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Subgrouping over large intervals of time, partial differential equations of $W (w^1, X|R (R^1))$ as $R^1=>R$, and fundamental theorems, 1850-2000 US econometric history, a theory of structural regression decompositions & wealth inequality

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Abstract
Recent studies have used regression decomposition to analyze recent data and found that over seventy percent of the black-white wealth differences remained unexplained (See, e.g., Gittleman and Wolff 2000; Altonji, Doraszelski and Segal 2000; and Blau and Graham 1990). However, their results are limited to the variation in recent data. This study contributes improved methodology and historical empirical results to the literature on economic discrimination. In this paper, (i) I present structural regression decompositions, which are modifications to methods developed by be cker (1957) and Oaxaca (1973); (ii) I present a basic empirical test when analyzing structural regression decompositions; (iii) I report the estimated sources of black-white differences in wealth directly before and after emancipation; and (iv) I link these findings to recent studies. Empirical estimates confirm that the size and persistence of modern black-white wealth differences have historical roots.

Keywords: Mathematical economics, structural regression decomposition, theory of economic discrimination

Introduction
The study of racial differences in factor market supply decisions and prices, as reflected in the extensive literature on labor supply, wages and income, presents only a subset of the variables that determine the accumulation and storage of assets over the lifetime of black and white households.

Literature Review
Overall, economic research shows that wealth differences were larger than income differences. Brimmer (1988) used 1984 US census data to find that blacks held 7.2 percent of US income and only three percent of US wealth. Additionally, Wolff (1992) [64] uses SCF, SFCC, and SIPP data from 1940 – 1988 to show that blacks possessed between 13-23 percent of white mean wealth and 4-10 percent of white median wealth. Wolff (1998) [65] affirms his earlier results using SCF data. He shows that the black-white ratio of mean net worth fell to 17 percent by 1995 while the ratio of median net worth rose to only twelve percent.

Blau and Graham (1990) [7] produced a foundational study of black-white wealth differences using regression decomposition. They employed data from the National Longitudinal Surveys (NLS) of young men and women in 1976 and 1978, respectively, to conduct regression decompositions of wealth by race. After controlling for income and demographic variables, they found that 78 percent of wealth gap remained unexplained. But they obtained different results when decomposing wealth based on white and black coefficient weights: 22 percent of black- white wealth differences (for couples and singles) were unexplained using white coefficients. However, 74 percent (for couples), and 97 percent (for singles) of black-white differences were unexplained using black coefficients. They state that “from a policy perspective, the more relevant question appears to be one addressed when black functions are employed: what would happen to black wealth if blacks were given the white means but retained there own functions?" (p. 332).

Based on the large unexplained differences in wealth, they proposed that barriers to businesses and housing, differences in labor market uncertainty and differences in inheritances may be possible sources of these differences. Altonji, Doraszelski and Segal (2000) [2] confirmed large unexplained differences in wealth among blacks and whites. They analyzed a sample of pooled data from the Panel Study of Income Dynamics (PSID) in 1984 [33], 1989 and 1994 by multiple models of regression

Ham (1982) [27] describes how labor market constraints impact worker decisions. Decomposition and found that 70 percent of the differences were still unexplained, based on the results using black coefficient weights. Their OLS decompositions show large unexplained black-white wealth differences using black and white coefficients: Explained wealth differences were six percent for couples and 27 percent for single males using black coefficients, and 67 percent for couples and 108 percent for single males using white coefficients. Gittleman and Wolff (2000) [25] also confirmed large unexplained black-white differences. They used PSID data from 1984-1994 and found at least 75 percent of differences remained unexplained depending on the coefficient. Using black coefficients, 32 percent of the wealth differences were explained in 1984, 44 percent in 1989, and 28 percent 1994, and using white coefficients, 81 percent (1984), 78 percent (1989), and 77 percent (1994) of the wealth differences were explained. Furthermore, using simulated counterfactuals and substituting observed savings and inheritance rates in black wealth function, the authors found the gap would remain and take another 72 years to close gap.

However, these studies lack a sufficient time dimension to understand the evolution of black-white differences in wealth. Comparing empirics from decompositions that are further apart in time, for instance, may help identify the underlying causes of black-white differences in wealth. Additionally, a thorough analysis of the variables that explain wealth accumulation patterns would provide readers additional confidence in the results. This study attempts to address these concerns by analyzing the source of the black-white wealth gap directly before and after emancipation of blacks in the United States, and comparing historical differences to modern differences.

Research Methodology

Wealth Identity

Wealth is determined by (i) wage rates offered by firms, (ii) individual choices of hours of work and commodity consumption, (iii) market prices of commodities, (iv) initial wealth of individuals and (v) market rates of returns on invested initial wealth and savings. (i) Wages. Consider the following single period model, formalized by Arrow (1972) [56], where owners of firms seek to maximize their utility, which includes short-run profits & types of labor. Using the following notation: output produced by the firm (y), competitive market output price (p y), fixed quantity of capital (K o), competitive rental rate of capital (r), labor demanded from group one (L 1), wage paid to group one laborer (w1), labor demanded from group two (L 2), wage paid to group two laborer (w2), profit earned by the firm (π), and utility (U) from earning profits (+) and employing members of group one (+) and group two (-).Firms choose L 1* and L 2* to maximize U(π, L 1, L 2) subject to π =p yy – rK o – w1L1 – w2L2 wherey = f(K o, L), L = L 1+L2, ∂U/∂L1 ≥ 0 and ∂U/∂L2 ≤ 0.

Thus, we can reduce the firm's problem to maximizing U (p yf(K o, L 1+L 2) – rK o

$$\frac{\partial U}{\partial L_1}: \frac{\partial U}{\partial \pi}\left[\frac{\partial \pi}{\partial y} \cdot \frac{\partial y}{\partial L} - \frac{\partial \pi}{\partial L_1}\right] + \frac{\partial U}{\partial L_1} = 0$$

$$\frac{\partial U}{\partial \pi}\left[(p_y)(MPL_1) - (w_1)\right] + \frac{\partial U}{\partial L_1} = 0$$

$$\left[(p_y)(MPL_1) - (w_1)\right] = -d_1$$

$$where \quad d_1 = \frac{\partial U}{\partial L_1} \Big/ \frac{\partial U}{\partial \pi} \leq 0$$

– w1L1 – w2L2, L 1, L2).The first order condition for group one (+):

Given p y MPL 1 – w1 = -d1, then p yMPL 1 = w1 - d1. For example, 50 = 60 –10. Thus, the value of the marginal product of labor for members of group one is less than the wage. Similarly, the first order conditions for group two

$$\frac{\partial U}{\partial L_2}: \frac{\partial U}{\partial p}\left[\frac{\partial p}{\partial y} \cdot \frac{\partial y}{\partial L_1} - \frac{\partial p}{\partial L_2}\right] + \frac{\partial U}{\partial L_2} = 0$$

$$\frac{\partial U}{\partial p}\left[(p_y)(MPL_2) - (w_2)\right] + \frac{\partial U}{\partial L_2} = 0$$

$$\left[(p_y)(MPL_2) - (w_2)\right] = -d_2$$

$$where \quad d_2 = \frac{\partial U}{\partial L_2} \Big/ \frac{\partial U}{\partial p} \leq 0$$

Given p y MPL 2 – w2 = -d2, then p yMPL 2 = w2 - d2. For example, 50 = 40 – (-10). Thus, the value of the marginal product of labor for members of group two is greater than the wage.

$$W_{tj} = W_{oj}\left(e^{\alpha_{1j}A_{tj} + \alpha_{2j}A_{tj}^2 + \beta_j X_{tj} + \varepsilon_{tj}}\right)^t + \Delta_j Y_{tj}\left(\frac{1}{1 - e^{\alpha_{1j}A_{tj} + \alpha_{2j}A_{tj}^2 + \beta_j X_{tj} + \varepsilon_{tj}}}\right)$$

Summarizing the impact of discrimination on wages and profits, we found that the discriminating firm offers wage rates such that w 1 > value of marginal product of labor > w 2 (i.e. 60 > 50 > 40) or the discriminating firm offers members of group one a wage that exceeds the wage offered to members of group two even though the value of their marginal products are the same.

Group specific rates of returns are not only determined by sale and purchase price of assets, but are also determined by the preferences of those that affect the price of the asset, similar to the discrimination coefficient that affects the size of wages paid to different groups.

Consider an alternative version of the wealth identity presented in equation (2’):

$$Y_{ij} = e^{\gamma_j + \mu_{1j} X_{ij} + \mu_{2j} X_{ij}^2 + \theta_j A_{ij} + u_{ij}}$$

Where the wealth (Wi,j) of person i in group j is determined by their initial assets (W0,,j) plus earnings (Yi,j) (scaled by the group-specific savings rate -Δj). Let Δj = e δ.

Both components are multiplied by the continuously compounded nominal, or effective, rate of return(r). When data on the rate of return, the purchase price of assets and the sale price of assets is not available, demand and supply factors for assets, which determine that rate of return, can be directly substituted in the equation. These factors include demographic variables, such as age (Ai,j), and occupational skill and regional variables (X i,j). Equation [D1] can be reduced by substituting factors that determine the rate of return (described above) and using the limit as T → ∞ on the latter half of the equation such that

$$W_{ijT} = W_{0j} e^{rT} + \Delta_j Y_{ij} \sum_{t=1}^{T} e^{r(t-1)}$$

Research Results

The equation can be estimated using natural log-linear approximation when data on earnings is available. However, when no data on earnings is available, we can first employ the structural model of earnings]. Mincer (1974) [44] proposed “the parabolic and Gompertz estimating equation (for earnings that) was specified to a quadratic approximation of a Taylor expansion” of the structural earnings model (p.90). This earnings equation is a reduced-form equation determined by a set of schooling (Si,j) and experience (E i,j) variables. When a sample does not include any income variables, we can estimate the equation by directly substituting the earnings into the wealth equation.

Furthermore, when schooling and experience variables are not available, proxy variables can be employed. The original formulation of experience is age minus years of schooling minus the beginning age of schooling (Mincer 1974, p.84) [44]. Schooling can be proxied by a literacy variable. Given the quasi-reduced regression equation and models of the proxy variables in equations, we can obtain the reduced-form regression equation. Margo (1990) [40] used data from Smith (1984) [23] to show that the average amount of schooling was seven years around the Civil War (Margo 1990, p.15) [40]. Since we are interested in identifying the parameter on age, given schooling, as a proxy for experience in the earnings equation in the Wealth identity, we focus on the most relevant parameter of the proxy variable, phi, to obtain mathematical economics proposition 1, proposition 2, and proposition 3.

$$\tilde{\phi}_{ij} = \frac{\partial W}{\partial r}\frac{\partial r}{\partial A} + \frac{\partial W}{\partial Y}\left(\frac{\partial Y}{\partial E}\frac{\partial E}{\partial A} - 2A\frac{\partial Y}{\partial E^2}\frac{\partial E^2}{\partial A}\right) \quad \text{[D11]}$$

$$= \underbrace{(\tau-1)\phi_{ij}}_{\alpha} + \underbrace{\delta_j(\mu_{1j}\lambda_{1j} - 2A\mu_{2j}\lambda_{2j})}_{\lambda}$$

Proposition 1: Let $W \to 0$. *Then* $\delta \to 1$ *and* $\alpha > \lambda$ *or* $\alpha < \lambda$. *Thus,* $\alpha \to \tilde{\phi}_{ij}$ *or* $\lambda \to \tilde{\phi}_{ij}$.

Proposition 2: Let $W \to \xi$ *where* $\xi \gg 0$. *Then* $\delta \to 0, \lambda \to 0$, *and* $\alpha \to \tilde{\phi}_{ij}$.

Proposition 3: Let $P = (\tilde{\phi}_{ij} - \tilde{\phi}_{ik})\bar{A}_{ijr}$. *If* $P > 0$, *then Proposition 2 holds for* j *and* $\alpha_j - \alpha_{-j} > \lambda_j - \lambda_{-j}$.

These propositions, or lemmas, describe the impact of changes in structural variables on leading economic indicators of utility happiness and the likely use of economic market tools by individuals, firms, social planners/universal planners to positively impact other individuals, firms, social planners/universal planners.

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