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Effects of some physical parameters on the fluid flow through a sandy soil

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Abstract
Effects of some physical parameters on the fluid flow through a sandy soil were investigated. The dimensional governing equations were reduced to non-dimensional form with the use of some dimensionless parameters which resulted into second order partial differential equations. These equations were then reduced to ordinary differential equation with the use of perturbation method and afterward solved analytically. Effects of the solar radiation parameter, internal heat generation parameter, thermal Grashof number and the Prandtl number were examined on the velocity of the fluid pass a sandy soil. The flow of the fluid was also investigated when the thermal conductivity and the permeability of the soil varied. The numerical results of these physical parameters were computed using Matlab and displayed on graphs. The increasing solar radiation, internal heat generation and thermal Grashof number speed up the rate of flow of fluid through the sandy soil, whereas the Prandtl number decreased the velocity at the boundary layer.

Keywords: Internal heat generation, perturbation method, Prandtl number, sandy soil, solar radiation, thermal Grashof number

1. Introduction

The study of heat transfer and fluid dynamics generally has attracted many researchers over the years because of its vital role in many areas which includes science, engineering and technology. Noran *et al* [1] worked on the effects of radiation and chemical reaction on magnetohydrodynamics (MHD) flow past an exponentially stretching sheet with heat sink. Their results show that the reaction rate parameter influenced the concentration profiles considerably while the concentration thickness of boundary layer decreases as reaction rate parameter increases. Moreover, Kho *et al* [2] researched on effect of thermal radiation on magnetohydrodynamics (MHD) flow and heat transfer analysis of Williamson nano fluid past over a stretching sheet with constant wall temperature. The work established that the rate of heat transfer is higher for Williamson nano fluid compared to the classical viscous fluid. Furthermore Akinpelu *et al* [3] considered the effects of some physical parameters on ground temperature with time-dependent suction velocity in the presence of internal heat generation. Reddy *et al* [4] studied a free convection heat and mass transfer flow of a fluid that is chemically reactive and radiation absorption in an aligned magnetic field. Akinpelu *et al* [5] studied variations in ground temperature in the presence of radiative heat flux and spatial dependent soil thermo physical property. Rajesh and Varma [6] researched on the effects of radiation and mass transfer on MHD flow past an exponentially accelerated vertical plate with variable temperature. This study therefore centered on the effects of some physical parameters on the fluid flow through a sandy soil.

2. Mathematical Analysis

A flow of an unsteady two dimensional viscous incompressible fluid through a sandy soil (porous medium) was considered. The flow is taken to be relatively large (infinite) along the horizontal axis (i.e. $y' - axis$) where the vertical axis which is normal to it is considered to be inside the soil (i.e. $z' - axis$). So, the flow field is a function of z' and t' alone [7], with a Dirichlet boundary condition. The soil is also taken to be an optically thin environment to allow the passage of the fluid flow through it. The radiation is directly towards the soil in a direction along the gravity. Under the assumptions, the governing equations under the usual Boussinesq's approximation became:

Continuity equation

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

Momentum Equation

$$\frac{\partial v'}{\partial t'} + w'\frac{\partial v'}{\partial z'} = \vartheta\frac{\partial^2 v'}{\partial z'^2} + g\beta\left(T' - T'_\infty\right) - \frac{w}{K'}\varphi v' \tag{2}$$

Energy equation

$$\frac{\partial T'}{\partial t'} + w'\frac{\partial T'}{\partial z'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial z'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial z} + \frac{Q_0(T' - T'_\infty)}{\rho C_p} \tag{3}$$

Subject to

$$v' = V'_p,\ T' = T'_w \text{ at } z = 0 \tag{4}$$

$$v' \to V'_\infty,\ T' \to T'_\infty \text{ as } z \to \infty \tag{5}$$

Where, z' is the dimensional depth of the soil which is perpendicular to y'. v' is the dimensional velocity of the flow along z' axis. Then t' and w' are the dimensional time and the suction velocity respectively. g, β, φ and K' are the acceleration due to gravity, volumetric coefficient of thermal expansion, porosity and dimensional permeability of the soil. T', T'_w and T'_∞ are the temperature, the wall temperature and the free stream temperature respectively. ρ, C_p, k and q'_r are density, specific heat capacity, thermal conductivity and the Radiative heat flux respectively.
Using the dimensionless parameters as used by Mohammed [7] and Seshaiah *et al* [8],

$$t = \frac{t' w_0^2}{w},\ z = \frac{w_0 z'}{w},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ \omega = \frac{w\omega'}{w_0^2},\ v = \frac{v'}{V'_\infty} \tag{6}$$

As used by Nwaigwe [9], the suction velocity given as

$$w' = -w_0(1 + \varepsilon A e^{i\omega' t'}) \tag{7}$$

Where w_0, A and ω are the initial suction velocity, the suction parameter and the frequency of oscillation respectively. A and ε are very small such that $\varepsilon A << 1$.
Also, as used by Krishna and Reddy [10] the heat flux is given as

$$\frac{\partial q'_r}{\partial z'} = 4\alpha^2(T' - T'_\infty) \tag{8}$$

α is the absorption coefficient.

Moreover, as used by Akinpelu *et al* [5], the time-dependent thermal conductivity is given as

$$k = k_0(1 + st) \tag{9}$$

s, t and k_0 Being the variable thermal conductivity parameter, time and the constant thermal conductivity respectively.

Equations (2)-(3) become:

$$\frac{\partial v}{\partial t}-(1+\varepsilon Ae^{i\omega t})\frac{\partial v}{\partial z}=\frac{\partial^2 v}{\partial z^2}+Gr\theta-\frac{v}{K} \tag{10}$$

$$\frac{\partial \theta}{\partial t}-(1+\varepsilon Ae^{i\omega t})\frac{\partial \theta}{\partial z}=\frac{(1+st)}{P_r}\left(\frac{\partial^2 \theta}{\partial z^2}\right)-R^2+Q\theta \tag{11}$$

Subject to:

$$v=V_p\,,\ \theta=1 \text{ at } z=0 \tag{12}$$

$$v\to 1\,,\ \theta\to 0 \text{ as } z\to\infty \tag{13}$$

Where,

$Gr=\dfrac{wg\beta(T'_w-T'_\infty)}{w_0^2V'_\infty}$ (the thermal Grashof number)

$K=\dfrac{K'w_0^2}{w^2\varphi}$ (the permeability of the soil type)

$P_r=\dfrac{w\rho C_p}{k_0}$ (the Prandtl number), $R^2=\dfrac{4\alpha^2\theta w}{w_0^2}$ (the radiation parameter), and

$Q=\dfrac{Q_0 w}{\rho C_p w_0^2}$ (the internal heat generation parameter)

3. Method of Solution

Equations (10)-(11) are coupled second order partial differential equation. These can be reduce to ordinary differential equation using regular perturbation method and then solved analytically. The assumed solutions are given are as follow:

$$v(z,t)=v_0(z)+\varepsilon e^{i\omega t}v_1(z) \tag{14}$$

$$\theta(z,t)=\theta_0(z)+\varepsilon e^{i\omega t}\theta_1(z) \tag{15}$$

Substituting equation (12) and its derivatives into (9), and neglecting the higher order terms $o(\varepsilon)^2$ with further simplifications, we obtain

$$v_0''+v_0'-\frac{1}{K}v_0=-Gr\theta_0 \tag{16}$$

$$v_1''+v_1'-\left(i\omega+\frac{1}{K}\right)v_1=-Av_0'-Gr\theta_1 \tag{17}$$

$$(1+st)\theta_0''+P_r\theta_0'+P_rQ\theta_0=P_rR^2 \tag{18}$$

$$(1+st)\theta_1''+P_r\theta_1'+(P_rQ-P_ri\omega)\theta_1=-P_rA\theta_0 \quad (19)$$

Where the primes represent ordinary differentiation with respect to z.
Using the assumed solutions (14) – (15), corresponding boundary conditions (12) and (13) can be rewritten as follows:

$$v_0=V_p, v_1=0,\ \theta_0=1,\ \theta_1=0 \text{ on } z=0 \quad (20)$$

$$v_0\to 1,\ v_1\to 0,\ \theta_0\to 0,\ \theta_1\to 0 \text{ as } z\to\infty \quad (21)$$

Solving equations (16)-(19) subject to the boundary conditions (20)-(21), the transient
With the assumed solution, the ground temperature becomes,

$$\begin{aligned} v &= C_8e^{m_5z}+C_9e^{m_6z}+C_{10}+C_{11}e^{m_1z}+C_{12}e^{m_2z} \\ &+\varepsilon e^{i\omega t}(C_{13}e^{m_7z}+C_{14}e^{m_8z}+C_{15}e^{m_1z}+C_{16}e^{m_2z}+C_{17}e^{m_3z} \\ &+C_{18}e^{m_4z}+C_{19}e^{m_5z}+C_{20}e^{m_6z}) \end{aligned} \quad (22)$$

$$\theta=C_1e^{m_1z}+C_2e^{m_2z}+C_3+\varepsilon e^{i\omega t}\left(C_4e^{m_3z}+C_5e^{m_4z}+C_6e^{m_1z}+C_7e^{m_2z}\right) \quad (23)$$

Where,

$$m_1=-\frac{P_r}{2(1+st)}+\sqrt{\frac{P_r^2}{4(1+st)^2}-\frac{P_rQ}{1+st}},\ m_2=-\left(\frac{P_r}{2(1+st)}+\sqrt{\frac{P_r^2}{4(1+st)^2}-\frac{P_rQ}{1+st}}\right)$$

$$m_3=-\frac{P_r}{2(1+st)}+\sqrt{\frac{P_r^2}{4(1+st)^2}-\frac{P_rQ}{1+st}+\frac{P_ri\omega}{1+st}}$$

$$m_4=-\left(\frac{P_r}{2(1+st)}+\sqrt{\frac{P_r^2}{4(1+st)^2}-\frac{P_rQ}{1+st}+\frac{P_ri\omega}{1+st}}\right)$$

$$m_5=-\frac{1}{2}+\sqrt{\frac{1}{K}+\frac{1}{4}} \qquad m_6=-\left(\frac{1}{2}+\sqrt{\frac{1}{K}+\frac{1}{4}}\right) \qquad m_7=-\frac{1}{2}+\sqrt{\frac{1}{K}+\frac{1}{4}+i\omega}$$

$$m_8=-\left(\frac{1}{2}+\sqrt{\frac{1}{K}+\frac{1}{4}+i\omega}\right) \qquad C_1=-C_3e^{-m_1z} \qquad C_2=1+C_3\left(e^{-m_1z}-1\right)$$

$$C_3=R^2/Q \qquad C_4=\frac{-C_6e^{m_1z}}{e^{m_3z}} \qquad C_5=-(C_4+C_6+C_7)$$

$$C_6=\frac{-P_rAm_1C_1}{m_1^2+stm_1^2+P_rm_1+P_rQ-P_ri\omega} \qquad C_7=\frac{-P_rAm_2C_2}{m_2^2+stm_2^2+P_rm_2+P_rQ-P_ri\omega}$$

$$C_8 = \frac{1 - C_{10} - C_{11} e^{m_1 z}}{e^{m_5 z}} \qquad C_9 = V_p - (C_8 + C_{10} + C_{11} + C_{12})$$

$$C_{10} = \frac{GrC_3}{\frac{1}{K}} \qquad C_{11} = \frac{-GrC_1}{m_1^2 + m_1 - \frac{1}{K}} \qquad C_{12} = \frac{-GrC_2}{m_2^2 + m_2 - \frac{1}{K}}$$

$$C_{15} = \frac{-(Am_1 C_{11} + GrC_6)}{m_1^2 + m_1 - \left(\frac{1}{K} + i\omega\right)} \qquad C_{16} = \frac{-(Am_2 C_{12} + GrC_7)}{m_2^2 + m_2 - \left(\frac{1}{K} + i\omega\right)}$$

$$C_{17} = \frac{-GrC_4}{m_3^2 + m_3 - \left(\frac{1}{K} + i\omega\right)} \qquad C_{18} = \frac{-GrC_5}{m_4^2 + m_4 - \left(\frac{1}{K} + i\omega\right)}$$

$$C_{19} = \frac{-Am_5 C_8}{m_5^2 + m_5 - \left(\frac{1}{K} + i\omega\right)} \qquad C_{20} = \frac{-Am_6 C_9}{m_6^2 + m_6 - \left(\frac{1}{K} + i\omega\right)}$$

$$C_{13} = \frac{-(C_{15} e^{m_1 z} + C_{17} e^{m_3 z} + C_{19} e^{m_5 z})}{e^{m_7 z}}$$

$$C_{14} = -(C_{13} + C_{15} + C_{16} + C_{17} + C_{18} + C_{19} + C_{20})$$

4. Results and Discussion

The numerical result of the transient velocity profile was computed using Matlab. For better illustrations, these results were later displayed on graphs. The Solar radiation parameter (R), the internal heat generation parameter (Q), the thermal Grashof number (Gr) and the Prandtl number (Pr) were examined on the velocity of the sandy soil. Also, the effects of an increasing permeability of the soil and its thermal conductivity were also investigated.

Tables 1 and 2 below show the values of the thermal conductivity and permeability of the sandy soil.

Table 1: Thermal conductivity of sandy soil

Description	Thermal Conductivity (Btu/ft hr ^{0}F)
Sand	0.44

Gary [11]

Table 2: Average permeability of sandy soil

Description	Permeability (cm/hour)
Sand	5.00

Retrieved at: [12]

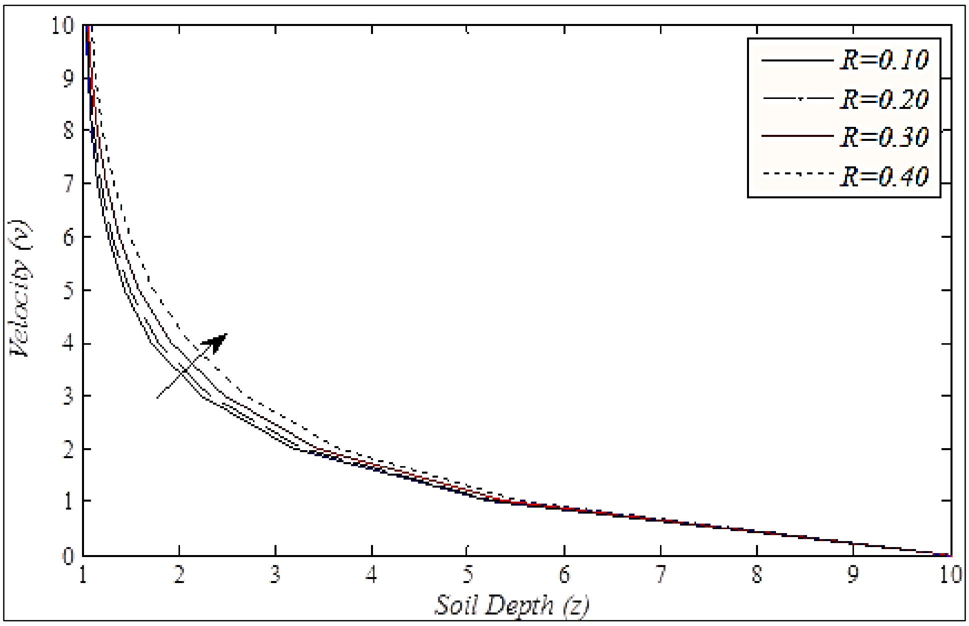

Fig 1: Velocity profile for various values of Radiation parameter for increasing depth of sandy soil.

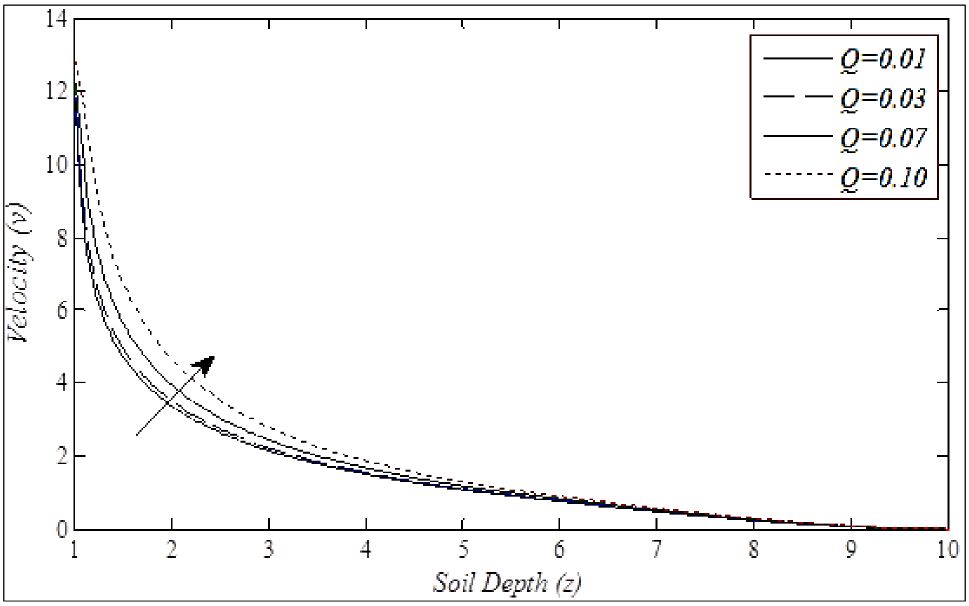

Fig 2: Velocity profile for various values of internal heat generation parameter for increasing depth of sandy soil

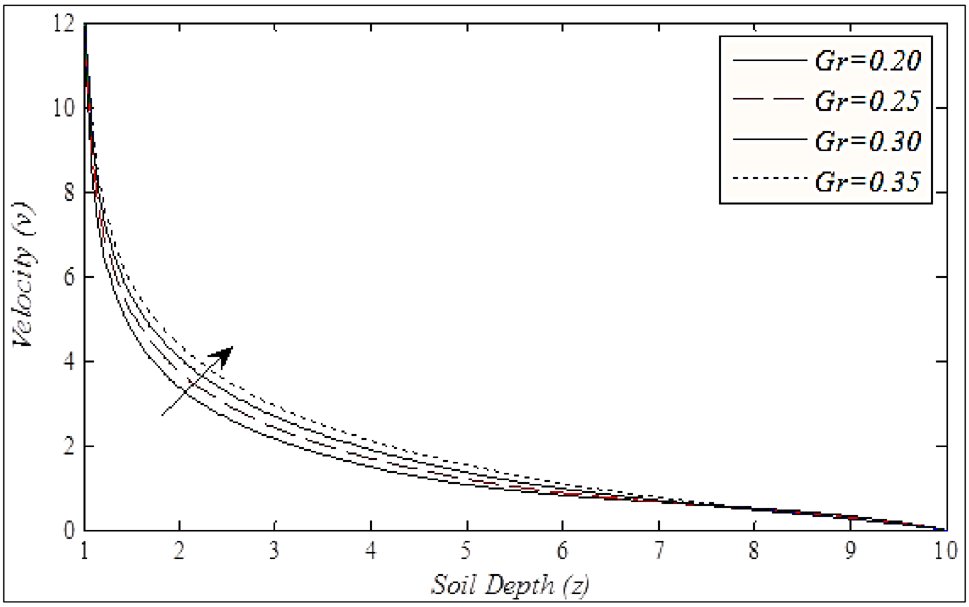

Fig 3: Velocity profile for various values of thermal Grashof number for increasing depth of sandy soil

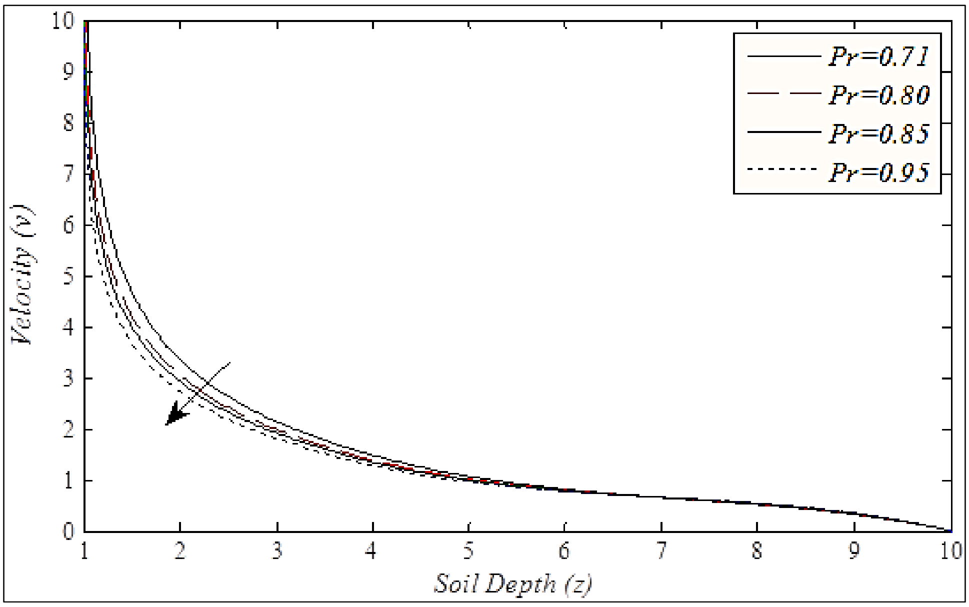

Fig 4: Velocity profile for various values of Prandtl number for increasing depth of sandy soil

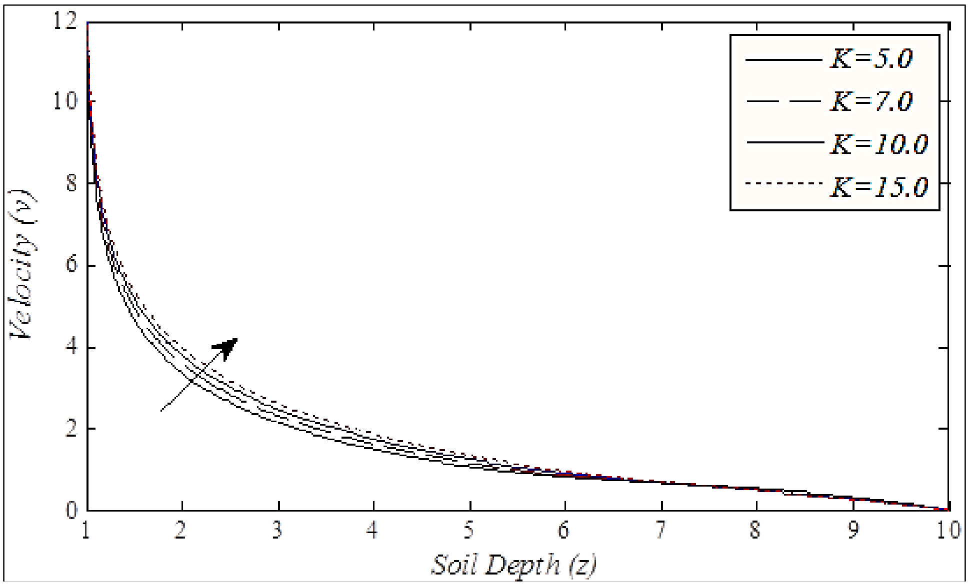

Fig 5: Velocity profile for various values of Permeability of soil

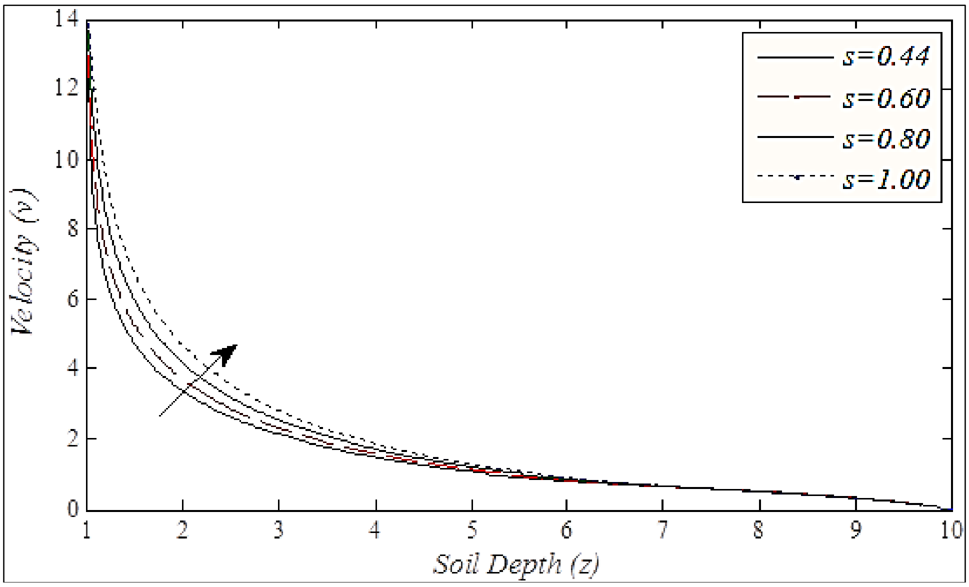

Fig 6: Velocity profile for various values of thermal conductivity of soil

Figure 1 depicts the velocity profile for different values of radiation parameter for sandy soil. The result shows that as the solar radiation increases, the velocity of sandy soil increase as well. Figure 2 represents the velocity profile for different values of internal heat generation parameter for the sandy soil. The increasing internal heat also speeds up the rate of flow through sandy soil. Figure 3 is the velocity profile of the thermal Grashof number at increasing depth of sand. When the thermal buoyancy force increases, the rate of flow of the fluid through the sandy soil also increases. In figure 4, as expected, the increasing value of the Prandtl number decreased the velocity of the fluid flow at the boundary layer of the sand. Moreover, the results in figures 5 and 6 show that when the permeability and the thermal conductivity of any soil increase, then the rate of flow of fluid is speed up.

5. Conclusion

Effects of some physical parameters on the fluid flow through a sandy soil have been considered. It is therefore evident that the solar radiation, the internal heat and the thermal buoyancy force have significant effects on the fluid flow. At any increase in these quantities, the rate of flow of fluid through a sandy soil is speed up. Moreover, the larger the value of thermal conductivity and permeability of any soil the faster the rate of flow of fluid through it.

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