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Turbulence effect on the stability of molecular cloud

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Abstract

Stability of self-gravitating, magnetized and warm molecular cloud is studied taking into account the turbulence effect. A logarithmic term is added in equation of state to include the effect of turbulent motion. A biquadratic dispersion relation is obtained and is discussed in a special case by ignoring friction between ions and neutrals. It is found that waves propagate with faster speed if perturbation wavelength is less than the Jeans length of the ion component and greater than the Jeans length of the neutral component.

Keywords: Molecular clouds, partially ionized plasma, MHD waves, star

1. Introduction

Evidences show that molecular clouds are the sites of star formation (Shu *et al.* 1987) ^[1]. Damped magnetohydrodynamic waves in molecular clouds are responsible for finite sized thermally dominated core regions (Curry and McKee 2000) ^[2]. The observations show that molecular clouds have a clumpy structure with clumps that may or may not be self-gravitating (Crutcher *et al.* 1978 and Blitz 1980) ^[3, 4]. The clumps often contain collapsing protostellar cores and young stars. Wave propagation due to supersonic motion in these clumps provides the dominant form of support against gravity. The important role of supersonic motion for star formation was first recognized by Simmons *et al.* (1978) ^[5]. Blitz (1983) ^[6] has reviewed several of the properties of molecular clouds. An effort has been made on analyzing the structure of molecular clouds in particular, to construct some automated algorithms (Williams and McKee 1997) ^[7]. In galactic discs clouds are supported against their self-gravity by a combination of thermal, mean magnetic pressure and turbulent wave pressure. In most of the work, the effect of gravity on wave propagation in molecular clouds has been ignored (Arons and Max 1975; Zweibel and Josafattson 1983) ^[8, 9]. However, Langer (1978) ^[10] included the gravitational effects and investigated the stability of molecular clouds. Mestel and Spitzer (1956) ^[11] analyzed the problem of star formation in molecular cloud considering magnetic field. In the stability analysis of molecular clouds, the magnetic field is generally assumed to be static but this can not be so as the presence of supersonic line widths in spectrum of molecular clouds is observed. There are many sources of excitation available by which hydromagnetic waves would be produced in such fields. Thus, the relevant static problem is not that of a static magnetic field in molecular gas, but of a gas which is traversed by a wave spectrum. A spectrum of Alfvén waves traversed a cloud produces a stress in the gas which can stabilize it against collapse (Dewar 1976) ^[12]. Pudritz (1990) ^[13] studied wave motion in isothermal molecular cloud filaments and sheets. Dutrey *et al.* (1991) ^[14] investigated gravitational instability in molecular cloud. Balsara (1990) ^[15] studied isothermal magnetized molecular filament. Matsumoto, Nakamura and Hanawa (1990) ^[16] extended this study to the rotation. A common feature of all the prior studies on molecular clouds and star formation are based on isothermal equation of state. Observations show that some properties of massive molecular clouds can not be explained by isothermal equation of state (Caselli and Myers 1995) ^[17]. Even some properties of low mass cores are explained by non-thermal equation of state (Myers *et al.* 1991) ^[18]. An alternative form of equation of state is logatropic equation of state. Lizano and Shu (1989) ^[19] first produced this equation of state by adding a correction term in classic equation of state to study the presence of non-thermal

or turbulent pressure inside the molecular clouds. Gehmen *et al.* (1996) ^[20] used turbulence equation of state to explain filamentary and sheetlike structures in molecular clouds. In the present paper, we study the wave propagation and instabilities in molecular clouds by taking logatrop equation of state which is softer than classic equation of state. Organization of the paper is as follows. In section 2, we discuss the basic equations governing the physical state of molecular clouds. In section 3, we obtain the dispersion relation and discuss the wave propagation in molecular clouds with and without the softer equation of state. Conclusions are drawn in section 4.

2. Formulation

We consider molecular cloud as a partially ionized composed of ions and neutrals, self-gravitating, warm and magnetized medium. To study the effect of turbulence observed in molecular cloud, a generalized equation of state

$$p = c_s^2 \rho + p_0 \log \frac{\rho}{\hat{\rho}} \quad (1)$$

where p_0 is constant which may be determined empirically and $\hat{\rho}$ is arbitrary reference density (Gehmen *et al.* 1996) ^[20]. The governing dynamical equations describing the motion of ion and neutral fluid components are

$$\frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n \mathbf{v}_n) = 0, \quad (2)$$

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = 0, \quad (3)$$

$$\rho_n \left[\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n \right] = \rho_n \nabla \phi - \nabla p_n - \mathbf{F}_{ni}, \quad (4)$$

$$\rho_i \left[\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla) \mathbf{v}_i \right] = \rho_i \nabla \phi - \nabla p_i + \mathbf{F}_{ni} + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}. \quad (5)$$

Here ρ_n is the neutral mass density, ρ_i is the ion mass density, p_n is the neutral gas pressure, p_i is the ion gas pressure, \mathbf{v}_n is the velocity of neutral gas, \mathbf{v}_i is the velocity of ion gas, ϕ is gravitational potential and \mathbf{B} is magnetic field, \mathbf{F}_{ni} is the frictional force arises due to collisions between ions and neutrals and enters with opposite signs in the two equations (4) and (5). This friction force is given by

$$\mathbf{F}_{ni} = \alpha \rho_i \rho_n (\mathbf{v}_n - \mathbf{v}_i), \quad (6)$$

where α is a constant and $\alpha = 3.5 \times 10^{13} \text{ cms}^3 \text{ gram}^{-1} \text{ sec}^{-1}$ (Draine *et al.* 1983) ^[21]. The presence of friction term causes damping of sufficiently short wavelength perturbation in the fluids. Poisson's equation for a two fluids medium is

$$\nabla^2 \phi = 4\pi G (\rho_i + \rho_n). \quad (7)$$

The response of the magnetic field to the ion motions as described by the induction equation, is given by

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_i \times \mathbf{B}). \quad (8)$$

This is supplemented by the condition

$$\nabla \cdot \mathbf{B} = 0. \quad (9)$$

In order to complete the equation, a generalized equation of state for neutrals and ions describing the turbulence effects in molecular clouds are given by

$$p_n = c_n^2 \rho_n + p_{n0} \log \left(\frac{\rho_n}{\rho_{n0}} \right), \quad (10)$$

$$p_i = c_i^2 \rho_i + p_{i0} \log \left(\frac{\rho_i}{\rho_{i0}} \right), \quad (11)$$

where c_n and c_i are sound speeds of neutrals and ions respectively, p_{n0} and p_{i0} are neutrals and ion gas pressures in unperturbed state. The logarithm nature of the turbulence has also been explained by Larson (1981) ^[22]. Equilibrium quantities ρ_{n0} , ρ_{i0} , p_{n0} , p_{i0} and \mathbf{B}_0 are taken to be uniform in the perturbation scheme

$$\begin{aligned} \rho_n &= \rho_{n0} + \rho_{n1}, \quad \rho_i = \rho_{i0} + \rho_{i1}, \\ p_n &= p_{n0} + p_{n1}, \quad p_i = p_{i0} + p_{i1} \\ \mathbf{v}_n &= \mathbf{v}_{n1}, \quad \mathbf{v}_i = \mathbf{v}_{i1}. \end{aligned} \quad (12)$$

Here initial flow velocities \mathbf{v}_{n0} and \mathbf{v}_{i0} are assumed to be zero. In order to satisfy both the conditions of a uniform and stationary medium and Poisson's equation, it is necessary to consider

$$\phi_0 = 0. \quad (13)$$

Using perturbation scheme (12) in equations (2) to (11), we obtain the following linearized equations about equilibrium state

$$\frac{\partial \rho_{n1}}{\partial t} + \rho_{n0} \nabla \cdot \mathbf{v}_{n1} = 0, \quad (14)$$

$$\frac{\partial \rho_{i1}}{\partial t} + \rho_{i0} \nabla \cdot \mathbf{v}_{i1} = 0, \quad (15)$$

$$\rho_{n0} \frac{\partial v_{n1}}{\partial t} = -\rho_{n0} \nabla \phi_1 - \nabla p_{n1} - \alpha \rho_{i0} \rho_{n0} (\mathbf{v}_{n1} - \mathbf{v}_{i1}), \quad (16)$$

$$\frac{\partial v_{i1}}{\partial t} = -\rho_{i0} \nabla \phi_1 - \nabla p_{i1} + \frac{1}{4\pi} (\nabla \times \mathbf{B}_1) \times \mathbf{B}_0 + \alpha \rho_{i0} \rho_{n0} (\mathbf{v}_{n1} - \mathbf{v}_{i1}), \quad (17)$$

$$\nabla^2 \phi_1 = 4\pi G (\rho_{n1} + \rho_{i1}), \quad (18)$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = -\mathbf{B}_0 (\nabla \cdot \mathbf{v}_{i1}) + (\mathbf{B}_0 \cdot \nabla) \mathbf{v}_{i1}, \quad (19)$$

$$\nabla \cdot \mathbf{B}_1 = 0. \quad (20)$$

$$p_{n1} = (c_n^2 + \frac{p_{n0}}{\rho_{n0}}) \rho_{n1}, \quad (21)$$

$$p_{i1} = (c_i^2 + \frac{p_{i0}}{\rho_{i0}}) \rho_{i1}, \quad (22)$$

Differentiating equations (14) and (15) partially with respect to t, we get

$$\frac{\partial^2 \rho_{n1}}{\partial t^2} + \rho_{n0} \frac{\partial}{\partial t} (\nabla \cdot \mathbf{v}_{n1}) = 0, \quad (23)$$

$$\frac{\partial^2 \rho_{i1}}{\partial t^2} + \rho_{i0} \frac{\partial}{\partial t} (\nabla \cdot \mathbf{v}_{i1}) = 0. \quad (24)$$

Divergence of equation (16) yields

$$\rho_{n0} \nabla \cdot \left(\frac{\partial \mathbf{v}_{n1}}{\partial t} \right) = -\rho_{n0} \nabla^2 \phi_1 - \nabla^2 p_{n1} - \alpha \rho_{i0} \rho_{n0} \nabla \cdot (\mathbf{v}_{n1} - \mathbf{v}_{i1}). \quad (25)$$

Using equations (18) and (21) in (25), we have

$$\rho_{n0} \nabla \cdot \left(\frac{\partial \mathbf{v}_{n1}}{\partial t} \right) = -\rho_{n0} 4\pi G (\rho_{n1} + \rho_{i1}) - \nabla^2 (c_n^2 + \frac{p_{n0}}{\rho_{n0}}) \rho_{n1} - \alpha \rho_{i0} \rho_{n0} \nabla \cdot (\mathbf{v}_{n1} - \mathbf{v}_{i1}) \quad (26)$$

From equations (14), (15) and (26), we get

$$\frac{\partial^2 \rho_{n1}}{\partial t^2} = \omega_{gn}^2 (\rho_{n1} + \rho_{i1}) + c_n^2 \nabla^2 \rho_{n1} + \frac{p_{n0}}{\rho_{n0}} \nabla^2 \rho_{n1} - \alpha (\rho_{i0} \frac{\partial \rho_{n1}}{\partial t} - \rho_{n0} \frac{\partial \rho_{i1}}{\partial t}). \quad (27)$$

Similarly equation (17) becomes

$$\frac{\partial^2 \rho_{i1}}{\partial t^2} = \omega_{gi}^2 (\rho_{n1} + \rho_{i1}) + c_i^2 \nabla^2 \rho_{i1} + \frac{p_{i0}}{\rho_{i0}} \nabla^2 \rho_{i1} + \alpha \left(\rho_{i0} \frac{\partial \rho_{n1}}{\partial t} - \rho_{n0} \frac{\partial \rho_{i1}}{\partial t} \right) - \mathbf{B}_0 \cdot \frac{\nabla \times (\nabla \times \mathbf{B}_1)}{4\pi}, \quad (28)$$

where ω_{gn}^2 and ω_{gi}^2 are gravitational response frequencies in the neutral and ion fluids respectively and are given as

$$\omega_{gn}^2 = 4\pi G \rho_{n0}, \quad \omega_{gi}^2 = 4\pi G \rho_{i0}.$$

Using equation (15) in (19), we get

$$\frac{\partial \mathbf{B}_1}{\partial t} = \frac{\mathbf{B}_0}{\rho_{i0}} \frac{\partial \rho_{i1}}{\partial t} + (\mathbf{B}_0 \cdot \nabla) \mathbf{v}_{i1}. \quad (29)$$

The magnetic force contributes an additional effective pressure to the ion pressure which stabilizes the cloud against gravitational collapse.

3. General dispersion relation

The linearized set of equations are solved using

$$\vartheta = \hat{\vartheta} e^{i(kx - \omega t)} \quad (30)$$

where ϑ represents the perturbed quantities ρ_{n1}, ρ_{i1} and \mathbf{B}_1 and $\hat{\vartheta}$ is associated constant. The amplitude of the magnetic perturbation is given by

$$\mathbf{B}_1 = \frac{\rho_{i1}}{\rho_{i0}} \mathbf{B}_0 - \frac{\mathbf{B}_0 \cdot \mathbf{k}}{\omega} \mathbf{v}_{i1}. \quad (31)$$

Substituting (30) in linearized equations, we get

$$\omega^2 \rho_{n1} = -\omega_{gn}^2 (\rho_{n1} + \rho_{i1}) + k^2 c_n^2 \rho_{n1} + \frac{p_{n0}}{\rho_{n0}} k^2 \rho_{n1} - i\alpha \omega (\rho_{i0} \rho_{n1} - \rho_{n0} \rho_{i1}). \quad (32)$$

$$\omega^2 \rho_{i1} = -\omega_{gi}^2 (\rho_{n1} + \rho_{i1}) + k^2 c_i^2 \rho_{i1} + \frac{p_{io}}{\rho_{io}} k^2 \rho_{i1} + i\alpha\omega(\rho_{io}\rho_{n1} - \rho_{no}\rho_{i1}) + k^2 v_A^2 [\rho_{i1} - (\mathbf{B}_0 \cdot \mathbf{k})(\widehat{\mathbf{B}}_0 \cdot \mathbf{v}_{i1}) \frac{\rho_{io}}{\omega}]. \quad (33)$$

Here $v_A^2 = \frac{B_0^2}{4\pi\rho_{io}}$ is the Alfvén speed in the ionic fluid and $\widehat{\mathbf{B}}_0$ is unit vector in the direction of magnetic field. The magnetic effects on the two fluid gas can be explained by the last term of the equation (33). Now, we define Alfvén speed in neutral medium and gravitational response frequencies as

$$v_{A,n}^2 = \frac{B_0^2}{4\pi\rho_{no}}, \omega_n^2 = \omega_{gn}^2 - k^2 c_n^2, \omega_i^2 = \omega_{gi}^2 - k^2 V_{Fi}^2,$$

where V_{Fi} ($V_{Fi}^2 = c_i^2 + v_A^2$) is the speed of the fast magnetosonic mode in the ions. Using above values in (31) and (32), we have

$$(\omega^2 + \omega_n^2 - \frac{p_{no}}{\rho_{no}} k^2 + i\omega x v_n) \rho_{n1} + (\omega_{gn}^2 - i\omega v_n) \rho_{i1} = 0, \quad (34)$$

$$(\omega_{gi}^2 - i\omega x v_n) \rho_{n1} + (\omega^2 + \omega_i^2 - \frac{p_{io}}{\rho_{io}} k^2 + i\omega v_n) \rho_{i1} = 0, \quad (35)$$

where $v_n = \alpha\rho_{no}$ and $x (= \frac{\rho_{io}}{\rho_{no}})$ is ionization fraction in the cloud.

Now setting the determinant of the matrix of the coefficients in equations (34) and (35) equal to zero, and putting $\omega_{gi}^2 = x\omega_{gn}^2$, we get

$$(\omega^2 + \omega_n^2 - \frac{p_{no}}{\rho_{no}} k^2 i\omega x v_n) (\omega^2 + \omega_i^2 - \frac{p_{io}}{\rho_{io}} k^2 + i\omega v_n) - (\omega_{gi}^2 - i\omega x v_n) (\omega_{gn}^2 - i\omega v_n) = 0 \quad (36)$$

Simplifying above relation we find biquadratic dispersion relation

$$[(\omega^2 + \omega_n^2)(\omega^2 + \omega_i^2) - x\omega_{gn}^4] + i\omega v_n [\omega^2(1+x) + (1+x)^2\omega_{gn}^2 - (k^2 c_n^2 + k^2 x V_{Fi}^2) - \frac{p_{io}}{\rho_{io}} k^2 x - \frac{p_{no}}{\rho_{no}} k^2] - \omega^2 k^2 (\frac{p_{io}}{\rho_{io}} + \frac{p_{no}}{\rho_{no}}) - k^2 [\omega_n^2 \frac{p_{io}}{\rho_{io}} + \omega_i^2 \frac{p_{no}}{\rho_{no}}] + k^4 \frac{p_{io} p_{no}}{\rho_{io} \rho_{no}} = 0 \quad (37)$$

This is a dispersion relation for two-component, partially ionized with arbitrary ionization fraction, warm, non isothermal, self gravitating and magnetized molecular clouds. This is quartic in ω , so the behaviour of the roots is not so obvious. We try to study this relation analytically in special case by neglecting friction between ion and neutrals which is justified as we are considering molecular clouds as partially ionized.

3.1. Negligible friction case

We discuss (37) in a special case when ion neutral collisions do not occur frequently by ignoring friction i.e. by setting $v_n = 0$. In this case, two fluid motion is coupled by gravity. Now, the dispersion relation (37) becomes

$$X^4 + X^2 [X_g^2 - (1 + \delta^2 + \kappa_n + c^2 \kappa_i)] - X_{gn}^2 (\delta^2 + c^2 \kappa_i) - X_{gi}^2 (1 + \kappa_n) + \delta^2 \kappa_n + c^2 \kappa_i + c^2 \kappa_i \kappa_n + \delta^2 = 0, \quad (38)$$

where $X = \frac{\omega}{kc_n}, \kappa_i = \frac{p_{io}}{c_i^2 \rho_{io}}, \kappa_n = \frac{p_{no}}{c_n^2 \rho_{no}}, X_{gn} = \frac{\omega_{gn}}{kc_n}, X_{gi} = \frac{\omega_{gi}}{kc_n}, \delta = \frac{V_{Fi}}{c_n}, X_g^2 = X_{gn}^2 + X_{gi}^2$, and $c^2 = \frac{c_i^2}{c_n^2}$.

where κ_i and κ_n are turbulence parameters in ion and neutrals respectively. If the turbulence is negligible i.e $\kappa_i = 0$ and $\kappa_n = 0$, equation (38) becomes

$$X^4 + X^2 [X_g^2 - (1 + \delta^2)] - X_{gn}^2 \delta^2 - X_{gi}^2 + \delta^2 = 0, \quad (39)$$

which is quartic equation in X . Its roots are given by

$$X^2 = \frac{1}{2} [1 + \delta^2 - X_g^2 \pm \sqrt{\{X_g^2 - (1 + \delta^2)\}^2 + 4(X_{gn}^2 \delta^2 + X_{gi}^2 - \delta^2)}]$$

After simplification, it becomes

$$X^2 = \frac{1}{2(kc_n)^2} [-(\omega_i^2 + \omega_n^2) \pm \sqrt{\{(\omega_n^2 - \omega_i^2)^2 + 4\omega_{gi}^2 \omega_{gn}^2\}}] \quad (40)$$

which can be written as

$$\omega^2 = \frac{1}{2} [-(\omega_i^2 + \omega_n^2) \pm \sqrt{\{(\omega_n^2 - \omega_i^2)^2 + 4\omega_{gi}^2 \omega_{gn}^2\}}]. \quad (41)$$

This dispersion relation is similar to that obtained by Pudritz (1990) [13].

To study the effect of turbulence on the stability of molecular clouds in the absence of friction, we consider the case when perturbation wavelength is less than the Jeans length of ion components and greater than the Jeans length of neutral components. This is a fruitful case for the study of molecular clouds as ions are very light. In this limit

$$\omega_{gi}^2 \ll k^2 V_{Fi}^2 \text{ and } k^2 c_n^2 \ll \omega_{gn}^2, \omega_n^2 = \omega_{gn}^2, \omega_i^2 = -k^2 V_{Fi}^2.$$

Under these limits dispersion relation (38) becomes

$$X^4 + X^2[X_{gn}^2 - (\delta^2 + \kappa_n + c^2 \kappa_i)] - X_{gn}^2(\delta^2 + c^2 \kappa_i) + \delta^2 \kappa_n + c^2 \kappa_i \kappa_n = 0. \quad (42)$$

This modified relation has the following roots

$$\omega_f^2 = k^2 V_{Fi}^2 + \kappa_i k^2 c_i^2, \quad (42,a)$$

$$\omega_s^2 = \kappa_n k^2 c_n^2 - \omega_{gn}^2. \quad (42,b)$$

Solution (42a) corresponds to a fast magnetosonic wave and propagates with faster speed than the wave discussed by Pudritz (1990) ^[13] in the absence of turbulence. Solution (42b) implies that wave propagation is only possible at gravitational response frequency in neutrals smaller than the critical frequency $\omega_{gn,c} = \kappa_n k^2 c_n^2$, otherwise it corresponds to Jeans instability in neutrals. In the absence of turbulence, equation (40) corresponds to only Jeans instability in neutrals having agreement with the results of Pudritz (1990) ^[13].

4. Conclusions

The turbulence observed in molecular clouds arises from small magnetohydrodynamic motions such as Alfvén waves within the molecular clouds. In the present paper, we have included the effects of such turbulence by incorporating a term in the equation of state that modifies the isothermal pressure. It is found that wave propagates with faster speed than the wave discussed by Pudritz (1990) ^[13] in the absence of turbulence. We can conclude that turbulence supports the molecular clouds against collapse, the result directly agrees with the results obtained by Gehman *et al.* (1996) ^[20]. The results obtained here might be useful in exploring the facts related to the stability of molecular cloud and star formation.

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