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Establishing a dynamic model based on stability theory of differential equations to study the predation relationship between biological populations and MATLAB implementation

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Abstract
During the development of biological populations, differential equation stability models are often used to analyze the effects of predation relationships between populations on population numbers. This paper uses MALAB to plot the stability of the phenomena described by the model, verifying its equilibrium point and the shape of the trajectory. In the same time, the model has been successfully promoted, providing effective rationalization suggestions for practical production applications.

Keywords: Stability theory of differential equations, MATLAB interspecific relationship predator model

Introduction
In real life, we describe the evolution of certain characteristics of actual objects over time or space, such as the evolution of population, weak meat and strong food among species, and sustainable development of renewable resources. Differential equation can be listed according to the inherent laws of the research object. We can use differential equations to analyze its changing laws, predict its future trends, and study its control methods. However, most of the differential equations cannot find its analytical solution, and for some practical problems, we do not necessarily require the solution of the differential equation to find the state of each moment, we only study the change trend of the whole dynamic process when the time is long enough. For example, under what conditions it will tend to a specific value. A simple and practical conclusion is drawn from the stability analysis of the equilibrium state.

2. Two types of models for population changes
2.1 exponential growth model
In the ideal case, the population growth model is based on an exponential growth model. Under this model, there is no stable stage for infinite population growth, which is difficult to exist in real life.

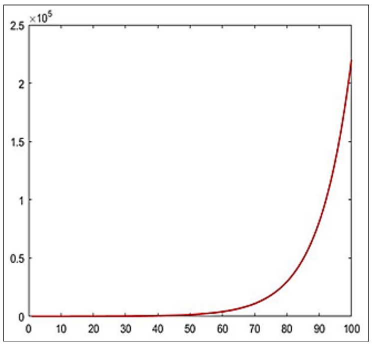

Fig 1: Exponential growth

2.2 Logistic growth model

However, in a natural environment, when only one biological group exists, the environmental resources are limited. The classic model describing the population change process is the logistic model, that is,

$$\dot{x}(t) = rx\left(1 - \frac{x}{N}\right) \quad (1)$$

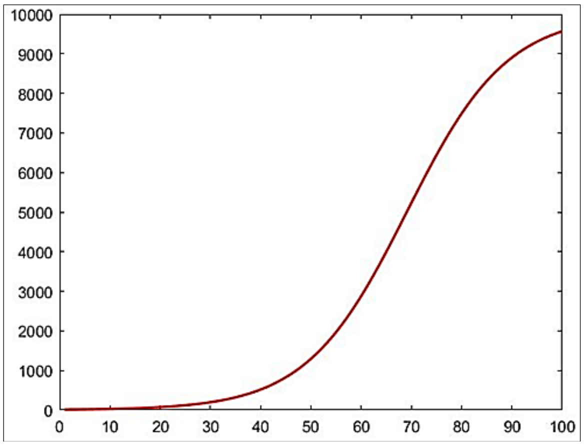

Fig 2: Logistic model

$\dot{x}(t)$ is the number of populations at time t, r is the inherent growth rate, and N is the maximum number of populations allowed by environmental resources? In fact, it can also be obtained directly from the logistic model equation.$x_0 = N$ is the stable equilibrium point, when $t \to\infty$,$\dot{x}(t)$ tends to N. This is an obvious result from the meaning of the model itself.

However, if two or more populations exist in a natural environment, there will be competition, interdependence, or predation among them. Next, the predation relationship among populations will be discussed based on the stability of differential equations.

2.3 Predator-prey relationship

2.3.1 Model assumptions

1. Suppose a predator cannot survive without a predator
2. Assume that the sea is rich in resources, and the predators grow exponentially when they survive independently.

2.3.2 Model building

Prey and predator at some point，The number of t is written as $x(t)$、$y(t)$，Because the sea is rich in resources, let the rate of growth of the predator be r, and the existence of the predator reduces the growth rate of the predator. Let the degree of reduction be proportional to the number of predators, and the proportionality coefficient a reflects the predator's predatory ability, so

$$\frac{dx}{dt} = x(r - ay) \quad (2)$$

Because the predator cannot survive without the predator, let it be d when it exists alone, that is,

$$\frac{dy}{dt} = y(-d + bx) \quad (3)$$

Since the above two equations do not have analytical solutions, first use MATLAB to find the numerical solutions of the equations, observe the numerical results and graphics, guess his analytical solution structure, and then theoretically study the shape of its equilibrium point and phase orbit to verify the previous guess.

The codes entered in MATLAB software are as follows

```
function xdot=shier(t,x)
r=1;d=0.5;a=0.1;b=0.02;
xdot=[(r-a*x(2)).*x(1);(-d+b*x(1)).*x(2)];
t1=0:0.1:15;
>> x0=[25,2];
>> [t,x]=ode45('shier',t1,x0);[t,x],

plot(t,x,'linewidth',1.5),grid,gtext('x(t)'),gtext('y(t)'),pause,plot(x(:,1),x(:,2),'linewidth',1.5),grid,
```

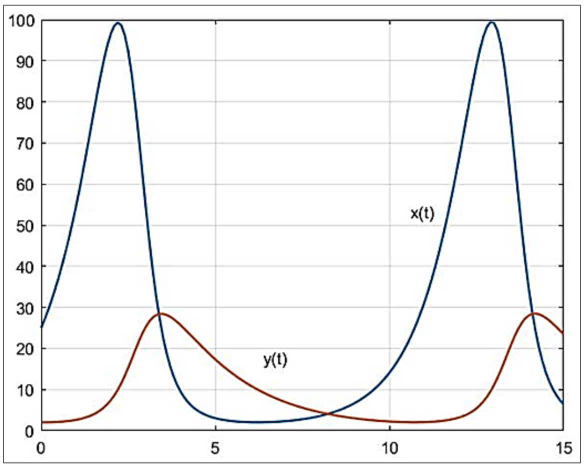

Fig 3: Graphs of numerical solutions $x(t)$, $y(t)$ (predator vs. predator time variation curve)

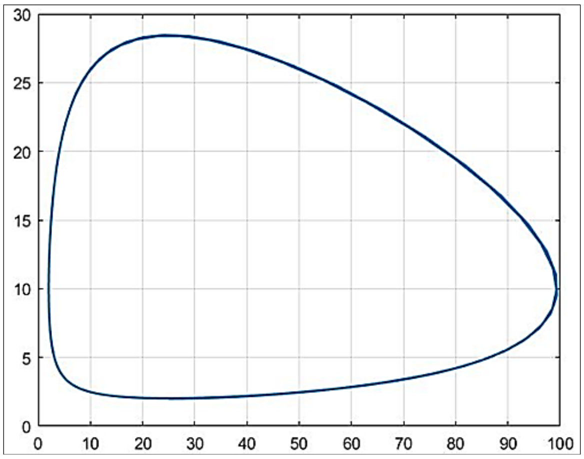

Fig 4: Graph of phase trajectory $y(x)$

3.Stability analysis

The predator and the prey are in a dynamic equilibrium, neither of which will be extinct, and their respective numbers will change periodically with time.

When blocking is considered (intraspecific competition)

$$\frac{dx}{dt} = r_1 x\left(1 - \frac{x}{n_1} - s_1\frac{y}{n_2}\right) \tag{4}$$

Where factor$\left(1-\frac{x}{n_1}\right)$ reflects the retardation of its own growth due to limited resources, $\frac{x}{n_1}$ can be explained that in terms of the maximum environmental capacity, the amount of food that is consumed by unit A to feed A. It should be noted that when two populations agree in the natural environment, B will consume the same limited resources for the growth of A. We can reasonably be in the factor$\left(1-\frac{x}{n_1}\right)$ minus one more, this term is directly proportional to the number of population B (relative to n_2).

$$\frac{dy}{dt} = r_2 y\left(1 - \frac{y}{n_2} + s_2\frac{x}{n_1}\right) \tag{5}$$

$$令 \begin{aligned} f(\mathrm{x},y) &\equiv r_1 x\left(1-\frac{\mathrm{x}}{n_1}-s_1\frac{y}{n_2}\right)=0 \\ g(\mathrm{x},y) &\equiv r_2 \mathrm{y}\left(1-s_2\frac{x}{n_1}-\frac{y}{n_2}\right)=0 \end{aligned} \tag{6}$$

Four balance points are as follows,

$$P_1(n_1,0), P_2(0,n_2), P_3\left(\frac{n_1(1-s_1)}{1-s_1s_2}, \frac{n_2(1-s_2)}{1-s_1s_2}\right), P_4(0,0)$$

Because it only has practical significance when the equilibrium point is in the first quadrant, for P_3, it is required that s_1, s_2 be greater than 1 at the same time, or less than 1 at the same time.

$$=\begin{bmatrix} f_{x_1} & f_{x_2} \\ g_{x_1} & g_{x_2} \end{bmatrix} = \begin{bmatrix} r_1\left(1-\frac{2x_1}{n_1}-\frac{s_1x_2}{n_2}\right) & -\frac{r_1s_1x_1}{n_2} \\ -\frac{r_2s_2x_2}{n_1} & r_2\left(1-\frac{s_2x_1}{n_1}-\frac{2x_2}{n_2}\right) \end{bmatrix} \quad (7)$$

Where ordered,

$$p = -(f_{x_1} + g_{x_2})|_{p_i}, i = 1,2,3,4 \quad (8)$$

$$q = \det A|_{P_i}, i = 1,2,3,4 \quad (9)$$

What we should care about is the global stability of the equilibrium point (that is, regardless of the initial value, the equilibrium point is stable). This needs to be supplemented by phase trajectory analysis based on the local stability obtained above. The equation is as follows:

$$\varphi(x_1,x_2) = 1 - \frac{x_1}{n_1} - s_1\frac{x_2}{n_2}$$
$$\psi(x_1,x_2) = 1 - s_2\frac{x_1}{n_1} - \frac{x_2}{n_2} \quad (10)$$

1. （1）When s_1<1, s_2>1, P_1 is stable, Because no matter where the trajectory starts, when t →∞, trajectory will tend to $P_1(n_1, 0)$
2. (2) When s_1>1, s_2<1, Similar to (1) analysis, $P_2(0, n_2)$ is stable
3. (3)When s_1<1, s_2<1, $P_3\left(\frac{n_1(1-s_1)}{1-s_1s_2}, \frac{n_2(1-s_2)}{1-s_1s_2}\right)$ is stable
4. (4) When s_1>1, s_2>1, because of q<0, P_3 is not stable（Saddle point）, the trajectory either approaches $P_1 or P_2$ is determined by the initial position of the trajectory, so it is only locally stable and not globally stable. Therefore, the conditions for global stability of P_1 need to be added to the conditions for global stability of P_1 plus $s_2 < 1$

To sum up, according to the combination of p > 0, q < 0 and phase trajectory diagram analysis, the stable condition analysis results are obtained.

Table 1: Equilibrium of predator-prey relationship

Balance point	**p**	**q**	**Stable condition**
$P_1(N_1, 0)$	r_1-r_2 (1-s_1)	-r_1r_2(1-s_2)	s_1<1, s_2>1
$P_2(0, N_2)$	-r_1 (1-s_1)+r_2	-r_1r_2(1-s_1)	s_1>1, s_2<1
$P_3\left(\frac{N_1(1-s_1)}{1-s_1s_2}, \frac{N_2(1-s_2)}{1-s_1s_2}\right)$	$\frac{r_1(1-s_1)+r_2(1-s_2)}{1-s_1s_2}$	$\frac{r_1r_2(1-s_1)(1-s_2)}{1-s_1s_2}$	s_1<1, s_2<1
$P_4(0,0)$	$-(r_1+r_2)$	r_1r_2	Unstable

4. Model inspection and analysis

the codes entered in MATLAB software are as follows

```
function y=fun(t,x)
y=[x(1).*(1-x(1)./3000-2*x(2)./400);0.3*x(2).*(-1+6.*x(1)./3000-x(2)./400)];
t1=0:0.1:20;
x0=[3000 60];
[t,x]=ode45('fun',[0,20],[3000,60])
plot(t,x),grid,gtext('x(t)'),gtext('y(t)')
plot(x(:,1),x(:,2)),grid,
```

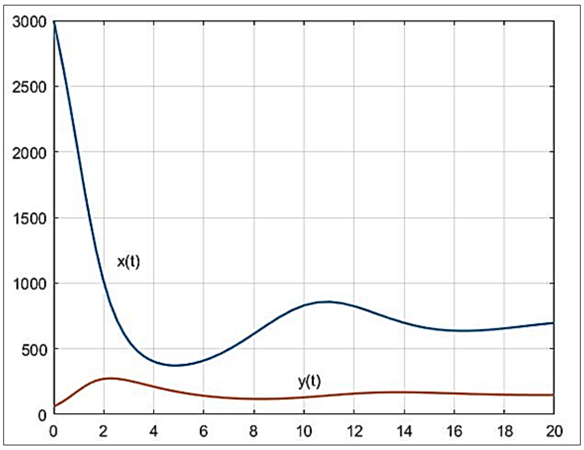

Fig 5: Graph of numerical solutions $x(t)$、$y(t)$

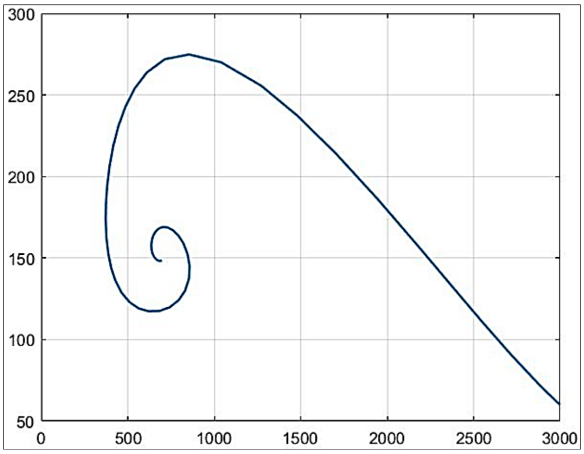

Fig 6: Graph of phase trajectory $y(x)$

From the graph of the numerical solution, it can be seen that their quantity changes, and they tend to a stable value over time. From the numerical solution, the stable value can be approximated (750, 150).

5. Conclusion

From the time-varying curve and phase diagram, it can be seen that although the system is blocked, the number of the two species gradually balances, but there is no blocking effect. The rate of stabilization is faster, and the comparison shows that the number of predators has not decreased, but has increased, but the number of predators has decreased. This has very important practical significance. In the development of fisheries, the number of predators can be increased through appropriate fishing and the fishery production can be increased.

1. Using MATLAB, a mathematical model of the system can be well established and graphics can be drawn, and numerical analysis can be performed to solve the differential equation problem with initial value conditions.
2. The system is considered to be an asymptotically stable ecological cycle without being affected by external influences and considering the effects of intra-species competition. This experiment only considers the two-species model, and we can extend it to the three-species case. The long-term cyclical ecological balance system in nature should be structurally stable, that is, after the system is unavoidably disturbed and deviates from the original periodic orbit Its internal constraints will automatically restore the system to its original state.
3. In fishery development, proper fishing can be used to increase the number of food fish, reduce the number of predatory fish, and increase fishery production. In the control of agricultural pests, the model of controlling insects by insects should be developed, which not only has better insect control effects, but also benefits environmental protection.

6. References

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