International Journal of Statistics and Applied Mathematics 2019; 4(6): 137-146

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2019; 4(6): 137-146

www.mathsjournal.com
Received: 14-09-2019
Accepted: 18-10-2019

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Thermosolutal instability of rotating Visco: Elastic fluid in a anisotropic porous medium

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Abstract
The Thermosolutal instability of Rivlin-Ericksen fluid in porous medium is considered in the presence of uniform vertical rotation. For the case of stationary convection, the stable solute gradient and rotation have stabilizing effects on the system, whereas, the medium permeability has stabilizing (or destabilizing) effect on the system under certain condition. The viscoelasticity effects disappear for stationary convection. The stable solute gradient, rotation, porosity and viscoelasticity introduce oscillatory modes in the system which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

Keywords: Thermosolutal instability, Rivlin-Ericksen rotating fluid, porous medium, viscoelasticity

Introduction

The theoretical and experimental results of the onset of thermal instability (Benard convection) in a fluid layer under varying assumptions of hydrodynamics has been treated in detail by Chandrasekhar [9] in his celebrated monograph. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [3]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually for removed from consideration of single component fluid and rigid boundaries and therefore, it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The thermosolutal convection problem arise in oceanography, limnology and engineering.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The Rivlin-Ericksen [8] fluid is one such fluid. Many research worker have paid their attention towards the study of Rivlin-Ericksen fluid. Johri [1] has discussed the viscoelastic Rivlin-Eriksen incompressible fluid under time dependent pressure gradient. Sisodia and Gupta [10] and Srivastava and Singh [7] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channel of different cross-sections in the presence of the time dependent pressure gradient. In another study, Garg *et al.* [4] have studied the rectilinear oscillations, of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid in the presence of a uniform magnetic field. Recently, Sharma and Kumar [6] have studied the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid acted on by a uniform rotation and found that rotation has a stabilizing-effect and introduces oscillatory modes in the system.

In all the above studies, the medium has been considered to be non-porous. When the fluid parameters a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous term in the equation of Rivlin-Ericksen fluid motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)q\right]$, where μ and μ' are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid. k_1 is the medium permeability and *q* is the Darcian (filter) velocity of the fluid. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dusty 'showball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of coments meteorites and interplanetary dust strongly suggest the importance of porosit in the astrophysical context Mc Donnel [5]. In many astrophysical situations, the effect of rotation on thermosolutal convection in porous medium is also important.

Keeping in mind the importance in geophysics, soil physics, astrophysics, ground-water hydrology and various applications mentioned above, the thermosolutal instability of a Rivlin-Ericksen fluid in porous medium in the presence of uniform horizontal rotation has been considered in the present paper.

2. Mathematical Formulation

In this problem, we consider an infinite, horizontal, incompressible Rivlin-Ericksen fluid-layer of thickness *d*, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface $z=0$ are T_0, ρ_0 and C_0 and at the upper surface $z=d$ are T_d, ρ_d and C_d respectively and that a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ and uniform solute $\beta' = \left|\frac{dc}{dz}\right|$ are maintained. The gravity field $g(0,0,-g)$, and a uniform horizontal rotation $W(\Omega, 0, 0)$ pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and medium permeability k_1.

The equations expressing the conservation of momentum, mass temperature, solute mass concentration and equation of state of Rivlin-Ericksen fluid are:

$$\frac{1}{\in}\left[\frac{\partial q}{\partial t}+\frac{1}{\in}(q.\nabla)q\right] = -\left(\frac{1}{\rho_0}\right)\nabla\rho + g\left(1+\frac{\rho}{\rho_0}\right)$$

$$-\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)q + \frac{2}{\in}(q\times w) \quad ...(1)$$

$$\nabla q = 0 \quad ...(2)$$

$$E\frac{\partial T}{\partial t}+(q.\nabla)T = k\nabla^2 T \quad ...(3)$$

$$E'\frac{\partial C}{\partial t}+(q.\nabla)C = k'\nabla^2 C \quad ...(4)$$

and $\rho_b = \rho_o[1-\alpha(T-T_0)+\alpha'(C-C_0)]$ ...(5)

Let $P, \rho, T, C, \alpha, \alpha', g$ and $q(u, v, w)$ denote, respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration and fluid velocity.

Where the suffix zero refers to values at the reference level $z=0$ and in writing equation (1), use has been made of the Boussinesq approximation. The kinematic viscosity *v*. Kinetic viscoelasticity ν', the thermal diffusivity *k* and the solute diffusivity k' are all assumed to be constant. $E = \in + (1-\in)\left(\frac{P_s C_s}{\rho_o C_i}\right)$ is a constant and E' is a constant analogues to *E* but corresponding to solute rather that heat ρ_S, C_S, ρ_0 and C_i denote the density and heat capacity of solid (porous matrix) material and fluid, respectively.

3. Basic State of the Problem

The basic state of the problem is taken as $q = q_b = (0,0,0), P = P_b(z), T = T_b(z), C = C_b(z)$ using above basic state, equation (1) to (5) becomes.

From (3)

$$T = T_b = -\beta z + T_0 \quad ...(6)$$

From (4)

$$C = C_b = -\beta' z + C_0 \quad ...(7)$$

From (5) $\rho_b = \rho_0[1 - \alpha(T_b - T_0) + \alpha'(C_b - C_0)]$

$$\rho_b = \rho_0[1 - \alpha(-\beta z) + \alpha'(-\beta' z)$$

$$\rho_b = \rho_0[1 + \alpha\beta z - \alpha'\beta' z] \quad ...(8)$$

4. Perturbation Equations

After perturbation, the new variable becomes

$$\vec{q} = q_b + q', \ \vec{P} = P_b + P', \ \rho = \rho_b + \rho' \ \ T = T_b + \theta, C = C_b + \gamma, \ W = W_b + W' \quad ...(9)$$

Where $\vec{q}, P', \rho', \theta, \gamma$ and W' are the perturbations in q, P, ρ, T, C and W respectively.

Using (6)-(9) equation (1)-(3) yield

$$\rho_b = \rho_0[1 - \alpha(T_b - T_0) + \alpha'(C_b - C_0)]$$

$$\rho_b + \rho' = \rho_0\,(1 - \alpha(T_b + \theta - T_0) + \alpha'(C_b + \gamma - C_0)]$$

$$\rho_b + \rho' = \rho_0\,[1 - \alpha(T_b - T_0) + \alpha'(C_b - C_0) - \alpha\theta + \alpha'\gamma]$$

$$\rho_b + \rho' = \rho_0\,[1 - \alpha(T_b - T_0) + \alpha'(C_b - C_0)] + \rho_0\,(-\alpha\theta + \alpha'\gamma)$$

$$\rho' = \rho_0\,(-\alpha\theta + \alpha'\gamma) \quad ...(10)$$

Then the linearized perturbation equation becomes from (1)

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla P - \vec{q}(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(v + v'\frac{\partial}{\partial t}\right)q + \frac{2}{\in}(q \times W_b) \quad ...(11)$$

From (2) $\nabla q = 0$ ...(12)

From (3) $E\dfrac{\partial\theta}{\partial t} = \beta W + k\nabla^2\theta$ ...(13)

From (4) $E'\dfrac{\partial\gamma}{\partial t} = \beta' W + k'\nabla^2\gamma$ ...(14)

5. Boundary Conditions

Here, we consider the case when both boundaries are free as well as maintained at constant temperatures and solute concentrations.

The appropriate boundary conditions, with respect

$W = D^2W = 0, \ \theta = 0, \ \Gamma = 0, \ Dz = 0$ at $Z = 0$ and 1 ...(15)

6. Dispersion Relation

Applying curl twice to equation (11) and taking z-component, we have

$$\frac{1}{\in}\frac{\partial}{\partial t}(\nabla \times \vec{q}) = -q\left[\left\{i\left(\alpha\frac{\partial\theta}{\partial y}\right) - \rho\left(\alpha\frac{\partial\theta}{\partial x}\right)\right\} - \left\{i\alpha'\frac{\partial\gamma}{\partial y} - j\alpha'\frac{\partial\gamma}{\partial x}\right\}\right]$$

$$-\frac{1}{k_1}\left(v+v\frac{\partial}{\partial t}\right)\nabla\times\vec{q}+\frac{2}{\in}\left(\Omega\frac{\partial\vec{q}}{\partial z}-(\vec{q}.\nabla).\vec{\Omega}\right)$$

$$\frac{1}{\in}\frac{\partial}{\partial t}(\nabla\times\vec{q})z=-\frac{1}{k_1}\left(v+v'\frac{\partial}{\partial t}\right)(\nabla\times\vec{q})_z+\frac{2}{\in}\left(\Omega\frac{\partial w}{\partial z}\right) \qquad ...(16)$$

$$\frac{1}{\in}\frac{\partial\xi}{\partial t}=-\frac{1}{k_1}\left(v+v'\frac{\partial}{\partial t}\right)\xi+\frac{2}{\in}\Omega\frac{\partial w}{\partial z} \qquad ...(17)$$

Again applying curl we obtain

$$\frac{1}{\in}\frac{\partial}{\partial t}(\nabla\times(\nabla\times\vec{q}))=-g\alpha\nabla\times\left(\frac{i\,\partial\theta}{\partial y}-j\frac{\partial\theta}{\partial x}\right)+q\alpha'\nabla\times\left(i\frac{\partial\gamma}{\partial y}-j\frac{\partial\gamma}{\partial x}\right)$$

$$-\frac{1}{k_1}\left(v+v'\frac{\partial}{\partial t}\right)\nabla\times(\nabla\times\vec{q})+\frac{2}{\in}\left[\Omega\frac{\partial}{\partial z}(\nabla\times\vec{q})-(\vec{q}.\nabla)\nabla\times\vec{\Omega}\right]$$

$$\frac{1}{\in}\frac{\partial}{\partial t}(-\nabla^2\vec{q})=-g\,\alpha\left[-\hat{i}\frac{\partial^2\theta}{\partial x\,\partial z}+\hat{j}\frac{\partial^2\theta}{\partial y\partial z}+\hat{k}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)\right]$$

$$+q\alpha'\left[-\hat{i}\frac{\partial^2\gamma}{\partial x\,\partial z}+\hat{j}\frac{\partial^2\gamma}{\partial y\partial z}+\hat{k}\left(\frac{\partial^2\gamma}{\partial x^2}+\frac{\partial^2\gamma}{\partial y^2}\right)\right]-\frac{1}{k_1}\left(v+v'\frac{\partial}{\partial t}\right)(-\nabla^2\vec{q})$$

$$+\frac{2\Omega}{\in}\frac{\partial}{\partial z}\left(i\left(\frac{\partial W}{\partial y}-\frac{\partial v}{\partial z}\right)-j\left(\frac{\partial W}{\partial x}-\frac{\partial u}{\partial z}\right)+k\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\right)\Bigg]$$

Taking *z*-component

$$-\frac{1}{\in}\frac{\partial}{\partial t}\nabla^2W=-q\,\alpha\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\theta+g\,\alpha'\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\gamma$$

$$+\frac{1}{k_1}\left(v+v'\frac{\partial}{\partial t}\right)(\nabla^2W)+\frac{2\Omega}{\in}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

$$-\frac{1}{\in}\frac{\partial}{\partial t}\nabla^2W=-q\,\alpha\nabla_1^2\theta+q\,\alpha'\nabla_1^2\gamma+\frac{1}{k_1}\left(v+v'\frac{\partial}{\partial t}\right)(\nabla^2W)+\frac{2\Omega}{\in}\frac{\partial\xi}{\partial z} \qquad ...(18)$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},\ \nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

$\xi_z=(\nabla\times\vec{q}_z)$ is the *z*-component, $W=\vec{q}.e_z,\quad D=\frac{\partial}{\partial z}$

and

$$E\frac{\partial\theta}{\partial t}=\beta W+k\,\nabla^2\theta \qquad ...(19)$$

$$E'\frac{\partial\gamma}{\partial t}=\beta'W+k'\nabla^2\gamma \qquad ...(20)$$

7. Normal Mode Analysis

The normal mode analysis can be defined as

$$[W,\ \theta,\ \gamma,\ \xi]=[W(z),\ \theta(z),\ \Gamma(z),\ Z(z)]\ \exp\ (ik_x+ik_y+nt) \qquad ...(21)$$

Where k_x and k_y are the wave numbers in the x and y directions respectively, $k = [k_x^2 + k_y^2]^{1/2}$ is the resultant wave number, and n is the growth rate which is, in general a complex constant $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ stands for the z-component of vorticity.

Expressing the coordinates x, y, z in the new unit of length d and letting $a = kd$, $\sigma = \frac{hd^2}{v}$, $P_1 = \frac{v}{k}$, $q = \frac{v}{k'}$, $F = \frac{v'}{k_1}$, $P_l = \frac{k_1}{d^2}$ and $D = \frac{d}{dz}$ equation (16)-(20), with the help of expression (21) in non-dimensional form become.

Analyzing above normal mode, we have from (18)

$$\frac{\sigma}{\in}(D^2 - a^2)W = -\frac{g\,a^2 d^2}{v}(\alpha\theta - \alpha'\Gamma) - \frac{d^2}{k_1}\left(1 + \frac{v'}{v}n\right)(D^2 - a^2)W - \frac{2\Omega d^3}{\in v}Dz$$

$$\frac{\sigma}{\in}(D^2 - a^2)W = -\frac{g\,a^2 d^2}{v}(\alpha\theta - \alpha'\Gamma) - \frac{1}{P_l}(1 + F\sigma)(D^2 - a^2)W - \frac{2\Omega d^3}{\in V}Dz$$

$$\frac{\sigma}{\in}(D^2 - a^2)W + \frac{g\,a^2 d^2}{v}(\alpha\Theta - \alpha'\Gamma) + \frac{1}{P_l}(1 + F\sigma)(D^2 - a^2)W + \frac{2\Omega d^3}{\in v}Dz = 0$$

From (17)

$$\left[\frac{\sigma}{\in} + \frac{1}{P_l}(1 + F\sigma)\right](D^2 - a^2)W + \frac{ga^2 d^2}{v}(\alpha\Theta - \alpha'\Gamma) + \frac{2\Omega d^3}{\in v}Dz = 0 \quad ...(22)$$

$$\frac{1}{\in}nZ = -\frac{v}{k_1}\left(1 + \frac{v'}{v}\frac{\partial}{\partial t}\right)z + \frac{2}{\in}\Omega\frac{\partial W}{\partial z}$$

$$\frac{\sigma z}{\in d^2} = -\frac{1}{k_1}\left(1 + \frac{v'}{v}n\right)z + \frac{2\Omega}{\in v}\frac{\partial W}{\partial z}$$

$$\frac{\sigma}{\in}z = -\frac{d^2}{k_1}\left(1 + \frac{v'}{v}n\right)z + \frac{2\Omega d}{\in v}DW$$

$$\left[\frac{\sigma}{\in} + \frac{1}{P_l}(1 + F\sigma)\right]z = \left(\frac{2\Omega d}{\in v}\right)DW \quad ...(23)$$

From (19)

$$E\,\Theta n = \beta W + \frac{K}{d^2}(D^2 - a^2)\Theta$$

$$\frac{End^2}{k}\Theta = \frac{\beta d^2 W}{k} + (D^2 - a^2)\,\Theta$$

$$E\sigma P_1\Theta = \frac{\beta\, d^2 W}{k} + (D^2 - a^2)\Theta$$

$$(D^2 - a^2 - EP_1\sigma)\Theta = -\left(\frac{\beta d^2}{k}\right)W \quad ...(24)$$

From (20)

$$E'\Gamma n = \beta' W + \frac{k'}{d^2}(D^2 - a^2)\Gamma$$

$$\frac{E'\Gamma n d^2}{k'} = \frac{\beta' d^2 W}{k'} + \left(D^2 - a^2\right)\Gamma$$

$$E'\Gamma q\sigma = \frac{\beta' d^2 W}{k'} + (D^2 - \alpha)\Gamma$$

$$(D^2 - a^2 - E'q\sigma)\,\Gamma = -\left(\frac{\beta d^2}{k'}\right)W \qquad ...(25)$$

Consider the case where both boundaries are free as well as maintained at constant temperatures and solve concentrations. The appropriate boundary conditions with respect to which equation (22)-(25). Must be solved, are Chandrasekhar' [9].

$$W = D^2 W = 0,\ \Theta = 0,\ \Gamma = 0,\ Dz = 0 \text{ at } z = 0 \text{ and } 1 \qquad ...(26)$$

The case of two free boundaries is a little artificial but it enables us to find analytical solution and to make some qualitative conclusions. This is the most appropriate for stellar atmosphere Spiegel [2].

Using the above boundary conditions, if can be shown that all the even order derivatives of W must vanish for $z = 0$ and 1 and hence the proper solution of W characterizing the lowest mode is:

$$W = W_0 \sin \pi z$$

Where W_0 is a constant

Eliminating Θ, Γ and z between equation (22)-(25) and substituting the proper solution $W = W_0 \sin \pi z$ in the resultant equation, we obtain the dispersion relation.

$$\alpha(D^2 - a^2 - Ep_1\sigma)\,(D^2 - a^2 - E'q\sigma)\theta = -\left(\frac{\beta\alpha d^2}{k}\right)(D^2 - a^2 - E'q\sigma)W$$

$$\alpha'(D^2 - a^2 - E'q\sigma)\,(D^2 - a^2 - Ep_1\sigma)\Gamma = -\left(\frac{\beta'\alpha' d^2}{k'}\right)(D^2 - a^2 - Ep_1\sigma)W$$

$$\left\{(D^2 - a^2 - Ep_1\sigma)\,(D^2 - a^2 - E'q\sigma)\right\}\left\{\alpha\theta - \alpha'\Gamma\right\}$$

$$= \left[-\left(\frac{\beta\alpha d^2}{k}\right)(D^2 - a^2 - E'q\sigma) + \frac{\beta'\alpha' d^2}{k'}(D^2 - a^2 - Ep_1\sigma)W\right.$$

From equation (22)

$$\left[\frac{\sigma}{\in} + \frac{1}{P_l}(1+\sigma F)\right]^2 [D^2 - a^2 - Ep_1\sigma]\,[D^2 - a^2 - E'q\sigma)\,D^2 - a^2)W$$

$$+\frac{qa^2d^2}{v}\left[-\left(\frac{\beta\alpha d^2}{k}\right)(D^2 - a^2 - E'q\sigma) + \frac{\beta'\alpha' d^2}{k'}(D^2 - a^2 - Ep_1\sigma)\right]\times$$

$$\left[\frac{\sigma}{\in} + \frac{1}{P_l}(1+F\sigma)\right]W + \frac{2\Omega d^3}{\in v}(D^2 - a^2 - Ep_1\sigma)\,(D^2 - a^2 - E'q\sigma)\,\frac{2\Omega d}{\in v}D^2 W = 0$$

$$\left[\frac{\sigma}{\in} + \frac{1}{P_l}(1+\sigma F)\right]^2\left[-\pi^2 - a^2 - Ep_1\sigma\right]\left[-\pi^2 a^2 - E'q\sigma\right]\left[-\pi^2 - a^2\right]$$

$$+\frac{g\,a^2 d^2}{v}\left[\frac{\sigma}{\in} + \frac{1}{P_l}(1+F\sigma)\right]\left[\begin{array}{l}\dfrac{-\beta\alpha d^2}{k}(-\pi^2 - a^2 - E'q\sigma)\\ +\dfrac{\beta'\alpha' d^2}{k'}(-\pi^2 - a^2 - Ep_1\sigma)\end{array}\right]$$

$$+\frac{4\Omega^2 d^4}{\in^2 v^2}(-\pi^2 - a^2 - Ep_1\sigma)\,(-\pi^2 - a^2 - E'q\,\sigma)\,(-\pi^2) = 0$$

$$-\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right]^2\left(1+\frac{a^2}{\pi^2}+\frac{Ep_1\sigma}{\pi^2}\right)\left(1+\frac{a^2}{\pi^2}+\frac{E'q\sigma}{\pi^2}\right)\left(1+\frac{a^2}{\pi^2}\right)$$

$$+\frac{g\,a^2 d^2}{v\pi^4}\left(\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right)\left[\begin{array}{l}\frac{\alpha\,\beta d^2}{k}\left(1+\frac{a^2}{\pi^2}+\frac{E'q\sigma}{\pi^2}\right)\\ -\frac{\alpha'\beta' d^2}{k'}\left(1+\frac{a^2}{\pi^2}+\frac{Ep_1\sigma}{\pi^2}\right)\end{array}\right]$$

$$-\frac{4\Omega^2 d^4}{\in^2 v^2}\left(1+\frac{a^2}{\pi^2}+\frac{Ep_1\sigma}{\pi^2}\right)\left(1+\frac{a^2}{\pi^2}+\frac{E'q\,\sigma}{\pi^2}\right)=0$$

$$\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right]^2\,[1+x+i\sigma_1\,EP_1]\,[1+x+E'q\;i\sigma_1]\,(1+x)$$

$$-\frac{ga^2 d^2}{v\pi^4}\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+F\sigma)\right]\left[\begin{array}{l}\frac{\alpha\beta d^2}{k}(1+x+i\sigma_1\,E'q)\\ -\frac{\alpha'\beta' d^2}{k'}(1+x+i\sigma_1 Ep_1)\end{array}\right]$$

$$+\frac{4\Omega^2 d^4}{\in^2 v^2}(1+x+i\sigma_1 EP)\,(1+x+i\sigma_1\,E'q)=0$$

$$\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right]^2(1+x+i\sigma_1 EP_1)(1+x+i\sigma_1\,E'q)(1+x)$$

$$-a^2\left(\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right)[R_1\,(1+x+i\sigma_1\,E'q)-S_1\,(1+x+i\sigma_1\,EP_1)$$

$$+T_{A_1}(1+x+i\sigma_1 Ep_1)(1+x+i\sigma_1 E'q)=0$$

$$R_1 a^2\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right](1+x+i\sigma_1\,E'q)=\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right]^2(1+x+i\sigma_1 EP_1)$$

$$(1+x+i\sigma_1 E'q)\,(1+x)+S_1 a^2\left(\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right)(1+x+i\sigma_1\,EP_1)$$

$$+T_{A_1}\,(1+x+i\sigma_1 Ep_1)(1+x+i\sigma_1 E'q)$$

$$Ra^2=(1+x)\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right](1+x+i\sigma_1\,EP_1)$$

$$+S_1 a^2\,\frac{(1+x+i\sigma_1\,EP_1)}{(1+x+i\sigma_1\,E'q)}+T_{A_1}\,\frac{(1+x+i\sigma_1\,Ep_1)}{\left(\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right)}$$

$$R_1=\frac{1+x}{x\pi^2}\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right](1+x+i\sigma_1\,EP_1)$$

$$+ S_1 \frac{(1+x+i\sigma_1 Ep_1)}{(1+x+i\sigma_1 E'q)} + \frac{T_{A_1}}{x} \frac{(1+x+i\sigma_1+EP_1)}{\frac{\left[\frac{\sigma}{\in}+\frac{1}{P_l}(1+\sigma F)\right]}{\pi^2}}$$

$$R_1 = \left(\frac{1+x}{x}\right)\left[\frac{i\sigma}{\in}+\frac{1}{P}+i\sigma_1 F\right](1+x+iEP_1\sigma_1)$$

$$+T_A \frac{(1+x+iEP_1\sigma_1)}{x\left(\frac{i\sigma_1}{\in}+\frac{1}{P}+i\sigma_1 F\right)} + S_1\left(\frac{(1+x+iEP_1\sigma_1}{(1+x+iE'q\sigma_1}\right) \quad ...(27)$$

$$R_1 = \frac{q\alpha\beta d^4}{vk\pi^4},\ S_1 = \frac{q\alpha' g\beta' d^4}{vk\,\pi^4},\ T_A = \frac{4\Omega^2 d^4}{\in^2 v^2} = \left(\frac{2\Omega d^2}{\in v}\right)^2$$

Where $x = \frac{a^2}{\pi^2},\ i\sigma_1 = \frac{\sigma}{\pi^2},\ P = \pi^2 P_l$ ...(28)

Equation (27) is the required dispersion relation studying the effects of rotation medium permeability, kinematic viscoelasticity and stable solute gradient on thermosolutal instability of Rivlin-Ericksen rotating fluid in porous medium.

8. The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0,$ the dispersion relation (27) reduces to:

$$R_1 = \frac{(1+x)^2}{xP} + T_{A_1} P \frac{(1+x)}{x} + S_1 \quad ...(29)$$

Which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number *x* and the parameters S_1, T_{A_1} and *P*. The parameter *F* accounting for the viscoelasticity effect disappears for the stationary convection.

To investigate the effects of stable solute gradient, rotation and medium permeability, we examine the behaviour of $\frac{dR_1}{dS_1}, \frac{dR}{dT_{A_1}}$ and $\frac{dR_1}{dP}$ analytically equation (29) yields.

$$\frac{dR_1}{dS_1} = +1 \quad ...(30)$$

Which implies that the stable solute gradient has a stabilizing effect on the thermosolutal convection. The advice solute gradient has destabilizing effect on the system. Since $\frac{dR_1}{dS_1}$ then becomes negative
Equation (29) yields

$$\frac{dR_1}{dT_{A_1}} = \left(\frac{1+x}{x}\right)P \quad ...(31)$$

The rotation, therefore, has always a stabilizing effect on the thermosolutal instability. Rivlin-Ericksen rotating fluid in porous medium.
It is evident from (29) that:

$$\frac{dR_1}{dP} = -\left(\frac{1+x}{x}\right)\left[\frac{1+x}{P^2} - T_{A_1}\right] \quad ...(31)$$

In the absence of rotation $(T_{A_1} \to 0)$, $\frac{dR_1}{dP}$ is given by

$$\frac{dR}{dP} = -\left[\frac{(1+x)^2}{xP^2}\right] \qquad ...(32)$$

which is always negative. The medium permeability, therefore, has a destabilizing effect on the thermosolutal instability of a fluid in the absence of rotation. In the presence of rotation the system is unstable or stable if:

$$T_{A_1} < \frac{1+x}{P^2} \text{ or } T_{A_1} > \frac{1+x}{P^2} \qquad ...(33)$$

9. The Case of Over Stability

Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the critical Rayleigh number for the onset of instability via. a state of pure oscillations, if suffices to find conditions for which (27) will admit of solutions with σ_1 real

If we equate real and imaginary parts of (27) eliminate R_1 between then, we obtain

$$a_0 s^2 + a_1 s + a_2 = 0 \qquad ...(34)$$

Where we have put $s = \sigma_1^2,\ b = 1+x$

$$a_0 = b\left(\frac{1+\in F}{P}\right)^2 E'^2 q^2 \left[b(1+\in F) + \frac{\in EP_1}{P}\right] \qquad ...(35)$$

$$a_1 = \left\{\left[\left(1+\frac{\in F}{P}\right)\left(1+\frac{2\in F}{P}\right)\right]b^4 + \left[\frac{\in}{P}EP_1\left(1+\in F\left(2+\frac{EF}{P}\right)\right)\right]b^3\right.$$

$$+\left[\frac{\in^2}{P^2}E'^2 q^2\left(1+\frac{\in F}{P}\right)\right]b^2 + \left[\in^2 E'^2 q^2\left(\frac{2FP_1}{P^3} - T_{A_1}\left(1+\frac{\in F}{P}\right)\right)\right.$$

$$\left.+\in(b-1)S_1\left(1+\frac{\in F}{P}\right)\left(EP_1 - E'q\left(1+\frac{\in F}{P}\right)\right)\right]b + \left.\left[\frac{\in^3}{P}T_{A_1}EP_1E'^2q^2\right]\right\} \qquad ...(36)$$

$$a_2 = \in^2 b\left\{\left[\frac{1}{P^2}\left(1+\frac{\in F}{P}\right)\right]b^3 + \left[\frac{\in EP_1}{P^3} - T_{A_1}\left(1+\frac{\in F}{P}\right)\right]b^2\right.$$

$$\left.+\left[\frac{\in}{P}T_{A_1}EP_1\right]b + \left[\frac{\in}{P^2}(b-1)\,S_1\,(EP_1 - E'q)\right]\right\} \qquad ...(37)$$

Since σ_1 is real for overstability, both the value of $s = \sigma_1^2$ are positive. Equation (34) is quadratic in *s* and does not involve any of its roots to be positive if

$$E_{P_1} > \frac{P^3 T_{A_1}}{\in}\left(1+\frac{\in F}{P}\right),\ EP_1 > E'q\left(1+\frac{\in F}{P}\right) \text{ and } EP_1 > E'q \qquad ...(38)$$

Which imply that

$$k' < \frac{\in Ev^3 d^2}{4\Omega^2 k_1(\pi^2 k_1^2 + \in v' d^2)} \text{ and } E'_k\left(1+\frac{\in v' d^2}{\pi^2 k_1^2}\right) < Ek' \qquad ...(39)$$

10. Conclusions

For stationary convection

1. $\frac{dR}{dS_1} > 0$, which implies that the stable solute gradient has a stabilizing effect on the thermosolutal convection.

2. $\frac{dR_1}{dT_{A_1}} > 0,$ which implies that the rotation has always a stabilizing effect on the thermosolutal instability Rivlin-Ericksen rotating fluid in porous medium.
3. (i) $\frac{dR}{dP} > 0,$ which implies that medium permeability has a stabilizing effect under above. Condition

   When $T_{A_1} > \frac{1+x}{x^2}$

   (ii) $\frac{dR}{dP} < 0$, the medium permeability has a destabilizing effect under condition

   When $T_{A_1} < \frac{1+x}{P^2}$

   (iii) In the absence of rotation $T_{A_1} \to 0$

$\frac{dR}{dP} < 0,$ which is always negative. The medium permeability has a destabilizing effect on the thermosolutal instability, of a fluid in the absence of rotation.

For oscillatory convection

The sufficient condition for the non-existent of overstability are given by the condition $k' < \frac{\in E v^3 d^2}{4\Omega^2 k_1 (\pi^2 k_1^2 + \in v' d^2)}$ and $E_k' \left(1 + \frac{\in v' d^2}{\pi^2 k_1^2}\right) < Ek'$

References

1. Johri AK. Acta Ciencia Indica. 1976; 24:377.
2. Spiegel EA. Astrophys. J. 1965; 141:1068.
3. Veronis G. J Marine Res. 1965, 23.
4. Garg Srivastava RK, Singh KK. Proc. Natn. Acad. Sci. 64A, 355.
5. McDonnel JAM. *Cosmic Dust.* John Wiley and Sons. Toronto, 1978, 330.
6. Sharma RC, Kumar P. Naturforsch. 1996; 51a:821.
7. Srivastava RK, Singh KK. Bull Cal. Math. Soc. 1988; 80:2.
8. Rivlin RS, Ericksen JL. J Rational Mech. Anal. 1955; 4:323.
9. Chandrasekhar S. Hydrodynamics and Hydromagnetic Stability, Dover Publications, New York, 1981.
10. Sisodia SS, Gupta M. Indian J Theor. Phys, 1984, 32.