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Ashok Kumar Pandey
Associate Prof. Ewing Christian College (An Autonomous Post graduate College of Allahabad University, Prayagraj), Prayagraj, Uttar Pradesh, India

Corresponding Author:
Ashok Kumar Pandey
Associate Prof. Ewing Christian College (An Autonomous Post graduate College of Allahabad University, Prayagraj), Prayagraj, Uttar Pradesh, India

Relation of radicals with strong vertex connectivity and ideal symmetry in near rings

Ashok Kumar Pandey

Abstract
The most interesting and active field of current research in mathematics is theory of near- ring due to its wide application in coding theory, group theory, geometry, cryptography and block- designing etc. Near rings are one of the generalized structure of rings, the study and research on near- rings is very systematic and continuous. Near- rings around in all directions of mathematics and continuous research is being conducted. In this paper we study about the relation of radicals in a Near ring with strong vertex connectivity of graphs.

Keywords: Nil radical, 3- semi prime ideal, ideal symmetric graph, zero- symmetric graph.

Introduction

The most interesting and active field of current research in mathematics is theory of near- ring due to its wide application in coding theory, group theory, geometry, cryptography and block- designing etc. Near rings are one of the generalized structure of rings, the study and research on near- rings is very systematic and continuous. Near- rings around in all directions of mathematics and continuous research is being conducted, which show that their structure has power and beauty all its own. Modern ring theory has very active mathematical discipline and studying rings in their own right. The key ideas important to near- ring were formed by L.E. Dickson in 1905. Actually he gave the concept of near- field. In 1930 Wieland studied near- rings which were not near- fields. In this paper we study about the relation of radicals in a Near ring with strong vertex connectivity of graphs.

Theorem 1.1: Let I be an ideal of N. If I be a non zero nil radical, then I be a strong vertex cut of $\left(G_I(N)\right)$. If I be a strong cut of $\left(G_I(N)\right)$ and for all $\epsilon\ I\ x^n = 0$, then I is a nil radical.

Proof: Suppose that I be a nil radical of N. If $I = N$, then there is nothing to prove. Let $I \neq N$ and $x, y \in N$ such that$x \neq y$ If possible, suppose that there exists an edge between the vertices x and y in $\left(G_I(N)\right)$, then either $xNy \subseteq I$ or $yNx \subseteq I$. without loss of generality, suppose that $xNy \subseteq I$. Since I is a nil radical, therefore, $xNy \subseteq I$ and either $x^n = 0$ or $y^n = 0$ implies that either $x \epsilon I$ or $y \epsilon I$ But this is a contradiction to the fact that $x, y \in N \backslash I$. Hence, I is a strong vertex cut.
Conversely, suppose that I is 3- semi-prime ideal and I is a strong vertex cut of $\left(G_I(N)\right)$.

Claim: I is a nil radical. We take $x, y \in N \backslash I$ such that $xNy \subseteq I$ and either $x^n = 0$ or, $y^n = 0$. Since I is 3- semi-prime ideal of N therefore $x = y$ implies $x \epsilon I$ Let $x \neq y$. If it possible then suppose $x \in N \backslash I$and $y \in N \backslash I. S$ince I is a strong vertex cut of $\left(G_I(N)\right)$, therefore, there is no edge between x and y in $\left(G_I(N)\right)$. It implies that $xNy \nsubseteq I$ and $yNx \nsubseteq I$. But this is a contradiction. Therefore, either $x \epsilon I$ or $y \epsilon I$. Hence, I is a nil radical.

Proposition 1.2: Let I be a non zero nil radical of N then $K\left(G_I(N)\right) = |\ I\ |$.

Proof: Since I be a non zero nil radical of N, therefore $I \neq \{0\}$. Since I is a strong vertex cut of N. Therefore, if we remove I from the graph $(G_I(N))$, then $(G_I(N))$ is totally disconnected. Hence, $K(G_I(N)) = |\ I\ |$.

Theorem 1.3: Let I be a 3- prime ideal of a near ring R, then $gr(G_I(R))$ = either ∞ or 3.

Proof: Suppose I= {0} then, there is no cycle. Hence $gr(G_I(R)) = \infty$. Suppose $I \neq 0$. Since we know that in a graph of a near ring with respect to an ideal the maximum distance between any two vertices of $(G_I(R))$ is at most 2. Since I is a 3- prime ideal. So, each element of I is connected to each element of N except itself. But by the definition of graph of a near ring with respect to an ideal, the distance between any two vertices of $(G_I(N))$ is at most 2. Therefore, $gr(G_I(R)) = 3$ or, $gr(G_I(R)) = \infty$.

Theorem 1.4: If N is a commutative zero symmetric near- ring with unity and I is a maximal ideal of N, then I is a strong vertex cut of $(G_I(N))$ and $(G_I(N))$ is ideal symmetric.

Proof: Since N is a commutative zero symmetric near- ring with unity and I is a maximal ideal of N, so, I is a prime ideal of N. Therefore, $xNy \subseteq I$ or $xNy \subseteq I$ implies either $x \epsilon I$ or $y \epsilon I$ for $x, y \in N$. This implies I is a 3- prime ideal. Therefore, I is a strong vertex cut. Since N is a commutative zero symmetric near- ring with unity and I is a 3- prime ideal, this implies $(G_I(N))$ is an ideal symmetric graph.

Theorem 1.5: Let I be a non- zero nil radical of N and x be a vertex in $(G_I(N))$, if deg(x) = deg(0), then $x \epsilon I$.

Proof: Since I be a non- zero nil radical of N and x be a vertex of $(G_I(N))$. Let us suppose that deg(x) = deg(0). Then $xNy \subseteq I$ or $yNx \subseteq I$ and either $x^n = 0$ or $y^n = 0$ for all $y \epsilon N$ such that $x \neq y$ Without loss of generality, let us assume $xNy \subseteq I$ and either $x^n = 0$ or $y^n = 0$ for all $y \epsilon N$ such that $x \neq y$. If I= N, then $x \epsilon N$ And hence it is proved.
Suppose that $I \neq N$ and let $y \in N \backslash I$. Since I is a nil radical of N and $xNy \subseteq I$ and either $x^n = 0$ or $y^n = 0$ this implies $x \epsilon I$. Hence it is proved.

Theorem 1.6: Let N be a zero symmetric near- ring and x is a vertex in $(G_I(N))$. If $x \epsilon I$, then deg(x) = deg(0).

Proof: Since N be a zero symmetric near- ring and x is a vertex in $(G_I(N))$.
Claim: deg(x) = deg(0)
Let $x \epsilon I$ If $x = 0$, then it is proved. Let $x \neq 0$. If it possible, suppose that deg(x) < deg(0), then there exists a vertex y such that y is not adjacent to x in$(G_I(N))$. This implies that $xNy \nsubseteq I$ and $yNx \nsubseteq I$. Now as $x \in I$ and I is an ideal of N, we have $xNy \subseteq I$ Therefore, $xNy \subseteq I_y$. But N is zero symmetric, we have $I_y \subseteq I$ Thus we get $xNy \subseteq I$. which is a contradiction.
Hence deg(x) = deg(0).

Theorem 1.7: Let N be a zero symmetric near- ring and I be a nil radical of N, then $x \in I$ if and only if deg(x) = deg(0).

Proof: Let N be a zero symmetric near- ring and I be a nil radical of N. Suppose that $(x \in G_I(N))$ also, let $x \in I$. If $x = 0$, then deg(x) = deg(0). If it possible, suppose that deg(x) < deg(0), then there exists a vertex y such that y is not adjacent to x in$(G_I(N))$. This implies that $xNy \subseteq I$ and $yNx \subseteq I$. Now as $x \in I$ and I is an ideal of N, we have $xN \subseteq I$. Therefore, $xNy \subseteq I_y$. But N is zero symmetric near ring, we have $I_y \subseteq I$. Thus we get $xNy \subseteq I$. Which is a contradiction. Hence deg(x) = deg(0).
Conversely, suppose that x be a vertex in $G_I(N)$ such that deg(x) = deg(0), let $xNy \subseteq I$ or, $yNx \subseteq I$ and either $x^n = 0$ for all $x \in N$ or $y^n = 0$ for all $y \in N$ such that $y \neq x$ without loss of generality, let us assume for all $y \in N$ and either $x^n = 0$ for all $x \in N$ or $y^n = 0$ for all $y \in N$ such that $y \neq x$. If $I = N$, then $x \in I$. Let $I \neq N$. Choose $y \in N \backslash I$. Since I is a nil radical of N and $xNy \subseteq I$ and $x^n = 0$, we get $x \in I$.

Theorem 1.8: Let I be an ideal of N. Suppose N is a zero symmetric near-ring and I is a nil radical of N, then $(G_I(N))$ is an ideal symmetric graph.

Proof: Since N is a zero symmetric near-ring and I is a nil radical of N. Let x, y be distinct vertices of $(G_I(N))$ with an edge between x and y, then $xNy \subseteq I$ or $yNx \subseteq I$ Assume that $xNy \subseteq I$. Since I is a nil radical of N, therefore, $xNy \subseteq I$ and $x^n = 0$ or $y^n = 0$ implies that either $x \in I$ or $y \in I$. Now as N is zero- symmetric and I is a nil radical therefore, we get that deg(x) = deg(0) or deg(y) = deg(0). Hence $(G_I(N))$ is an ideal symmetric graph.

Theorem 1.9: Let $(G_I(N))$ is an ideal symmetric graph and I is 3- semiprime ideal of N. For every $x \in N$, deg(x) = deg(0) in $(G_I(N))$ implies $x \in I$, then I is nil radical and I is a strong vertex cut of $(G_I(N))$.

Proof: Suppose that $(G_I(N))$ is an ideal symmetric graph, I is a 3- semi-prime ideal of N and for every $x \in N$, deg(x) = deg(0) in $(G_I(N))$ implies $x \in I$ also let $x, y \in N$ and $xNy \subseteq I$. Since I is a $3-$ semi-prime ideal of N, therefore, $x = y$ implies $x \in I$ and $x^n = 0$. Let $\neq y$. Now there is an edge between x and y in $(G_I(N))$. Since $(G_I(N))$ is an ideal symmetric graph, therefore deg(x)

= deg(0) or deg(y) = deg(0) that is $x^n = 0$ or $y^n = 0$. This implies that $x \in I$ or $y \in I$. Thus, I is a nil radical of N. Hence, I is a strong vertex cut of $(G_I(N))$.

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