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On local polynomial regression estimators in finite populations

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Abstract
This article examines the local polynomial regression estimators for the mean regression functions $\bar{m}_0(x_j)$ and $\bar{m}_1(x_j)$ in finite populations under circumstances when the order of the local polynomial being fit p is 0 and 1 respectively. The study utilizes a measure of performance that centres on the efficiency of the estimators of the mean regression function in survey sampling theory. To a greater extent, the examination considers analytical comparisons of the estimators in line with the concept of asymptotic relative efficiency. Particularly, asymptotic properties of the local constant regression estimator of the mean regression function are studied in a model based framework. The results of the local constant regression estimator of the mean regression function $\bar{m}_0(x_j)$ are compared with those of the local linear regression estimator of the mean regression function $\bar{m}_1(x_j)$ studied by Kikechi *et al.* (2017). Variance comparisons are made using the local constant regression estimator $\bar{m}_0(x_j)$ and the local linear regression estimator $\bar{m}_1(x_j)$ which show that the estimators are asymptotically equivalently efficient.

Keywords: finite population, local constant regression, local linear regression, model based framework

1. Introduction

The theory of sample survey aims at developing sampling strategies that result in the selection of a representative sample from a finite population of N units. In addition, the sampling survey theory provides procedures for making statistical inferences about the study variable of interest and it is used in determining the criteria for comparing different strategies in order to obtain optimal results from the survey. See Royall (1970a) [20], Royall (1970b), [21] Royall (1971), [22] Smith (1976) [25] and Pfeffermann (1993). This study reviews the local polynomial regression estimator for the mean regression function, $m(x)$ when $p = 0$ and derives the asymptotic properties of this estimator in a finite population. Local polynomial regression is a nonparametric technique which is a generalization of kernel regression and is used for smoothing scatter plots and modeling functions. The idea of nonparametric regression is introduced by Nadaraya (1964) [17] and Watson (1964) [27]. Several types of nonparametric regression methods such as the kernels, penalized splines and orthogonal series are in existence (see Dorfman (1992), [5] Hardle (1989) and Zeng & Little (2003)). In many estimation problems, the sample is used to describe and analyze the target population from which it was selected by estimating population parameters and other descriptive and analytic inferences such as correlations. Some common parameters of interest for the finite population $Y = (x_1, x_2, \dots, x_N)^/$ are the finite population total, the finite population mean, the finite population variance and the finite population proportion.

Inferences may explore properties of the process that generate the population values (see Bolfarine and Zacks (1991)). We assume that the finite population has been generated by a super population model $\xi = f(x, y, \varphi)$ and we are interested in estimating the population parameters $\varphi = (\alpha, \beta)$, where $y_i = \alpha + \beta x_i$. The super population model can be applied to predict the unobserved values $y_i{'}s$ after obtaining estimates of α and β using the known auxiliary information $x_i,\ i = 1,2 \dots, N$ (see Montanari & Ranalli (2005) [16] and Sanchez Borrego (2009) [24]. Using the model ξ, the nonparametric estimator of totals, T has been derived by Dorfman (1992) [5] who has been able to prove the asymptotic unbiasedness and MSE consistency of this estimator. The estimator, however suffers from sparse sample problem, and more work needs to be done to come up with another technique that can overcome this problem. This is where the local polynomial procedure comes in according to studies carried out by Kikechi *et al.* (2017) [11], Kikechi *et al.* (2018) [12], Kikechi and Simwa (2018)) [13] and Kikechi *et al.* (2019) [14].

This study therefore considers a model based approach to robust finite population estimation for the mean regression function using the procedure of local polynomial regression. It is typically of interest to estimate $m(x)$, using Taylor's expansion. The weighted least squares principle to be explored in the local polynomial approximation procedure, opens a wealth of statistical knowledge and thus providing easy computations and generalizations (see Fan and Gijbels (1996)). [8] The local polynomial regression is one of the most successfully applied design adaptive non parametric regression. This estimation procedure is an attractive choice due to its flexibility and asymptotic performance. Because of its simplicity, it can be tailored to work for many different distributional assumptions. It does not require smoothness and regularity conditions required by other methods such as boundary kernels. The procedure has also the advantage of adapting well to bias problems at boundaries and in regions of high curvature. Furthermore, it is easy to understand and interpret. The estimate is also linear in the response, provided the fitting criterion is least squares and model selection does not depend on the response. See Stone (1977) [26], Fan (1992), [6] Fan (1993) [7] and Ruppert and Wand (1994) [23] among others.

Analytical comparisons are carried out between the local constant regression estimator $\bar{m}_0(x_j)$ and the local linear regression estimator $\bar{m}_1(x_j)$ studied by Kikechi *et al.* (2017) [11] which indicate that the estimators are asymptotically equivalently efficient.

2. The Local Constant regression estimator $\bar{m}_0(x_j)$ and the Local Linear Regression Estimator $\bar{m}_1(x_j)$

Consider the superpopulation model of the form,

$$Y_i = m(X_i) + \sigma(X_i)\varepsilon_i \qquad (1)$$

The following specific assumptions hold for the model considered in the nonparametric regression estimation of $m(x_i)$:

$$E(Y_i|X_i = x_i) = m(x_i)$$
$$Cov(Y_i, Y_j|X_i = x_i, X_j = x_j) = \begin{cases} \sigma^2(x_i) & , \quad i = j \\ 0 & \quad i \neq j \end{cases} \quad i = 1, 2, 3, \ldots, N \quad j = 1,2,3, \ldots, N. \qquad (2)$$

The properties of the error are given by,

$$E(\varepsilon_i|X_i = x_i) = m(x_i)$$
$$Cov(\varepsilon_i, \varepsilon_j|X_i = x_i, X_j = x_j) = \begin{cases} \sigma^2(x_i), & i = j \\ 0 & i \neq j \end{cases} \quad i = 1, 2, 3, \ldots, N \quad j = 1,2,3, \ldots, N \qquad (3)$$

The functions $m(x_i)$ and $\sigma^2(x_i)$ are assumed to be smooth and strictly positive. Consider the Taylor series expansion of $m(x_i)$ expressed as,

$$\begin{aligned} m(x_i) &= m(x_j + ht) = m(x_j) + htm^{/}(x_j) + \frac{h^2t^2}{2!}m^{//}(x_j) + \frac{h^3t^3}{3!}m^{///}(x_j) + \cdots \\ &= m(x_j) + (x_i - x_j)m^{/}(x_j) + \frac{(x_i - x_j)^2}{2!}m^{//}(x_j) + \frac{(x_i - x_j)^3}{3!}m^{///}(x_j) + \cdots \qquad (4) \end{aligned}$$

The Taylor series expansion is written in a general form expressed as,

$$y_i = \alpha + (x_i - x_j)\beta + \varepsilon_i \qquad (5)$$

Where x_i lies in the interval $[x_j - h,\ x_j + h]$ and

$$\varepsilon_i = \frac{(x_i - x_j)^2}{2!}m^{//}(x_j) + \frac{(x_i - x_j)^3}{3!}m^{///}(x_j) + \cdots$$

The constants α and β are solved using the least squares procedure by making ε_i the subject of the formulae, squaring both sides, summing over all possible sample values and applying the weights to obtain a solution to the weighted least squares problem. For the constructions and derivations of the estimators $\bar{m}_0(x_j)$ and $\bar{m}_1(x_j)$, see Kikechi *et al.* (2017), [11] Kikechi *et al.* (2018), [12] Kikechi and Simwa (2018) [13] and Kikechi *et al.* (2019) [19].

3. Properties of the local constant regression estimator $\bar{m}_0(x_j)$

Fan (1993) [7] and Ruppert and Wand (1994) [23] exploit the assumptions below in investigating the properties of the conditional bias, variance and mean square error of the estimator for the mean regression function:

(i) The x_j variables lie in the interval $(0, 1)$.

(ii) The function $m^{//}(.)$ is bounded and continuous on $(0, 1)$.

(iii) The kernel $K(t)$ is symmetric and supported on $(-1,1)$. Also $K(t)$ is bounded and continuous satisfying the following: $\int_{-\infty}^{\infty} K(x)\,dx = 1$, $\int_{-\infty}^{\infty} xK(x)\,dx = 0$, $\int_{-\infty}^{\infty} x^2K(x)\,dx > 0$, $\int_{-\infty}^{\infty} K^2x\,dx < \infty$, $d_k = \int_{-\infty}^{\infty} K^2(t)\,dt$

(iv) The bandwidth h is a sequence of values which depend on the sample size n and satisfying $h \to 0$ and $nh \to \infty$, as $n \to \infty$.

(v) The point x_j at which the estimation is taking place satisfies $h < x_j < 1 - h$.

Applying similar assumptions in this article, the properties of the local constant regression estimator $\overline{m}_0(x_j)$ are investigated. Fan (1993) [7] imposed conditions on $K(.)$ and can only be used for convenience in terms of the technical arguments and therefore can be relaxed.

3.1 The expectation of the local constant regression estimator $\overline{\mathbf{m}}_0(\mathbf{x_j})$

The expectation of $\overline{m}_0(x_j)$ is

$$E(\overline{m}_0(x_j)) = \sum_{i\in S} w_i(x_j)E(y_i)$$
$$= \sum_{i\in S}\left\{\frac{\left(S_2(x_j;h) - S_1(x_j;h)(x_i - x_j)\right)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2} k\left(\frac{x_i - x_j}{h}\right) m(x_i)E(y_i)\right\} \quad (6)$$

Using the Taylor series expansion of the form,

$$m(x_i) = m(x_j) + htm^{/}(x_j) + \frac{h^2t^2}{2!}m^{//}(x_j) + \cdots, \quad (7)$$

Then theorem III in Fan and Gijbels (1996) [8] is such that under the conditions given in (i)-(v), allows

$$E\left(\overline{m}_0(x_j)\right) = \sum_{i\in S} w_i(x_j)\left(m(x_j) + htm^{/}(x_j) + \frac{h^2t^2}{2!}m^{//}(x_j) + \cdots\right)$$
$$= \frac{S_2(x_j;h)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2}\left(S_0(x_j;h)m(x_j) + S_1(x_j;h)m^{/}(x_j) + \frac{S_2(x_j;h)}{2!}m^{//}(x_j) + \cdots\right)$$
$$- \frac{S_1(x_j;h)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2}\left(S_1(x_j;h)m(x_j) + S_2(x_j;h)m^{/}(x_j) + \frac{S_3(x_j;h)}{2!}m^{//}(x_j) + \cdots\right)$$
$$= \frac{\left(S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2\right)m(x_j)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2} + \frac{\left(S_1(x_j;h)S_2(x_j;h) - S_1(x_j;h)S_2(x_j;h)\right)m^{/}(x_j)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2}$$
$$+ \frac{\left(S_2(x_j;h)^2 - S_{n,1}S_{n,3}\right)m^{//}(x_j)}{2\left(S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2\right)}$$
$$= m(x_j) + \frac{S_2(x_j;h)^2 - S_1(x_j;h)S_3(x_j;h)}{\left(S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2\right)}\frac{m^{//}(x_j)}{2} \quad (8)$$

3.2 The Bias of the Local Constant Regression Estimator $\overline{\mathbf{m}}_0(\mathbf{x_j})$

The bias of $\overline{m}_0(x_j)$ is given by

$$Bias(\overline{m}_0(x_j)) = \left(\frac{S_2(x_j;h)^2 - S_1(x_j;h)S_3(x_j;h)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2}\right)\frac{m^{//}(x_j)}{2}. \quad (9)$$

Therefore, the asymptotic expression of the bias of the local polynomial regression estimator $\overline{m}_0(x_j)$ can be obtained by making the assumption that, $x_i's$ are fixed uniform design points in $(0,1)$ (Masry, 1996; Eubank and Speckman, 1993). Therefore,

$$\sum_{i\in S}(x_i - x_j)^l k\left(\frac{x_i - x_j}{h}\right) = nh^{l+1}k_l + o(nh^{l+3}), \quad (10)$$

is almost uniform for $x \in (0,1)$ and $h \in H_n$, where $H_n = [C_1n^{-E_1}, C_2n^{-E_2}]$, $0 < E_2 < E_1 < 1$, and $C_1, C_2 > 0$.
This implies that,

$$S_0(x_j;h) = nh + o(nh^3), \qquad S_1(x_j;h) = o(nh^4), \qquad S_2(x_j;h) = nh^3k_2 + o(nh^5),$$
$$S_3(x_j;h) = nh^4k_3 + o(nh^6) \quad \text{and} \quad S_4(x_j;h) = nh^5k_4 + o(nh^7)$$

Such that,

$$S_0(x_j;h)S_2(x_j;h)-S_1(x_j;h)^2=\{nh+o(nh^3)\}\{nh^3k_2+o(nh^5)\}\{o(nh^4)\}^2$$
$$=n^2h^4k_2+o(n^2h^6). \quad (11)$$
$$S_2(x_j;h)^2-S_1(x_j;h)S_3(x_j;h)=\{nh^3k_2+o(nh^5)\}^2-\{o(nh^4)\}\{nh^4k_3+o(nh^6)\}$$
$$=n^2h^6{k_2}^2+o(n^2h^8). \quad (12)$$
$$Bias_{asy}(\bar{m}_0(x_j))=\frac{\left(n^2h^6{k_2}^2+o(n^2h^8)\right)}{\left(n^2h^4k_2+o(n^2h^6)\right)}\frac{m^{//}(x_j)}{2}$$
$$=\frac{1}{2}h^2k_2m^{//}(x_j) \quad (13)$$

3.3 The Variance of the Local Constant Regression Estimator $\bar{\mathbf{m}}_0(\mathbf{x}_j)$

The variance of the local polynomial regression estimator $\bar{m}_0(x_j)$ is given by

$$Var(\bar{m}_0(x_j))=Var\left\{\sum_{i\in S}w_i(x_j)y_i\right\}$$
$$=\sum_{i\in S}w_i^2(x_j)\,\sigma^2(x_i) \quad (14)$$

Where,

$$w_i^2(x_j)=\left\{\frac{\left(S_2(x_j;h)-S_1(x_j;h)(x_i-x_j)\right)}{S_0(x_j;h)S_2(x_j;h)-S_1(x_j;h)^2}K\left(\frac{x_i-x_j}{h}\right)\right\}^2.$$

Letting

$$w_i^2(x_j)=\left\{\frac{\left(S_2(x_j;h)-S_1(x_j;h)(x_i-x_j)\right)}{S_0(x_j;h)S_2(x_j;h)-S_1(x_j;h)^2}K\left(\frac{x_i-x_j}{h}\right)\right\}^2$$
$$\approx\left\{\frac{1}{nh}K\left(\frac{x_i-x_j}{h}\right)\frac{\left(n^2h^4k_2+o(n^2h^6)\right)}{\left(n^2h^4k_2+o(n^2h^6)\right)}\right\}^2$$
$$\approx\frac{1}{n^2h^2}K^2\left(\frac{x_i-x_j}{h}\right)$$

Then the asymptotic expression for the variance of $\bar{m}_0(x_j)$ is given by the expression of the form

$$Var_{asy}\left(\bar{m}_0(x_j)\right)=\frac{1}{nh}\sum_{i\in S}\left\{K^2\left(\frac{x_i-x_j}{h}\right)\sigma^2(x_i)\left(\frac{x_i-x_{i-1}}{h}\right)\right\}$$
$$=\frac{d_k}{nh}\sigma^2(x_j). \quad (15)$$

Where $d_k=\int K^2(t)dt$

3.4 The MSE of the Local Constant Regression Estimator $\bar{\mathbf{m}}_0(\mathbf{x}_j)$

Theorem I in Fan (1993) [7] allows that under condition (ii) gives,

$$MSE\left(\bar{m}_0(x_j)\right)=\left\{Bias\left(\bar{m}_0(x_j)\right)\right\}^2+Var\left(\bar{m}_0(x_j)\right)$$
$$=\left\{\left(\frac{S_2(x_j;h)^2-S_1(x_j;h)S_3(x_j;h)}{S_0(x_j;h)S_2(x_j;h)-S_1(x_j;h)^2}\right)\frac{m^{//}(x_j)}{2}\right\}^2+\sum_{i\in S}w_i^2(x_j)\,\sigma^2(x_i) \quad (16)$$

The asymptotic expression of the MSE of the local polynomial regression estimator $\bar{m}_0(x_j)$ is obtained using the asymptotic bias and asymptotic variance expressions of $\bar{m}_0(x_j)$. Thus,

$$MSE_{asy}\left(\bar{m}_0(x_j)\right)=\left\{\frac{1}{2}\,h^2k_2m^{//}(x_j)\right\}^2+\frac{d_k}{nh}\sigma^2(x_j) \quad (17)$$

Note that results for the local polynomial regression estimator of finite population total $\bar{m}_1(x_j)$ have been derived by Kikechi *et al.* (2017). [11]

3.5 The asymptotic relative efficiency of the estimators

The relative efficiency of two procedures is the ratio of their efficiencies, but it is often possible to use the asymptotic relative efficiency, defined as the limit of the relative efficiencies as the sample size grows, as the principal measure of comparison. Let $\bar{m}_0(x_j)$ be the local polynomial regression estimator in a finite population for P = 0 and $\bar{m}_1(x_j)$ be the local polynomial regression estimator in a finite population for P = 1 as studied by Kikechi *et al.* (2017). [11]

If $\bar{m}_0(x_j)$ and $\bar{m}_1(x_j)$ are both unbiased estimators of $m(x)$, then the relative efficiency of $\bar{m}_0(x_j)$ to $\bar{m}_1(x_j)$ is given by,

$$Eff\left(\bar{m}_0(x_j), \bar{m}_1(x_j)\right) = \frac{Var\left(\bar{m}_1(x_j)\right)}{Var\left(\bar{m}_0(x_j)\right)}. \qquad (18)$$

If $\bar{m}_0(x_j)$ and $\bar{m}_1(x_j)$ are both asymptotically unbiased estimators of $m(x)$, then the asymptotic relative efficiency of $\bar{m}_0(x_j)$ to $\bar{m}_1(x_j)$ is given by,

$$ARE\left(\bar{m}_0(x_j), \bar{m}_1(x_j)\right) = \lim_{n\to\infty} Eff\left(\bar{m}_0(x_j), \bar{m}_1(x_j)\right) = \lim_{n\to\infty} \frac{Var(\bar{m}_1(x_j))}{Var(\bar{m}_0(x_j))}. \qquad (19)$$

Therefore, the mean regression functions for the local polynomial regression estimators $\bar{m}_0(x_j)$ and $\bar{m}_1(x_j)$ are respectively given by;

$$\bar{m}_0(x_j) = \sum_{i\in S}\left\{\frac{\left(S_2(x_j;h) - S_1(x_j;h)(x_i - x_j)\right)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2} K\left(\frac{x_i - x_j}{h}\right) y_i\right\} \qquad (20)$$

$$\bar{m}_1(x_j) = \sum_{i\in S}\left\{\frac{\left(S_2(x_j;h) - S_1(x_j;h)(x_i - x_j)\right)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2} k\left(\frac{x_i - x_j}{h}\right) y_i\right\} + (x_i - x_j)\sum_{i\in S}\left\{\left(\frac{\left(S_0(x_j;h)(x_i - x_j) - S_1(x_j;h)\right)}{S_0(x_j;h)S_2(x_j;h) - S_1(x_j;h)^2}\right) k\left(\frac{x_i - x_j}{h}\right) y_i\right\} \qquad (21)$$

The variance of the local polynomial regression estimator $\bar{m}_0(x_j)$ is given by,

$$Var\left(\bar{m}_0(x_j)\right) = \sum_{i\in S} w_i^2(x_j)\,\sigma^2(x_i) \qquad (22)$$

The asymptotic expression for the variance of the local polynomial regression estimator $\bar{m}_0(x_j)$is estimated by,

$$Var_{asy}\left(\bar{m}_0(x_j)\right) = \frac{d_k}{nh}\sigma^2(x_j) \qquad (23)$$

The variance of the local polynomial regression estimator $\bar{m}_1(x_j)$ is given by,

$$Var\left(\bar{m}_1(x_j)\right) = \sum_{i\in S} w_i^2(x_j)\,\sigma^2(x_i) + (x_i - x_j)^2 \sum_{i\in S} w_i^{/2}(x_j)\sigma^2(x_i) \qquad (24)$$

The asymptotic expression for the variance of the local polynomial regression estimator $\bar{m}_1(x_j)$ is estimated by,

$$Var_{asy}\left(\bar{m}_1(x_j)\right) = \frac{d_k}{nh}\sigma^2(x_j). \qquad (25)$$

Note that in Kikechi e tal (2017), [11] $Var_{asy}\left(\bar{m}_{LL}(x_j)\right) = \frac{d_k}{nh}\sigma^2(x_j)$ and $Var_{asy}\left(\bar{m}_{NW}(x_j)\right) = \frac{d_k}{nh}\sigma^2(x_j)$

Thus the asymptotic relative efficiency of the local polynomial regression estimator $\bar{m}_0(x_j)$ to the local polynomial regression estimator $\bar{m}_1(x_j)$ derived by Kikechi *et al.* (2017) [11] is given by,

$$ARE\left(\bar{m}_0(x_j), \bar{m}_1(x_j)\right) = \lim_{n\to\infty} Eff\left(\bar{m}_0(x_j), \bar{m}_1(x_j)\right) = \lim_{n\to\infty}\left\{\frac{Var_{asy}\left(\bar{m}_1(x_j)\right)}{Var_{asy}\left(\bar{m}_0(x_j)\right)}\right\} = \lim_{n\to\infty}\frac{\frac{d_k}{nh}\sigma^2(x_j)}{\frac{d_k}{nh}\sigma^2(x_j)} = 1 \quad (26)$$

4. Conclusion

In this article the local constant regression estimator $\bar{m}_0(x_j)$ and the local linear regression estimator $\bar{m}_1(x_j)$ in finite populations have been studied in a model based framework. In particular, properties of the local constant regression estimator $\bar{m}_0(x_j)$ have been investigated. Analytically, variance comparisons are explored using the local constant regression estimator $\bar{m}_0(x_j)$ and the local linear regression estimator $\bar{m}_1(x_j)$ in which results indicate that the estimators are asymptotically equivalently efficient as shown by equation(26).

5. References

1. Breidt FJ, Opsomer JD. Local Polynomial Regression Estimation in Survey Sampling. Annals of statistics. 2000; 28:1026-1053.
2. Chambers RL, Dorfman AH, Wehrly TE. Bias robust estimation in finite populations using nonparametric calibration. J. Amer Statist Assoc. 1993; 88:268-277.
3. Cleveland WS. Robust Locally Weighted Regression and Smoothing Scatter Plots. J. Amer. Statist. Assoc. 1979; 74:829-836.
4. Cleveland WS, Devlin S. Locally Weighted Regression: an approach to regression analysis by local fitting. J. Amer. Statist. Assoc. 1988; 83:596-610.
5. Dorfman A. Nonparametric Regression for Estimating Totals in Finite Populations, Proceedings of the Section on Survey Research Methods. American Statistical Association, 1992, 622-625.
6. Fan J. Design Adaptive Nonparametric Regression. Journal of American Statistical Association. 1992; 87:998-1004.
7. Fan J. Local linear regression smoothers and their minimax efficiencies. Annals of Statistics. 1993; 21:196-216. https://doi.org/10.1214/aos/1176349022
8. Fan J, Gijbels I. Local Polynomial Modeling and its Applications. London: Chapman and Hall. 1996.
9. Gasser T, Muller HG. Kernel Estimation in Regression Functions. Smoothing Techniques for Curve Estimation, 1979, pp.23-68.
10. Horvitz DG, Thompson DJ. A Generalization of Sampling without Replacement from a Finite Universe. Journal of American Statistical Association. 1952; 47:663-685. https://doi.org/10.1080/01621459.1952.10483446
11. Kikechi CB, Simwa RO, Pokhariyal GP. On Local Linear Regression Estimation in Sampling Surveys. Far East Journal of Theoretical Statistics. 2017; 53(5):291-311. http://dx.doi.org/10.17654/TS053050291
12. Kikechi CB, Simwa RO, Pokhariyal GP. On Local Linear Regression Estimation of Finite Population Totals in Model Based Surveys. American Journal of Theoretical and Applied Statistics. 2018; 7(3):92-101. https://doi: 10.11648/j.ajtas.20180703.11
13. Kikechi CB, Simwa RO. On Comparison of Local Polynomial Regression Estimators for $P = 0$ and $P = 1$ in a Model Based Framework. International Journal of Statistics and Probability. 2018; 7(4):104-114. https://doi.org/10.5539/ijsp.v7n4p104
14. Kikechi CB, Simwa RO, Pokhariyal GP. On Prediction Based Robust Estimators of Finite Population Totals. International Journal of Statistics and Applied Mathematics. 2019; 4(6):101-107.
15. Montanari GE, Ranalli MG. Nonparametric Methods in Survey Sampling. In: Vinci, M., Monari, P., Mignani, S. and Montanari, A., Eds., New Developments in Classification and Data Analysis, Springer, Berlin. 2003, 203-210.
16. Montanari GE, Ranalli MG. Nonparametric Model Calibration Estimation in Survey Sampling. Journal of the American Statistical Association, 2005; 100:1429-1442. https://doi.org/10.1198/016214505000000141
17. Nadaraya EA. On Estimating Regression. Theory of Probability Applications. 1964; 10:186-190.
18. Odhiambo RO, Mwalili T. Nonparametric Regression for Finite Population Estimation. East African Journal of Science. 2000; II(2):107-112.
19. Priestley MB, Chao MT. Nonparametric Function Fitting. Journal of the Royal Statistical Society. 1972; B34:384-392.
20. Royall RM. On Finite Population Sampling under certain Linear Regression Models. Biometrika. 1970a; 57:377-387.
21. Royall RM. Finite Population Sampling-On Labels in Estimation. Journal of the Annals of Mathematical Statistics. 1970b; 41:1774-1779.
22. Royall RM. Linear Regression Models in Finite Population Sampling Theory Holt, Rinhart and Winston, Toronto, Canada, 1971; 54:499-513.
23. Ruppert D, Wand MP. Multivariate Locally Weighted Least Squares Regression. Annals of Statistics. 1994; 22:1346-1370. https://doi.org/10.1214/aos/1176325632
24. Sanchez-Borrego IR, Rueda M. A Predictive Estimator of Finite Population Mean Using Nonparametric Regression. Computational Statistics. 2009; 24:1-14. https://doi.org/10.1007/s00180-008-0140-x
25. Smith TM. The Foundations of Survey Sampling. Journal of Royal Statistical Society Association. 1976; 139(2):183-204.
26. Stone C. Consistent Nonparametric Regression. Annals of Statistics. 1977; 5:595-645.
27. Watson G. Smooth Regression Analysis. Sankhya Series A. 1964; 26:359-372.