International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2020; 5(1): 28-31

www.mathsjournal.com
Received: 09-11-2019
Accepted: 13-12-2019

Sultanov Bekzod Maqsud Ugli
PhD candidate of department of Geometry and Topology at National University of Uzbekistan, Uzbekistan

Ismailov Sherzodbek Shokirjon Ugli
Lecturer of department of Geometry and Topology National University of Uzbekistan, Uzbekistan

Corresponding Author:
Sultanov Bekzod Maqsud Ugli
PhD candidate of department of Geometry and Topology at National University of Uzbekistan, Uzbekistan

Cyclic surfaces in pseudo-euclidean space

Sultanov Bekzod Maqsud Ugli and Ismailov Sherzodbek Shokirjon Ugli

Abstract
Article observed concept of a cyclic surface defined in a Galilean space is generalized for a five-dimensional pseudo-Euclidean space of index two, and proved the existence of a full cyclic surface in pseudo-Euclidean spaces. The dimension of the smallest pseudo-Euclidean space is defined, where there exists a full cyclic surface.

Keywords: cyclic point, cyclic surface, full surface, galilean space, pseudo-euclidean space, isotropic space

1. Introduction

Theory of cyclic surfaces was introduced by Artikbaev A. in Galilean space [1]. When classifying surface points in Galilean space, points with indicatrix curve hyperbola are called cyclic points. Cyclic points are called surfaces with cyclic surfaces. Cyclic surfaces in the Galilean space were studied by Kurbanov E. [2]. The differential characteristics of cyclic surfaces in Galilean space are given in [3].
Aim of this article is to determine which pseudo-Euclidean space contains a cyclic surface.

2. Methods

We are given A_n an affine space and two $\vec{X}(x_1, x_2, x_3, ..., x_n)$, $\vec{Y}(y_1, y_2, y_3, ..., y_n)$ vectors in this space.

Definition 1. Scalar product of these vectors $\vec{X}, \vec{Y} \in A_n$ is as follows:

$$(\vec{X}, \vec{Y}) = -x_1y_1 - x_2y_2 - ... - x_ly_l + x_{l+1}y_{l+1} + ... + x_ny_n$$

affine space is identified; space is n - dimensional; l- indexed Pseudo-Euclidean space is denoted by: ${}^lR_n$ (l - the number of negative thresholds) [4].
The norm of vectors is the square root obtained from the scalar product of these vectors, i.e.

$$|\vec{X}| = \sqrt{(\vec{X}, \vec{X})}$$

Obviously, scalar product vector itself can be negative, positive, and zero.

a) $(\vec{X}, \vec{X}) > 0$ $|\vec{X}|$ - real number, $\vec{X}$ - spatial vector;

b) $(\vec{X}, \vec{X}) < 0$ $|\vec{X}|$ - abstract number. $\vec{X}$ - abstract vector;

c) $(\vec{X}, \vec{X}) = 0$ $|\vec{X}| \neq 0$, $\vec{X}$ - are called isotropic vectors.

Distance between two points $\overrightarrow{AB}$ equal to the vector norm. If so $A(x_1, x_2, x_3, x_4, x_5)$ $B(y_1, y_2, y_3, y_4, y_5)$

$$d_{AB} = |\overrightarrow{AB}| = \sqrt{-(y_1 - x_1)^2 - (y_2 - x_2)^2 - ... - (y_l - x_l)^2 + (y_{l+1} - x_{l+1})^2 + ... + (y_n - x_n)^2}$$

Let ${}^2R_5$ be a five-dimensional two-indexed pseudo-Euclidean space. Then we look at a part space $U(x,y,z,y,z) \subset {}^2R_5$. This part in space $\vec{X}(x_1,x_2,x_3,x_2,x_3)$ and $\vec{Y}(y_1,y_2,y_3,y_2,y_3)$ scalar product vector's

$$(\vec{X},\vec{Y})_1 = x_1y_1 + x_2y_2 + x_3y_3 - x_2y_2 - x_3y_3 = x_1y_1$$

Also, the distance between two points is calculated as follows $A(x_1,x_2,x_3,x_3,x_4)$ and $B(y_1,y_2,y_3,y_2,y_3)$.

$$AB_1 = |\overrightarrow{AB}| = \sqrt{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2-(y_2-x_2)^2-(y_3-x_3)^2} = |y_1-x_1|$$

If $AB_1 = |\overrightarrow{AB}| = 0$ is so, $y_1 = x_1$ but these points do not intersect, the distance between them

$$AB_2 = |\overrightarrow{AB}| = \sqrt{(y_2-x_2)^2+(y_3-x_3)^2}$$

are equal.

$U(x,y,z,y,z) \subset {}^2R_5$ in the part of space $x_2 = x_4$, $x_3 = x_5$ the equality

$$\begin{cases} \vec{e}_1 = \vec{i} \\ \dfrac{\vec{e}_2+\vec{e}_4}{2} = \vec{j} \\ \dfrac{\vec{e}_3+\vec{e}_5}{2} = \vec{k} \end{cases} \qquad (1)$$

If we make this change $\{\vec{i},\vec{j},\vec{k}\}$, the vector forms the base vectors in three-dimensional space. The consequential space is a Galilean space.

Page Size and Layout

Look at the surface in this space:

Say $U(x,y,z,x,y) \subset {}^2R_5$ in a part of space $\vec{r} = \vec{r}(u,v)$ let the surface be given in the form of a vector form. Vector formulas for any two-dimensional surface can be written as follows.

$$\vec{r} = \vec{r}(u,v) = u\vec{e}_1 + x_2(u,v)\vec{e}_2 + x_3(u,v)\vec{e}_3 + x_2(u,v)\vec{e}_4 + x_3(u,v)\vec{e}_5 \qquad (2)$$

If we transformation (1), then the equation

$$\bar{r} = \bar{r}(u,v) = u\vec{i} + y(u,v)\vec{j} + z(u,v)\vec{k} \qquad (3)$$

takes shape. Here $y(u,v) = \frac{1}{2}(x_2(u,v)+x_2(u,v)) = x_2(u,v);\ z(u,v) = \frac{1}{2}(x_3(u,v)+x_3(u,v)) = x_3(u,v)$ are equal. This gives a surface equation defined as a vector in Galilean space.

Thus, ${}^2R_5 \supset \Gamma_3$ this phase has a dense surface.

Say ${}^2R_5$ let the space be a regular surface defined by equation (2).

Theorem 1. ${}^2R_5$ with this equation in space

$$\begin{cases} x_1(u,v) = u \\ x_2(u,v) = F_1(u)v + F_3(u) \\ x_3(u,v) = F_2(u)v + F_4(u) \\ x_4(u,v) = F_1(u)v + F_3(u) \\ x_5(u,v) = F_2(u)v + F_4(u) \end{cases} \qquad (4)$$

Any given surface is a cyclic surface.

Proof. The surface in question is included in this space because the three-dimensional part belongs to the space $\{\vec{i},\vec{j},\vec{k}\}$ the main use. The second principal form for the surface to be a point of speed N coefficient 0 Gat should be the best and sufficient [1] i.e

$$N = \frac{y_{vv}z_v - z_{vv}y_v}{\sqrt{y_v^2+z_v^2}} = 0, \quad M = \frac{y_{uv}z_v - z_{uv}y_v}{\sqrt{y_v^2+z_v^2}} \neq 0$$

find the surface equation.

We will see the general form of the function for finding the surface equation $y(u,v)$, $z(u,v)$. From the foregoing, satisfying this condition

$y_{vv}z_v - z_{vv}y_v = 0$ and $y_{uv}z_v - z_{uv}y_v \neq 0$ if $z^2{}_v(\frac{y_v}{z_v})_v = 0$ and $z^2{}_v(\frac{y_v}{z_v})_u \neq 0$

We have. This relationship shows $\frac{y_v}{z_v}$ the relationship is and only u that is, depending on the parameter $\frac{y_v}{z_v} = F(u)$. From here y_v, z_v we define

$$y_v = f_1(u)g(u),\ z_v = f_2(u)g(u) \quad \text{here } f_1, f_2, g \in C^2$$

From this $y(u,v)$ and $z(u,v)$ when we find the functions, we obtain the following equation:

$$y = F_1(u)v + F_3(u),\ z = F_2(u)v + F_4(u)$$

Based on the results, we have the following equation:

$$\vec{r} = \vec{r}(u,v) = u\vec{i} + (F_1(u)v + F_3(u))\vec{j} + (F_2(u)v + F_4(u))\vec{k}$$

This cycle is a general view of the surface. From this surface is a cyclic surface

$$\vec{r} = \vec{r}(u,v) = ue_1 + (F_1(u)v+F_3(u))\vec{e}_2 + (F_2(u)v+F_4(u))\vec{e}_3 + (F_1(u)v+F_3(u))\vec{e}_4 + (F_2(u)v+F_4(u))\vec{e}_5$$

will come from. Thus, the theorem is completely proved.

Result 1. If surface represented by equation (4) is a surface of the surface, the following equation for the surface

$$x_2^2(u,v) - x_4^2(u,v) + x_3^2(u,v) - x_5^2(u,v) = 0$$

capacity.

Note. A double surface is a linear surface, and when we cut it with a special plane, a straight line forms on the surface. That is, the creators have a linear surface parallel to the special plane, and have the following equation:

$$\begin{cases} r_u^2 = 1 \\ r_v^2 = \varphi(u) \end{cases} \tag{6}$$

Cyclic surface in this theorem ${}^2R_5$ is in part three-dimensional space. We ${}^2R_5$ find that there are cyclic surfaces that do not fit into the four-dimensional space of space. For this, we refer to two-dimensional cyclic surfaces that do not belong to the four-dimensional space of space.

To us ${}^2R_5$ to get the two-dimensional equation of the full surface in space:

$$\vec{r} = \vec{r}(u,v) = u\vec{e}_1 + x_1(u,v)\vec{e}_2 + x_3(u,v)\vec{e}_3 + x_3(u,v)\vec{e}_3 + x_4(u,v)\vec{e}_5 \tag{7}$$

Theorem 2. Pseudo-Euclidean space ${}^2R_5$ there is a two-dimensional of full cyclic surface.

Proof: Surface defined by equation (7) must satisfy equation (6) to be from surface of cyclic points (6) to equality:

$$\begin{cases} x^2_{1_u}(u,v) + x^2_{2_u}(u,v) - x^2_{3_u}(u,v) - x^2_{4_u}(u,v) = 0 \\ x^2_{1_v}(u,v) + x^2_{2_v}(u,v) - x^2_{3_v}(u,v) - x^2_{4_v}(u,v) = \varphi(u) \end{cases} \tag{8}$$

come from $r_v = (0, \varphi_{1_v}(u), \varphi_{2_v}(u), \varphi_{3_v}(u), \varphi_{4_v}(u))$ integrate v next

$$r = (u, v\varphi_1(u), v\varphi_2(u), v\varphi_3(u), v\varphi_4(u))$$

So be it. Substituting this into equation (8), we can obtain following equation:

$$v^2\varphi^2_{1_u}(u) + v^2\varphi^2_{2_u}(u) = v^2\varphi^2_{3_u}(u) + v^2\varphi^2_{4_u}(u)$$

From this

$$\varphi^2_{1_u}(u) + \varphi^2_{2_u}(u) = \varphi^2_{3_u}(u) + \varphi^2_{4_u}(u) \tag{9}$$

satisfaction our equality (9) $\varphi_1(u), \varphi_2(u), \varphi_3(u), \varphi_4(u)$ turns out to be cyclic surface. The equation:

$$\begin{cases} x_1(u,v) = u \\ x_2(u,v) = \varphi_1(u)v \\ x_3(u,v) = \varphi_2(u)v \\ x_4(u,v) = \varphi_3(u)v \\ x_5(u,v) = \varphi_4(u)v \end{cases}$$

Example 1

$$\begin{cases} \varphi_{1_u}(u,v) = \varphi(u)_u \cos u \\ \varphi_{2_u}(u,v) = \varphi(u)_u \cdot \sqrt{\cos 2u} \cdot shu \\ \varphi_{3_u}(u,v) = \varphi(u)_u \sin u \\ \varphi_{4_u}(u,v) = \varphi(u)_u \cdot \sqrt{\cos 2u} \cdot chu \end{cases}$$

If we choose this equality $\varphi^2_{1_u}(u) + \varphi^2_{2_u}(u) = \varphi^2_{3_u}(u) + \varphi^2_{4_u}(u)$ satisfies the condition:

$$\varphi^2_{1_u}(u) - \varphi^2_{3_u}(u) + \varphi^2_{2_u}(u) - \varphi^2_{4_u}(u) =$$

$$= \varphi^2(u)_u \cdot \sin^2 u - \varphi^2(u)_u \cdot \cos^2 u + \cos 2u \cdot ch^2 u \cdot \varphi^2(u)_u - \cos 2u \cdot sh^2 u \cdot \varphi^2(u)_u = \varphi^2(u)(-\cos 2u + \cos 2u) = \varphi^2(u)_u \cdot 0 = 0$$

Based on this, the surface is the equation of our surface

$$\begin{cases} x_1(u,v) = u \\ x_2(u,v) = \int_0^u (\varphi(t)_u \cos t)dt \cdot v \\ x_3(u,v) = \int_0^u (\varphi(t)_u \cdot \sqrt{\cos 2t} \cdot sht)dt \cdot v \\ x_4(u,v) = \int_0^u (\varphi(t)_u \sin t)dt \cdot v \\ x_5(u,v) = \int_0^u (\varphi(t)_u \cdot \sqrt{\cos 2t} \cdot cht)dt \cdot v \end{cases}$$

So will look.

We investigate the presence of full cyclic surfaces in Pseudo-Euclidean ${}^2R_5$ space in a part of space. For example, look at the following sections: ${}^1R_3, R_3^2, {}^1R_4, {}^2R_4, {}^1R_5$.

Theorem 3. There is a two – dimensional full surface in space ${}^1R_4$.

Proof: ${}^1R_4$ Two-dimensional surfaces in space must be equidistant (6) to be full surfaces. (6) condition

$$\begin{cases} x^2_{1_u}(u,v) + x^2_{2_u}(u,v) = x^2_{3_u}(u,v) \\ r_v^2 = \varphi(u) \end{cases} \tag{10}$$

$r_v = (0, \varphi_{1_v}(u), \varphi_{2_v}(u), \varphi_{3_v}(u))$ as follows let's integrate, $r = (u, v\varphi_1(u), v\varphi_2(u), v\varphi_3(u))$ will be. If we establish it in equation (10), we can have

$$v^2\varphi^2_{1_u}(u) + v^2\varphi^2_{2_u}(u) = v^2\varphi^2_{3_u}(u)$$

From this

$$\varphi^2_{1_u}(u) + \varphi^2_{2_u}(u) = \varphi^2_{3_u}(u) \tag{11}$$

Equality is forming. $\varphi^2{}_{1_u}(u),\ \varphi^2{}_{2_u}(u)$ We select functions as follows:

$$\begin{cases}\varphi_{1_u}(u)=\varphi_{3_u}(u)\cos u\\ \varphi_{2_u}(u)=\varphi_{3_u}(u)\sin u\end{cases},$$

$$\begin{cases}\varphi_1(u)=\int_0^u\varphi_{3_u}(t)\cos t\\ \varphi_1(u)=\int_0^u\varphi_{3_u}(t)\sin t\end{cases}$$

Here ${}^1R_4$, the equation of a cyclic surface satisfying the condition in space (6) has the following form

$$\begin{cases}x_1(u,v)=u\\ x_2(u,v)=v\cdot\int_0^u(\varphi_{3_u}(t)\cos t)dt\\ x_3(u)=v\cdot\int_0^u(\varphi_{3_u}(t)\sin t)dt\\ x_4(u)=v\cdot\varphi_3(u)\end{cases}$$

Here $\varphi_3(u)\in C^2$.

Example 2. ${}^1R_4$ a surface in space with the following equation:

$$\begin{cases}x_1(u,v)=u\\ x_2(u,v)=v\cdot(\frac{1}{2}e^u\cos u+\frac{1}{2}e^u\sin u-\frac{1}{2})\\ x_3(u)=v\cdot(\frac{1}{2}e^u\cos u-\frac{1}{2}e^u\sin u+\frac{1}{2})\\ x_4(u)=v\cdot e^u\end{cases}$$

this is an example of a cyclic surface.

${}^2R_4$ space ${}^1R_4$ space can be replaced by coordinate axes, that is, condition (10) is as follows:

$$\begin{cases}x^2{}_{1_u}(u,v)=x^2{}_{2_u}(u,v)+x^2{}_{3_u}(u,v)\\ r_v^2=\varphi(u)\end{cases}$$

This means that in space there is ${}^2R_4$ the of a full cyclic surface.

Result 2. As it is ${}^1R_4\subset{}^1R_5$ there is a cyclic surface ${}^1R_5$.

When we lookout at space ${}^1R_3$, surface for point in this tangent plane should be Galilean plane ${}^1R_3$. So in space there is a cyclic point ${}^1R_3$.

Theorem 4. a) There is no cyclic surface in space ${}^1R_3$.

Proof: Since the entire surface of the surface is a tangent plane, it follows that F the surface is plane. A plane is a trivial state of a surface. It does not match the full cyclic surface conditions we are looking for. Therefore, we assume that there is no cyclic surface in this space.

b) There is no cyclic surface in isotropic space R_3^2

Proof: The isotropic space R_3^2 is space of parts R_4^1 of Minkowski, and in this space the Galileo plane $\vec{n}(0,0,1)$ is a plane parallel to the vector. The Galilean plane is the plane of the points, and the asymptotic $\vec{n}(0,0,1)$ is parallel to the vector, there are cyclic points, such as lines of cyclic points, but a set of points represents a cylindrical surface. The cylindrical surface is not a cyclic surface. This means that in space R_3^2 there is no cyclic surface.

3. Conclusion

Research results show that - ${}^lR_n$ should be $n\geq 4$ and $l\geq 1$ enough to have a full cyclic surface in space.

4. References

1. Artikbaev A, Sokolov DD. Geometry as a whole in flat space-time. Tashkent, Fan, 1991, pp.180.
2. Kurbanov EK. Cyclic surfaces of Galilean space Uzbekistan mathematics journal. 2001; 2:51-57.
3. Sultanov BM. The existence of a cyclic surface with respect to a given function of full curvature Bulletin National University of Uzbekistan. 2017; 2(2):201-204.
4. Rosenfeld BA. Non-Euclidean spaces. Publishing house Science, 1969, pp.548.