International Journal of Statistics and Applied Mathematics 2019; 4(6): 152-153

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2019; 4(6): 152-153

www.mathsjournal.com
Received: 25-09-2019
Accepted: 27-10-2019

Anu Chohla
Department of Mathematics,
Guru Jambheshwar University,
Hisar, Haryana, India

Corresponding Author:
Anu Chohla
Department of Mathematics,
Guru Jambheshwar University,
Hisar, Haryana, India

Analytical treatment of Fresnel's theory

Anu Chohla

Abstract
In this paper, problem of wave theory of light solved analytically by applying mathematical analysis to Fresnel's theories. This paper based on principal that plane polarizes light on entering crystal along optical axis decomposed into two circular polarized vibration s rotating in clockwise and anticlockwise direction with same frequency and having zero initial phase difference and find angle of rotation analytically by applying mathematical computation.

Keywords: Circular motion, Fresnel's theory, vibration

1. Introduction
A brief review is given of the relevant point about the role of mathematics in physics so applying mathematical analysis to Fresnel theories we removed many of problem of wave theory of light. Shappard and Hrynevch (1992) [6] studied the generalization of Fresnel diffraction theory. BB Baker *et al.* (2003) give the mathematical theory of Huygenis principal in optics and its application to theory of diffraction. BA Kamp (2011) [3] studied the implications for electromagnetic wave theory light in matter. V.Lakshmiharyanan (2012) studied the mathematical optics. In the present paper we show the problem of wave theory of light solved analytically by applying mathematical analysis to Fresnel's theories.

2. Formulation of Problem
In an optically inactive crystal, we obtained the result by considering the equation of vibrations.
Let the incident vibration Z'OZ be represented by

$x=2b \cos wt$

where 2b is amplitude and w is angular velocity. These incident vibration is separated into two equal and opposite circular motion which is given by

$$y_1=b \sin wt$$
$$z_1=b \cos wt \quad (1.1)$$

and

$$y_2=-b \sin wt$$
$$z_2=b \cos wt \quad (1.2)$$

here equation (1.1) represents the right handed circular motion and equation (1.2) represents the Left handed circular motion .These equation represent individually in circular motion because pair gives

$$y_1^2+z_1^2=b^2$$
$$y_2^2+z_2^2=b^2 \quad (1.3)$$

On superposition they give rise Resultant displacement along y-axis

$y=y_1+y_2$
$=b \sin wt - b \sin wt$
$=0$ (1.4)

Resultant displacement along z-axis

$z=z_1+z_2$
$=b \cos wt + b \cos wt$
$=2b \cos wt$ (1.5)

These (1.4) and (1.5) components give the resultant displacement

$z=2b \cos wt$

when the vibration transmitted through crystal plate they travel with different velocities, and when they emerge from crystal plate they emerge with phase difference Φ between them. If the anti-clockwise components advance in front of other.
The emergent circular components given by

$y_1=b \sin wt$
$z_1=b \cos wt$ (1.6)

and $y_2=-b \sin(wt+\Phi)$
$z_2=b \cos(wt+\Phi)$ (1.7)

equation (1.6) represent the right hand system and equation(1.7) represent the left hand system the resultant displacement along x axis and y axis are

$y=y_1+y_2$
$=b (\sin wt - \sin(wt+\Phi))$
$=2 b \sin(\Phi/2) \cos(wt+(\Phi/2))$ (1.8)

And take

$z= z_1+z_2$
$=b (\cos wt + \cos(wt+\Phi))$
$=2 b \cos(\Phi/2) \cos(wt+(\Phi/2))$ (1.9)

These two equation (1.8) and (1.9) represent the mutually perpendicular vibration having same phase, when compounded together give resultant linear vibration make an angle δ with y-axis,
Such that

$\tan\delta=(y/z)=\tan(\Phi/2)$ (2.0)

Here $\delta=(\Phi/2)$ represent the angle of rotation.
Equation(2.0) represents the vibration plane of incident plane polarized light has been rotated in the anticlockwise direction by an angle which is equal to half the phase difference between the two emergent circular polarized vibration. Similarly same relation is proved for right handed system.

2.1 Angle of rotation

The magnitude of angle of rotation is calculated in term of refractive indices of right and left handed vibration represent μ_S and μ_T respectively.
If d is thickness of the crystal along the optic al axis then the time of rotation between two circular components traveling left-handed crystal is.

$\Delta T=T_S-T_T$
$=(t/v_S)-(t/v_T)$ (2.1)

V_b is the velocity of light in air ,then path of retardation

$= V_b\, \Delta T=t((t/v_S)-(t/v_T))$
$=t(\mu_S- \mu_T)$ (2.2)

Where μ_S and μ_T are the refractive index of substance for right handed and left handed vibration respectively.

$\Phi=(2\pi/\lambda)d(\mu_S- \mu_T)$ (2.3)

Where λ is wavelength of light in air.
Therefore for the crystal $\mu_s> \mu_T$, the angle of rotation of plane of vibration is

$\delta=(\Phi/2)= (\pi\, d\, (\mu_S- \mu_T))/\lambda$ (2.4)

3. Conclusions

This paper is based on the principal that a linear vibration may be taken as resultant of two opposite circular motion of same frequency .The rotation of vibration plane will be right or left according as right handed or left handed circular vibration is faster and equal to half of the phase difference on emergence between two circular motion.

4. Acknowledgments

Author is thankful to university grants commission, New Delhi, for financial support.

5. References

1. Anton H, Busby R. Contemporary linear algebra, 2003.
2. John Wiley, Sons NJ, Aoyagi N. Theoretical study of optical Fresnel transformation, Dr. Thesis., Tokyo Institute of Technology, Tokyo, 1973.
3. Kemp BA. Resolution of Abraham–minkowski debate: Implication for the electromagnetic wave theory in matter. Journal of Applied Physics. 2011, 109, 111101.
4. Kondo J. Integral Equation, Baifukan Tokyo, 1954.
5. Reed M, Simon B. Method of modern mathematical physics, Aceadmic Press, New York, 1972, 1.
6. Shappard, Hrynevych. Diffraction by a Circular Aperature A Generalization of Fresnel Diffraction Theory. J Opt. Soc. Am. 1992, 9.