_x = k\cos 45^0, \in_0 = 0.5, k_{10} = 6$ for two values of $V_A^2 =$15 and 55 respectively. The graph shows that for fixed wave-numbers, the growth rate increases with the increase in the square of the Alfvén velocity V_A^2 for certain wave number which indicates the destabilizing influence of the square of the Alfvén velocity, whereas growth rate decreases for certain wave numbers, implying thereby the stabilizing effect of the square of the Alfvén velocity on the system in the presence of medium permeability and medium porosity.

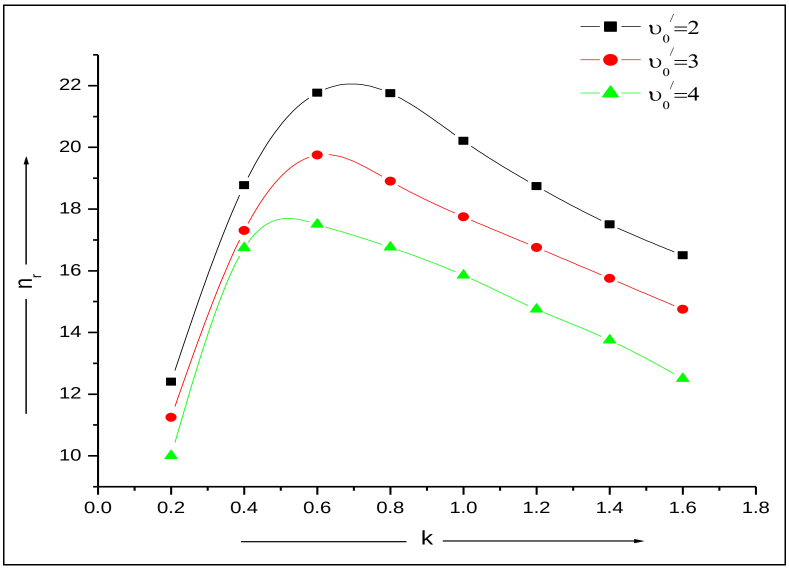

Fig 1: The variation of n_r with wave-number k for three values of $\upsilon_0' = 2,3,4$.

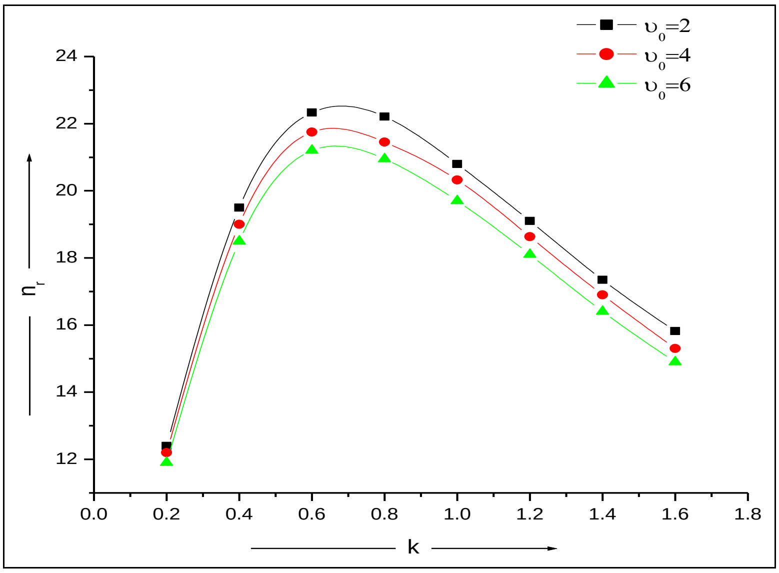

Fig 2: The variation of n_r with wave-number k for three values of $\upsilon_0 = 2, 4, 6$.

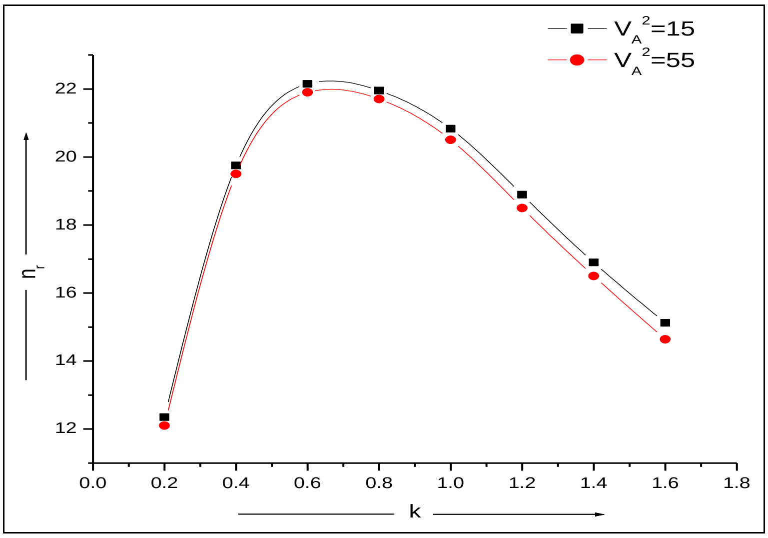

Fig 3: The variation of n_r with wave-number k for two values of $V_A^2 = 15, 55$.

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