International Journal of Statistics and Applied Mathematics 2019; 4(5): 143-153

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2019; 4(5): 143-153

www.mathsjournal.com
Received: 24-07-2019
Accepted: 28-08-2019

Dr. Abhishek Kumar Srivastava
Assistant Professor Mathematics, Sant Gani Nath Government P.G. College, Muhammadabad Gohna, Mau, Uttar Pradesh, India

Correspondence
Dr. Abhishek Kumar Srivastava
Assistant Professor Mathematics, Sant Gani Nath Government P.G. College, Muhammadabad Gohna, Mau, Uttar Pradesh, India

Mathematical modeling numerical simulation of air pollution

Dr. Abhishek Kumar Srivastava

Abstract
Environment, which etymologically means surrounding is considered as a composite term for the condition in which organism live and thus consist of air, water, food and sunlight which are the basic needs of all living beings and plant life, to carry on their life functions. Environment creates favorable conditions for the existence and development of living organisms. Pollution of the environment is one of the most horrible ecological crisis to which we are subjected today. Air Pollution is generally disequilibrium condition of air caused due to the introduction of foreign elements from natural and manmade sources to the air so that it become injurious to biological communities. So air pollution may be described as "The imbalance in the quality of air so as to cause adverse effects on the living organisms existing on earth," or we can say that "The presence of one or more contaminant such as dust, gas, odour, smoke, smog or vapour in the outdoor atmosphere, in quantities of characteristics, and of duration so as to be injurious to human plant or animal life or to property or which unreasonably interferes with the comfortable enjoyment of life and property is known as air pollution".

Keywords: Mathematical modeling, applied mathematics, differential equations

Introduction

Applied mathematics is a branch of mathematics that concern itself with the mathematical techniques typically used in the application of mathematical knowledge to other domain. A mathematical model uses the language of mathematics to produce a more refined and precise description of the system.

With the rapid development of modern technology, industry and agriculture, it is of much interest to consider the effects of toxicants (substances that are harmful to living organisms as a result of physical or chemical interests) on ecological communities from both an environmental and conservational point of view. Pollutants (toxicants) released from natural sources include dust storms, earthquakes, volcanic eruptions, forest fires, etc. A major source of human caused pollution is the burning of fossil fuels (coal, oil, natural gas, petrol) for transportation, electricity generation and industrial use. In order to use and regulate toxic substances wisely, we must assess the risk of the populations exposed to toxicants. In recent years, the effects of toxicants emitted into the environment from industrial and household resources on biological species have received much attention of researchers.

Systems of delay differential equations now occupy a place of central importance in all areas of science and particularly in the biological sciences. Delay differential equations differ from ordinary differential equations in that the derivative at any time depends on the solution at prior time. Structured population models describe the distribution of individuals in the population among the possible categories of important individual differences (e.g., age, size etc.). In the natural world, many species have a life history that takes their individual members through two stages: immature and mature with a time lag. In particular, we have in mind mammalian populations, which exhibit these two distinct stages.

We know that species do not exist alone in nature. They interact with other species in their surroundings for their survival. Competition between species may be one kind of interaction. Competition is usually defined as the simultaneous demand by two or more organisms for limited environmental resources, such as food, nutrients, living space or light.

Competition occurs naturally between living organisms which co – exist in the same environment when limited amount of resources are available, and several species depend on these resources. Competition can occur between individuals of same species, called intra specific competition, or between different species, called inter specific competition. Competition invariably results in a reduction in the numbers of one or both competitors, and in evolution contributes both to the decline of certain species and to the evolution of adaptations.
A mathematical model has been proposed and analyzed to study the effects of toxicants on biological population. It is assumed that toxicant is emitted into the environment by natural sources and also by various industrial waste. Further, its concentration is increased due to precursor of the population. The existence of non-trivial equilibrium has been proved and the sufficient conditions for its stability behavior have been determined. It has been found that density of toxicant increases very rapidly into the environment with increase in the density of industrialization. Also, it is found that the magnitude of equilibrium values of population and precursor decreases with increase in the emission rate of toxicant due to industrialization. Further, it is noted that the precursor density increases more with increase in density of industrialization than increase in population density.

Modelling the effects of toxicant and precursor on biological population in industrial environment
The advent of industrial and technological revolution has been accompanied by a growing negative impact on the environment in terms of pollution and degradation. Rapid industrialization and urbanization not only deplete the stock of natural resources but also add stress to the environmental system by accumulating the stock of wastes which ultimately results into the formation of air pollutants. This implies that while assessing the adverse impacts of air pollution, one must be careful to assess not only directly emitted pollutants (e.g. industries, vehicles etc.) but pollutant precursor as well.
Pollutants are assumed to be emitted from external sources into the environment and emission rate have no relation with population. In real situations, as discussed earlier toxicant are emitted into the environment either directly by external sources or through its precursor which are population density dependent and various industries set up by population. Rescigno (1977) [11] has studied the effect of a toxicant, which is formed only by precursor of the population and affecting itself. The simultaneous effect of toxicants and precursor of the population on the survival of resource dependent population has been done by Shukla *et al*. [2009] [12].

$$\Omega = \{(N, P, I, T):\ 0 \le N \le K, 0 \le P \le P_m, 0 \le I \le I_m, 0 \le T \le T_m\}$$

In view of above consideration, in this chapter, a nonlinear mathematical model is proposed and analyzed to study the effects of toxicants on biological population. It is assumed that toxicant is emitted into the environment by natural sources and also by industrialization. Further, its concentration is increased due to precursor of the population and various industrial wastes. This situation is modeled by the system of four ordinary differential equations. Stability theory of nonlinear differential equation and fourth order Runge-Kutta method are used to analyze and predict the behavior of the model.

The mathematical model
The following system of differential equation is considered to study the effect of toxicant emitted into the environment from external sources as well as formed by precursor of population and industrial waste on the survival of biological species.

$$\frac{dN}{dt} = sN\left(1 - \frac{N}{K}\right) - \pi s_1 NT, \tag{7.1.1}$$

$$\frac{dP}{dt} = \gamma N - (\gamma_0 + \theta)P + \gamma_1 I,$$

$$\frac{dI}{dt} = rI\left(1 - \frac{I}{L}\right) + \alpha_1 IN,$$

$$\frac{dT}{dt} = Q(I) + \pi_1 \theta P - \delta_0 T - s_1 NT.$$

$$N(0) \ge 0, P(0) \ge 0, I(0) \ge 0, T(0) \ge 0.$$

In model (1.1.1), N is the density of the population, P is the density of precursor, I is the density of industrialization and T is the density of toxicant present in the environment. The constants s and r are the growth rate of population and Industrialization respectively, under consideration. It is assumed that the rate of increase of the precursor density is proportional to the population density and industrial waste producing by it. γ and γ_1 are the growth rate coefficients of precursor formed by population and industrial wastes. γ_0 is the natural depletion rate coefficients of precursor and θ the fraction of the precursor, part of which is used in forming of toxicant. π_1 is the coefficient of augmentation of the concentration of the toxicant being emitted into the environment due to precursor; the constant δ_0 is the natural depletion rate coefficient of toxicant in the environment and s_1 denotes the rate of depletion of toxicant in the environment due to its inhalation by the population and π the fraction of toxicant, part of which depletes the population due to toxicant in the environment. α_1 is the growth rate coefficient of industrialization due to its interaction by population. Parameters π_1 and δ_0 are assumed to be constants. odel (1.1.1), the function $Q(I)$ is an increasing function of I which shows the rate of introduction of toxicant into the environment. Here, we take

$$Q(0) = Q_0 \geq 0, \ \frac{\partial Q(I)}{\partial I} \geq 0, \text{ For } I \geq 0. \tag{7.1.2}$$

To analyze the model (7.1.1), we need the bounds of dependent variable involved. For this we find the region of attraction in the following lemma.

Boundedness of solutions
Lemma (1.2.1)
The set
is the region of attraction for all solutions of model (1.1.1) initiating in the interior of the positive orthant, where

$$I_m = \frac{L(r + \alpha_1 K)}{r}, \quad P_m = \frac{\gamma K + \gamma_1 I_m}{\gamma_0 + \theta}, \quad T_m = \frac{Q(I_m) + \pi_1 \theta P_m}{\delta_0}.$$

Proof: Proof is analogous to the proof of lemma (3.1.1) of chapter 3.

Equilibrium analysis
The system (1.1.1) has four nonnegative equilibria namely,

$$E_0\left(0,0,0,\frac{Q_0}{\delta_0}\right), \ E_1\left(0, \frac{\gamma_1 L}{\gamma_0 + \theta}, L, \frac{Q(L) + \frac{\pi_1 \theta \gamma_1 L}{\gamma_0 + \theta}}{\delta_0}\right), \ E_2(\overline{N}, \overline{P}, 0, \overline{T}), \ E^*(N^*, P^*, I^*, T^*)$$

The existence of E_0 and E_1 is obvious. We prove the existence of the other equilibria.

Existence of $E_2(\overline{N}, \overline{P}, 0, \overline{T})$:

In this case, $\overline{N}, \overline{P}$ and $\overline{T}$ are the positive solutions of the following equations.

$$s\left(1 - \frac{\overline{N}}{K}\right) - \pi s_1 \overline{T} = 0, \tag{1.3.1}$$

$$\overline{P} = \frac{\gamma \overline{N}}{\gamma_0 + \theta} = f_1(\overline{N}), \text{ say} \tag{1.3.2}$$

$$\overline{T} = \frac{Q_0 + \pi_1 \theta \ f_1(\overline{N})}{\delta_0 + s_1 \overline{N}} = f_2(\overline{N}), \text{ say} \tag{1.3.3}$$

It is noted from equations (7.3.2) and (7.3.3) that $\overline{P}$ and $\overline{T}$ are functions of $\overline{N}$ only. To show the existence of E_2, we define a function $F(\overline{N})$ as follows

$$F(\overline{N}) = s\left(1 - \frac{\overline{N}}{K}\right) - \pi \ s_1 f_2(\overline{N}). \tag{1.3.4}$$

From equation (7.3.4), we note that

$$F(0) = s - \pi \ s_1 \frac{Q_0}{\delta_0} > 0.$$

$$F(K) = -\pi s_1 f_2(K) < 0,$$

Where

$$f_2(K) = \frac{Q_0(\gamma_0 + \theta) + \pi_1 \theta \gamma K}{(\delta_0 + s_1 K)(\gamma_0 + \theta)}.$$

Thus there exists a root $\overline{N}$ in the interval $0 < \overline{N} < K$ given by $F(\overline{N}) = 0.$

Now, a sufficient condition for E_2 to be unique is

$F'(\overline{N}) < 0,$ Where

$$F'(\overline{N}) = -\left(\frac{Q_0 s_1(\gamma_0 + \theta) - \delta_0 \pi_1 \theta \gamma}{(\gamma_0 + \theta)(\delta_0 + s_1 \overline{N})^2}\right) > 0. \quad (1.3.5)$$

With this value of $\overline{N}$, values of $\overline{P}$ and $\overline{T}$ can be found from equations (1.3.2) and (1.3.3) respectively. This completes the existence of E_2.

Existence of $E^*(N^*, P^*, I^*, T^*)$:

In this case, N^*, P^*, I^*, T^* are the solutions of following equations.

$$s\left(1 - \frac{N^*}{K}\right) - \pi s_1 T^* = 0, \quad (1.3.6)$$

$$I^* = \frac{L(r + \alpha_1 N^*)}{r} = h_1(N^*), \text{ say} \quad (1.3.7)$$

$$P^* = \frac{\gamma N^* + \gamma_1 h_1(N^*)}{\gamma_0 + \theta} = h_2(N^*), \text{ say} \quad (1.3.8)$$

$$T^* = \frac{Q(h_1(N^*)) + \pi_1 \theta h_2(N^*)}{\delta_0 + s_1 N^*} = h_3(N^*), \text{ say} \quad (1.3.9)$$

It is noted from equations (1.3.7), (1.3.8) and (1.3.9) that I^*, P^* and T^* are functions of N^* only. To show the existence of E^*, we define a function $G(N^*)$ as follows

$$G(N^*) = s\left(1 - \frac{N^*}{K}\right) - \pi s_1 h_3(N^*). \quad (1.3.10)$$

From equation (1.3.10), we note that

$$G(0) = s - \pi s_1 \frac{(Q_0 + Q_1 L)(\gamma_0 + \theta) + \pi_1 \theta \gamma_1 L}{\delta_0(\gamma_0 + \theta)} > 0.$$

$$G(K) = -\pi s_1 h_3(K) < 0,$$

Where

$$h_3(K)=\frac{(rQ_0+LQ_1(r+\alpha_1 K))(\gamma_0+\theta)+(r\gamma K+L\gamma_1(r+\alpha_1 K))\pi_1\theta}{r(\gamma_0+\theta)(\delta_0+s_1 K)}.$$

Thus there exists a root N^* in the interval $0<N^*<K$ given by $G(N^*)=0.$

Now, a sufficient condition for E^* to be unique is $G'(N^*)<0,$ where

$$G'(N^*)=-\left[\frac{s}{K}+\frac{\pi s_1}{r}\begin{Bmatrix}(\delta_0+s_1N^*)Q_1L\alpha_1+\dfrac{\pi_1\theta}{(\gamma_0+\theta)}(\delta_0\gamma r+\gamma_1L(\alpha_1-s_1r)) \\ -s_1Q(L)r-Q_1L\alpha_1s_1N^*\end{Bmatrix}\right]$$

With this value of N^*, values of I^*, P^* and T^* can be found from equations (1.3.7), (1.3.8) and (1.3.9), respectively. This completes the existence of E^*.

Stability analysis

Local Stability

To discuss the local stability of system (1.1.1), we compute the variational matrix of system (1.1.1).The entries of general variational matrix are given by differentiating the right hand side of system (1.1.1) with respect to $N, P, I,$ and $T,$ i.e

$$M(E)=\begin{bmatrix} s-\dfrac{2sN}{K}-\pi s_1T & 0 & 0 & -\pi s_1N \\ \gamma & -(\gamma_0+\theta) & \gamma_1 & 0 \\ \alpha_1 I & 0 & r-\dfrac{2rI}{L}+\alpha_1N & 0 \\ -s_1T & \pi_1\theta & Q_1 & -(\delta_0+s_1N)\end{bmatrix}$$

The variational matrix $M(E_0)$ at equilibrium point E_0 is given by

$$M(E_0)=\begin{bmatrix} s-\dfrac{\pi s_1Q_0}{\delta_0} & 0 & 0 & 0 \\ \gamma & -(\gamma_0+\theta) & \gamma_1 & 0 \\ 0 & 0 & r & 0 \\ -\dfrac{s_1Q_0}{\delta_0} & \pi_1\theta & Q_1 & -\delta_0\end{bmatrix},$$

From $M(E_0)$, we note that characteristic roots namely, $s-\dfrac{\pi s_1Q_0}{\delta_0}$ and r are positive, giving a saddle point which is stable in the $P-T$ plane and unstable in the $N-I$ plane. Therefore, E_0 is unstable. The variational matrix $M(E_1)$ at equilibrium point E_1 is given by

$$M(E_1)=\begin{bmatrix} s-\pi s_1H & 0 & 0 & 0 \\ \gamma & -(\gamma_0+\theta) & \gamma_1 & 0 \\ \alpha_1L & 0 & -r & 0 \\ -s_1H & \pi_1\theta & Q_1 & -\delta_0\end{bmatrix},$$

Where

$$H=\frac{Q(L)(\gamma_0+\theta)+\pi_1\theta\gamma_1L}{\delta_0(\gamma_0+\theta)}.$$

From $M(E_1)$, we note that one characteristic root namely, $s-\pi s_1H$ is positive, giving a saddle point which is stable in the $P-I-T$ space and unstable in the N direction. Therefore, E_1 is unstable. The variational matrix $M(E_2)$ at equilibrium point E_2 is given by In the following theorem we show that E^* is locally asymptotically stable.

$$M(E_2)=\begin{bmatrix} \frac{-s\bar{N}}{K} & 0 & 0 & -\pi s_1\bar{N} \\ \gamma & -(\gamma_0+\theta) & \gamma_1 & 0 \\ 0 & 0 & r+\alpha_1\bar{N} & 0 \\ -s_1\bar{T} & \pi_1\theta & Q_1 & -(\delta_0+s_1\bar{N}) \end{bmatrix},$$

Theorem (1.4.1): In addition to equation (7.1.2), let the following inequalities hold

$$\gamma+\alpha_1 I^*+s_1T^* < \frac{sN^*}{K}, \tag{1.4.1}$$

$$\pi_1\theta < \gamma_0+\theta, \tag{1.4.2}$$

$$\gamma_1+Q_1 < \frac{rI^*}{L}, \tag{1.4.3}$$

$$\pi s_1 N^* < \delta_0 + s_1 N^*. \tag{1.4.4}$$

Then E^* is locally asymptotically stable.

Proof: If inequalities (1.4.1) – (1.4.4) hold, then by Gerschgorin’s theorem (Lancaster and Tismenetsky, 1985), all eigenvalues of $M(E^*)$ have negative real parts and interior equilibrium E^*, is locally asymptotically stable.

Global Stability

Theorem (1.4.2): In addition to assumptions (1.2.1), let $Q(I)$ satisfy the condition

$0 \le \frac{\partial Q}{\partial I} \le \rho,$ in positive orthant Ω for some positive constants ρ. (1.4.5)

Then if the following inequalities hold

$$\gamma^2 < \frac{4}{9}\frac{s}{K}(\gamma_0+\theta), \tag{1.4.6}$$

$$\alpha_1^2 < \frac{4rs}{9KL}, \tag{1.4.7}$$

$$s_1^2(\pi+T_m)^2 < \frac{4}{9}\frac{s}{K}(\delta_0+s_1N^*), \tag{1.4.8}$$

$$\gamma_1^2 < \frac{4}{9}(\gamma_0+\theta)\frac{r}{L}, \tag{1.4.9}$$

$$(\pi_1\theta)^2 < \frac{4}{9}(\gamma_0+\theta)(\delta_0+s_1N^*), \tag{1.4.10}$$

$$\rho^2 < \frac{4}{9}\frac{r}{L}(\delta_0+s_1N^*). \tag{1.4.11}$$

Then E^* is globally asymptotically stable with respect to all solutions initiating in the positive orthant Ω..

Proof: Consider the following positive definite function about E^*

$$V(N,P,I,T)=\left(N-N^*-N^*\ln\frac{N}{N^*}\right)+\frac{1}{2}(P-P^*)^2+\left(I-I^*-I^*\ln\frac{I}{I^*}\right)$$
$$+\frac{1}{2}(T-T^*)^2.$$

Differentiating V with respect to time t, we get

$$\frac{dV}{dt}=\left(\frac{N-N^*}{N}\right)\frac{dN}{dt}+(P-P*)\frac{dP}{dt}+\left(\frac{I-I^*}{I}\right)\frac{dI}{dt}+(T-T*)\frac{dT}{dt}.$$

Substituting the values of $\frac{dN}{dt},\frac{dP}{dt},\frac{dI}{dt}$ and $\frac{dT}{dt}$ from the system of equations (1.1.1) in the above equation and after doing some algebraic manipulations and defining the functions

$$\eta(I)=\begin{cases}\dfrac{Q(I)-Q(I*)}{I-I^*}, & I\neq I^*,\\ \dfrac{\partial Q(I*)}{\partial I}, & I=I^*,\end{cases} \tag{1.4.12}$$

We get

$$\frac{dV}{dt}=-\frac{1}{3}a_{11}(N-N*)^2+a_{12}(N-N*)(P-P*)-\frac{1}{3}a_{22}(P-P*)^2$$

$$=-\frac{1}{3}a_{11}(N-N*)^2+a_{13}(N-N*)(I-I*)-\frac{1}{3}a_{33}(I-I*)^2$$

$$=-\frac{1}{3}a_{11}(N-N*)^2+a_{14}(N-N*)(T-T*)-\frac{1}{3}a_{44}(T-T*)^2$$

$$=-\frac{1}{3}a_{22}(P-P*)^2+a_{23}(P-P*)(I-I*)-\frac{1}{3}a_{33}(I-I*)^2$$

$$=-\frac{1}{3}a_{22}(P-P*)^2+a_{24}(P-P*)(T-T*)-\frac{1}{3}a_{44}(T-T*)^2$$

$$=-\frac{1}{3}a_{33}(I-I*)^2+a_{34}(I-I*)(T-T*)-\frac{1}{3}a_{44}(T-T*)^2.$$

Where

$$a_{11}=\frac{s}{K},\quad a_{22}=\gamma_0+\theta\ ,\ a_{33}=\frac{r}{L},\ \ a_{44}=\delta_0+s_1N^*,\ \ a_{12}=\gamma,\ \ a_{13}=\alpha_1$$

$$a_{14}=-s_1(\pi+T_m),\quad a_{23}=\gamma,\quad a_{24}=\pi_1\theta,\quad a_{34}=\eta. \tag{1.4.13}$$

Inequalities hold

$$a_{12}^2<\frac{4}{9}a_{11}a_{22},\quad a_{13}^2<\frac{1}{4}a_{11}a_{33},\quad a_{14}^2<\frac{1}{3}a_{11}a_{44},\quad a_{23}^2<\frac{1}{4}a_{22}a_{33},\quad a_{24}^2<\frac{1}{3}a_{22}a_{44},\quad a_{34}^2<\frac{1}{3}a_{33}a_{44}. \tag{1.4.14}$$

Now, from (7.4.13) and the mean value theorem, we note that

$$|\eta(I)|\leq\rho. \tag{1.4.15}$$

Further, we note that the stability conditions (1.4.6) - (1.4.11) as stated in theorem (1.4.2), can be obtained by maximizing the left-hand side of inequalities (1.4.14). This completes the proof of Theorem (1.4.2).

Numerical simulations and discussion

To facilitate the interpretation of our mathematical findings by numerical simulation, we integrate system (7.1.1) using a fourth order Runge-Kutta method. We take the following particular form of the functions involved in model (7.1.1):

$$Q(I)=Q_0+Q_1I. \tag{1.5.1}$$

Now we choose the following set of values of the parameters in model (1.1.1) and equation (1.5.1).

$$s = 5,\quad K = 6,\quad \pi = 0.1,\quad s_1 = 0.2,\quad \gamma = 0.3,\quad \gamma_0 = 0.2,\quad \theta = 0.3,\quad \gamma_1 = 0.2,$$
$$r = 3,\quad l = 9,\quad \alpha_1 = 0.02,\quad Q_0 = 4,\quad Q_1 = 1,\quad \pi_1 = 0.01,\quad \delta_0 = 2.5,\quad \rho_1 = 0.1. \tag{1.5.2}$$

With the above values of the parameters, we note that the condition for the existence of E^* are satisfied, and E^* is given by

$$N^* = 5.9128,\quad P^* = 7.2896,\quad I^* = 9.3548,\quad T^* = 3.6324.$$

It is further noted that all conditions of local stability (1.4.1) – (1.4.4) as well as global stability (1.4.6) – (1.4.11) are satisfied for the set of values of the parameters given in (1.5.2). Figures (1) and (2), graphically illustrates the global stability of the interior equilibrium point in $P - I$ and $N - P$ plane respectively.

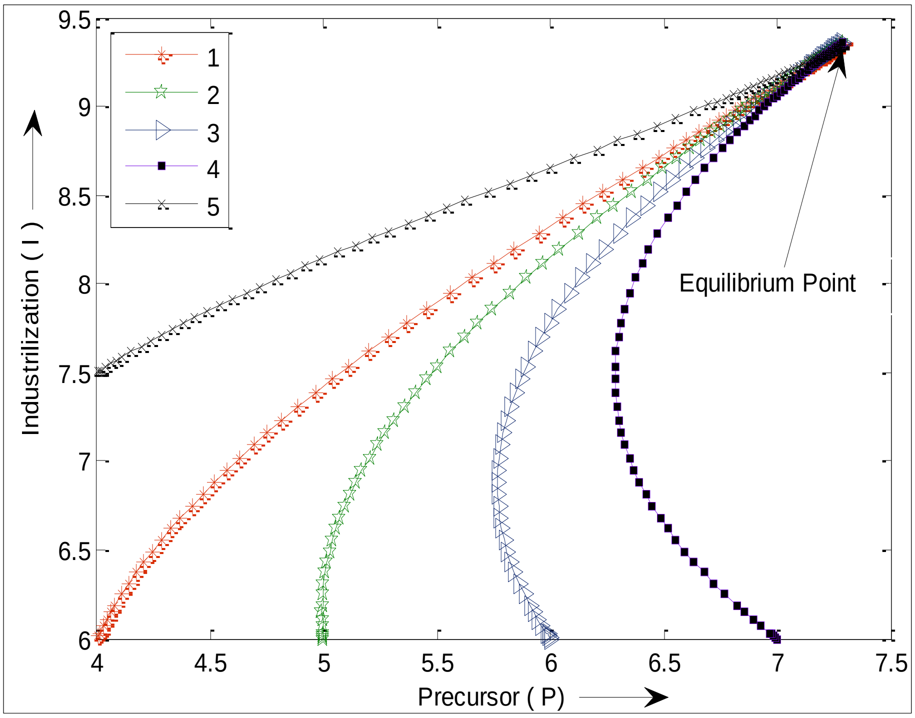

Fig 1: Variation of Precursor with industrialization for different initial start for the set of parameter values given in (1.5.2)

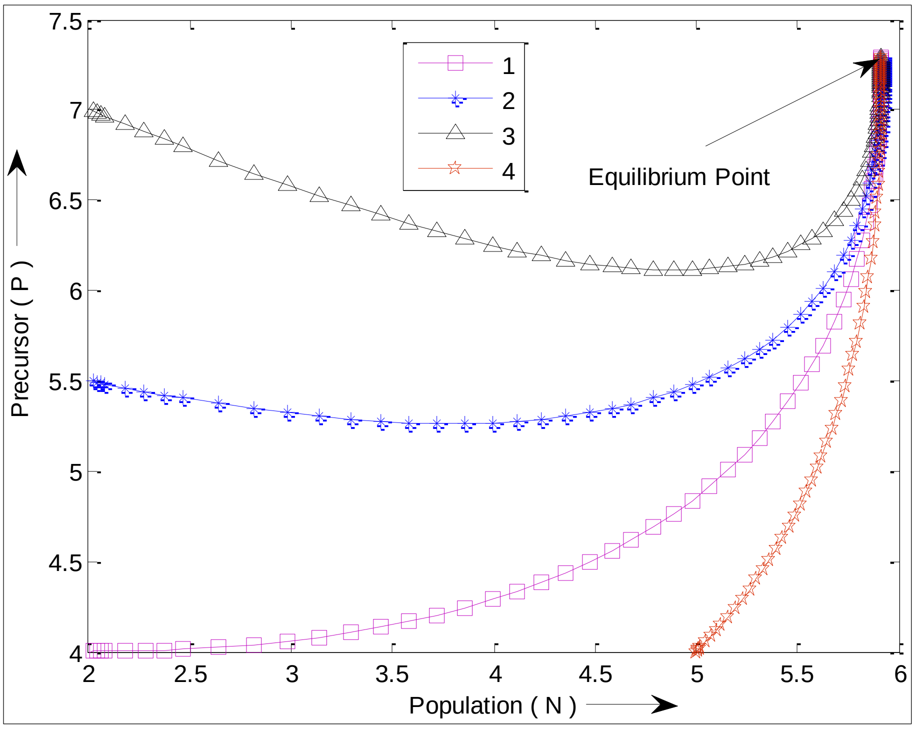

Fig 2: Variation of Population with Precursor for different initial starts for the set of parameter values given in (1.5.2)

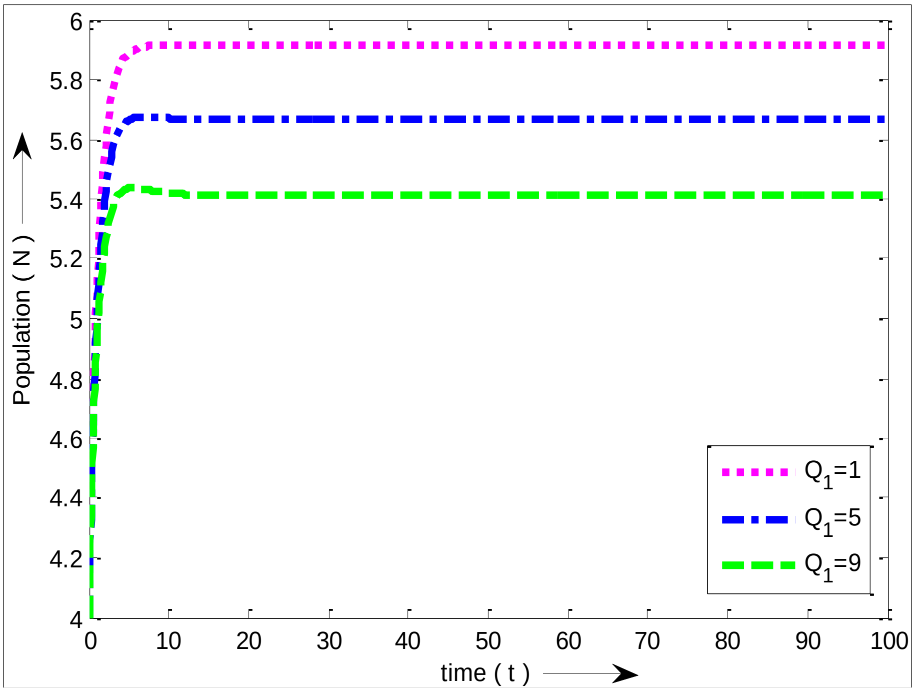

Fig 3: Graph of N versus t for different Q_1 and other values of parameters are same as in (7.5.2).

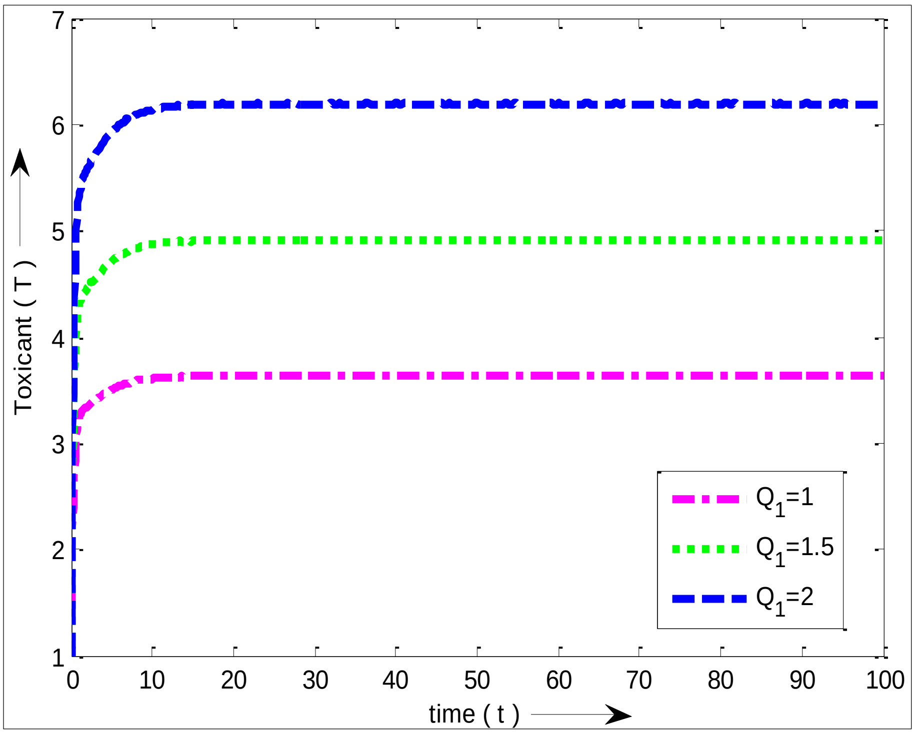

Fig 4: Graph of T versus t for different Q_1 and other values of parameters are same as in (1.5.2).

In figure (3) and (4), the variation of N and T with time 't' are shown for increasing value of Q_1, toxicant due to industrialization. From figure (3), it is observed that density of population decreases with increase in concentration of toxicant due to industrialization. From figure (4), it is observed that by very slight increment in the value of Q_1, toxicant density increases very rapidly into the environment.

The variation of precursor, P, with time 't' for different values of γ and γ_1 is shown in figure (5). It is analyzed from the figure that the qualitative behavior of γ and γ_1 are same but precursor density is more prone to γ_1 than γ, i.e. precursor density increases more into the environment with increase in density of industrialization than increase in population density.

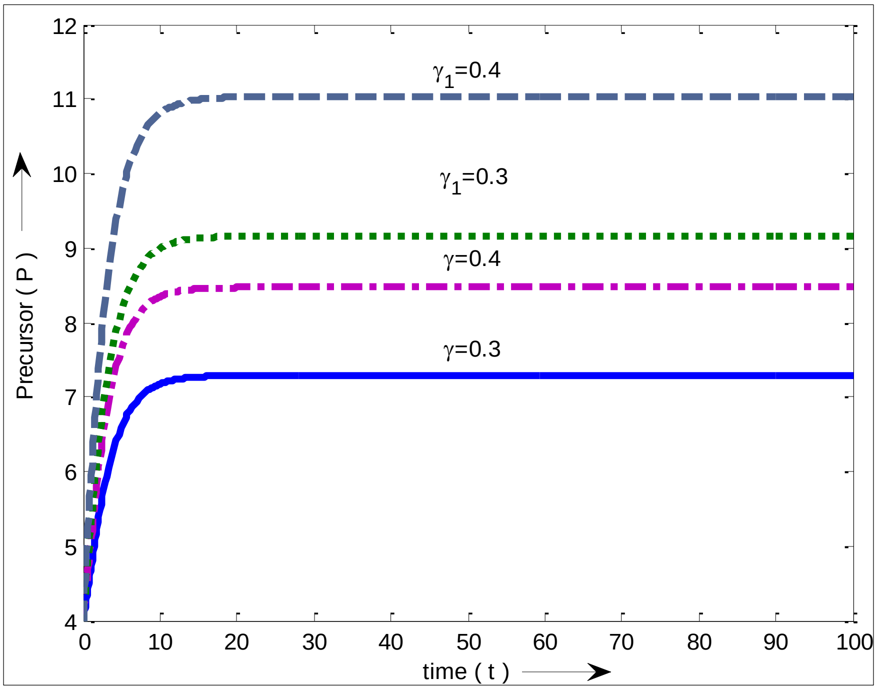

Fig 5: Graph of P versus t for different values of γ and γ_1 and other values of parameters are same as in equation (1.5.2)

In table 2.5.1 the variation of equilibrium values of precursor with different emission rates of toxicant Q_1, keeping Q_0, to be fixed is shown.

Table 1:

Q_0	Q_1	Population (N)	Toxicant (T)	Precursor (P)
4	1	5.9128	3.6314	7.2896
4	3	5.7896	8.7640	7.2127
4	5	5.6649	13.9591	7.1349

From above table, it can be found that if the emission of toxicant due to industrialization is higher in the environment, then its concentration taken by population will increases, leading to decrease in the density of population and it leads to decrease in the formation of precursor. The variation of I, with time 't' for different values of, α_1, is shown in figure (6). From this figure it can be seen that density of industrialization increases with increase in density of population.

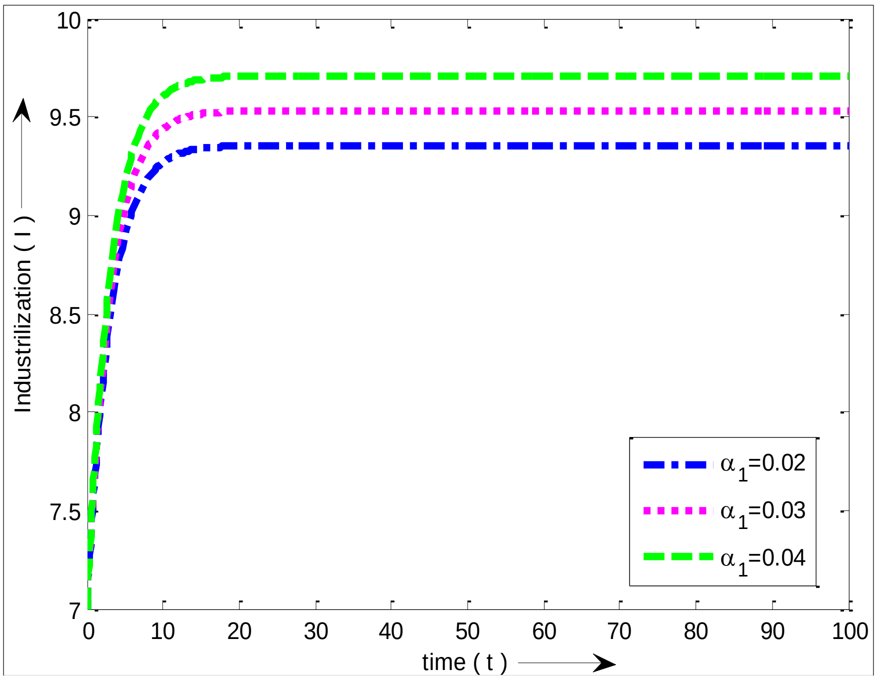

Fig 6: Graph of I versus t for different α_1 and other values of parameters are same as in equation (1.5.2)

Conclusion

A mathematical model has been proposed and analyzed to study the effects of toxicants on biological population. It is assumed that toxicant is emitted into the environment by natural sources and also by various industrial waste. Further, its concentration is increased due to precursor of the population. The existence of non-trivial equilibrium has been proved and the sufficient conditions for its stability behavior have been determined. It has been found that density of toxicant increases very rapidly into the environment with increase in the density of industrialization. Also, it is found that the magnitude of equilibrium values of population and precursor decreases with increase in the emission rate of toxicant due to industrialization. Further, it is noted that the precursor density increases more with increase in density of industrialization than increase in population density.

References

1. Agarwal AK, Sinha P, Dubey B, Shukla JB. Effects of two or more toxicants on a biological species: A nonlinear mathematical model and its analysis, in mathematical analysis and application (A. P. Dwivedi, ed.), Narosa Publication House, New Delhi, India; c2000. p. 97-113.
2. Agarwal M. Depletion of forestry resource biomass due to industrialization pressure: a ratio–dependent mathematical model, Journal of Biological dynamics. 2010 Jul;4(4):381-396.
3. Agarwal M, Sapna Devi. A time delay model for the effect of toxicant in a single species growth with stage- structure, Nonlinear Analysis: Real World Applications, Elsevier Publications; c2010. p. 2376-2389.
4. Aiello WG, Freedman HI. A time – delay model of single species growth with stage - structure, Math. Bio sci. 1990;101:139-153.
5. Aiello WG, Freedman HI, Wu J. Analysis of a model representing stage – structured population growth with state – dependent time delay, SIAM J Appl. Math. 1992;52(3):855-869.
6. Albrecht F, Gatzke H, Haddad A, Wax N. The dynamics of two interacting populations, J Math. Anal. Appl. 1974;46:658-670.
7. Arditi R, Berryman AA. The biological control paradox, Trends Ecol. Evol. 1991;6:32.
8. Arino O, Sanchez E, Fathallah A. A state – dependent delay differential equations in population dynamics: modelling and analysis. Topics in functional differential and difference equations (Lisbon), Fields Inst. Commun., 29, Amer. Math. Soc., Providence, RI 2001; c1999. p. 19-36.
9. Barclay HJ, Driessche PVD. A model for a single species with two life history stages and added mortality, Ecol. Model. 1980;11:157-166.
10. Birkhoff G, Rota GC. Ordinary Differential Equations, Ginn, Boston; c1982.
11. Rescigno A, DeLisi C. Immune surveillance and neoplasia-II a two-stage mathematical model. Bulletin of Mathematical Biology. 1977 Jul;39(4):487-97.
12. Shukla VK, Doyon Y, Miller JC, DeKelver RC, Moehle EA, Worden SE, *et al*. Precise genome modification in the crop species Zea mays using zinc-finger nucleases. Nature. 2009 May;459(7245):437-41.