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Finding a compound weibull distribution

Mohammed Mohammed Ahmed Almazah

Abstract
This study has proposed a new distribution based on the three Weibull distributions, which is called Weibull combined distribution. The real data was collected from the Department of Radiology at the General Hospital, Mahayil, Kingdom of Saudi Arabia. A combination parameter (η), which represents the percentage of each one of these three distributions in the new combined distribution using real values of the merged parameter and all probability functions for the new distribution. The study has found more flexibility when the merged parameter is at 0.60.

Keywords: Weibull distribution, probability density function, distribution function, survival function, failure function, combined distribution

Introduction

The probabilistic Weibull model was first introduced by Waloddi Weibull to represent the distribution of the breaking strength of materials and later to describe the behavior of systems or events that have some degree of variability. It is a flexible distribution that can encompass characteristics of several other distributions. This property has given rise to widespread applications. Today, it is commonly used to assess product reliability, analyze life data and model failure times (Bartolucci *et al*., 1999) [2].

The Weibull distribution is one of the continuing probability distributions and one of the most commonly used failure models. In the past 40 years, Weibull's distribution has a status and importance in the field of reliability and life testing. It is also an important probability distribution in the theory of reliability because it has a great effect in producing accurate results when used and has important applications in the study of failure models (Teimouri, Hoseini & Nadarajah, 2013) [20]. The most popular distribution for failure data analysis is the Weibull distribution. The process of modeling failure data is commonly known as Weibull analysis, even when the underlying failure distribution is not Weibull. The Weibull distribution is widely applied in radar systems to model the dispersion of the received signal level caused by clutters. It is also used in wireless communications (Bartolucci *et al*., 1999) [2]. Other application areas include tensile characteristics of examination of tyre rubber cure (Barreto-Souza, Santos & Cordeiro, 2010; Genc *et al*., 2005) [17, 9], estimation of wind power potential (Chiang *et al*., 2004; Hasumi, Akimoto & Aizawa, 2009) [3, 10], food products drying technology (Collett, 2015; Jiang & Murthy, 2011) [4, 11], significant wave height simulation and prediction (Corzo *et al*., 2008; Johnson, Kotz & Balakrishnan, 1996) [5, 12], modeling air drying of coroba slices (Diaconu, 2009; Meeker & Escobar, 2014) [6, 13], model for growth/decline in product sales (Dodson, 2006; Muraleedharan *et al*., 2007) [7, 14], and intercurrence times of earthquakes (García-Pascual *et al*., 2006) [8]. This distribution can be shortened to exponential distribution when the shape parameter is equal to one ($\beta = 1$), and Weibull's distribution has an increasing failure rate when the shape parameter is larger than one true ($\beta > 1$) and has a decreasing failure rate when the shape parameter is less than one true ($\beta < 1$) (Murthy, Xie & Jiang, 2004) [15].

Recent studies are looking to find compound distributions by merging previous distributions to obtain new distributions that are useful in scientific applications and; therefore, resulting in combined distributions, discrete and continuous (Nelson, 2005; Paranaíba *et al*., 2011; Rinne, 2008; Tahir *et al*., 2016) [16- 19]. Therefore, the paper aims to find the distribution of the merged Weibull and extracting the new distribution functions using the merging parameter.

By combining a number of Weibull distributions, each distribution contains a number of parameters to produce a new distribution.

Weibull Distribution

It is known that the Weibull distribution may contain one, two or three parameters. These parameters are Shape Parameter (β), Scale Parameter (α) and there may be a third parameter, the parameter Location (γ) [20]. Thus, the Probability density function of the one -parameter Weibull distribution is as in the following formula:

$$f_1(t) = \frac{c}{\alpha_1}\left(\frac{t}{\alpha_1}\right)^{c-1} e^{-\left(\frac{t}{\alpha_1}\right)^{c}}, t>0 \quad (1)$$

Where the only parameter is the scale parameter α_1 again this is obtained by setting $\gamma = 0$ and assuming $\beta = c = \text{Constant}$.

In the formulation of the 1-parameter Weibull, this study has assumed that the shape parameter β is known a priori from past experience with identical or similar products. The advantage for this assumption is tthat data sets with few or no failures can be analyzed.

The distribution function is written in the formula:

$$F_1(t) = 1 - e^{-\left(\frac{t}{\alpha_1}\right)^{c}} \quad (2)$$

The survival function for the one-parameter Weibull distribution is written in the formula:

$$R_1(t) = 1 - F_1(t) = e^{-\left(\frac{t}{\alpha_1}\right)^{c}} \quad (3)$$

The failure function for the one-parameter Weibull distribution is written in the formula:

$$h_1(t) = \frac{c\, t^{c-1}}{{\alpha_1}^{c}} \quad (4)$$

Thus, the Probability density function of the two-parameter Weibull distribution is denoted in formula:

$$f_2(t) = \frac{\beta_1}{\alpha_2}\left(\frac{t}{\alpha_2}\right)^{\beta_1-1} e^{-\left(\frac{t}{\alpha_2}\right)^{\beta_1}}, t > 0 \quad (5)$$

Where: $\beta_1 > 0$ is the shape parameter and $\alpha_2 > 0$ is the scale parameter of the distribution. Its complementary cumulative distribution function is a stretched exponential function. The Weibull distribution is related to a number of other probability distributions; in particular, it interpolates between the exponential distribution (β= 1) and the Rayleigh distribution (β = 2 and $\alpha = \sqrt{2}\sigma$ (Wakeel, 2010).

The form of the density function of the Weibull distribution changes drastically with the value of k. For 0 < β < 1, the density function tends to ∞ as X approaches zero from above and is strictly decreasing. For β = 1, the density function tends to 1/α as x approaches zero from above and is strictly decreasing. For k > 1, the density function tends to zero as x approaches zero from above, increases until its mode and decreases after it. It is interesting to note that the density function has infinite negative slope at x = 0 if 0 < β < 1, infinite positive slope at x = 0 if 1 < β < 2 and null slope at x = 0 if β > 2. For k = 2 the density has a finite positive slope at x = 0. As β goes to infinity, the Weibull distribution converges to a Dirac delta distribution centered at x = α. Moreover, the skewness and coefficient of variation depend only on the shape parameter (Wakeel, 2010) [21].

The distribution function is written is in the formula:

$$F_2(t) = 1 - e^{-\left(\frac{t}{\alpha_2}\right)^{\beta_1}} \quad (6)$$

The survival function for the one-parameter Weibull distribution is written in the formula:

$$R_2(t) = 1 - F_2(t) = e^{-\left(\frac{t}{\alpha_2}\right)^{\beta_1}} \quad (7)$$

The failure function for the one-parameter Weibull distribution is written in the formula:

$$h_2(t) = \frac{\beta_1\left(\frac{t}{\alpha_2}\right)^{\beta_1-1}}{{\alpha_2}^{\beta_1}} \quad (8)$$

It is possible to obtain the three-parameters Weibull distribution depending on the two-parameter, in addition to a third parameter, the Location parameter, which is symbolized by the symbol γ. The probability density function of the three-parameter Weibull distribution is given by (Muraleedharan *et al*., 2007; Rinne, 2008; Wakeel, 2010) [14, 18, 21].

$$f_3(t) = \frac{\beta_2}{\alpha_3}\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2-1} e^{\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2}}, \gamma < t < \infty \quad (9)$$

Where:

$f(t) > 0, t > \gamma, \beta > 0, \alpha > 0, -\infty < \gamma < \infty$

And:

- α is the ascal parameter, also known as the Weibull slope
- β is the Shape parameter (or slope)
- γ is the Loction parameter (or failure life)

The distribution function is written is in the formula:

$$F_3(t) = 1 - e^{-\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2}}, t > \gamma \quad (10)$$

The survival function for the one-parameter Weibull distribution is written in the formula:

$$R_3(t) = 1 - F_3(t) = e^{-\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2}} \quad (11)$$

The failure function for the one-parameter Weibull distribution is written in the formula

$$h_3(t) = \frac{\beta_2(t-\gamma)^{\beta_2-1}}{\alpha_3{}^{\beta_2}} \quad (12)$$

The one-parameter Weibull distribution, the two parameters, and the three parameters can be used to find a new distribution in the Weibull combined distribution.

The New (Merged) Weibull distribution

For a Weibull merged distribution, a new parameter or parameters must be available to link the distributions together, and these parameters are the merged or mixture. Whereas, $0 < \eta_i < 1$. As well $\sum_{i=1}^{n} \eta_i = 1$. The number of parameters must be equal to the number of distributions used minus one to determine the proportion of the contribution of each distribution in the merged distribution, whether these distributions of the same or different types, in general, the probability density function for the merged distribution is in the following formula:

$$f(t) = \eta_1 f_1(t) + \eta_2 f_2(t) + \eta_3 f_3(t) + \cdots + \eta_n f_n(t) \quad (13)$$

Where: $f(t)$: Represents probabilty densty Function Compled distribution
$\eta_i, i = 1, 2, 3, \ldots, n$, represents the merge parameter and
$f_1(t), f_2(t), f_3(t), \ldots, f_n(t)$: This represent the probabilty density functions involved in the merged distribution.
The cumulative distribution function of the merged distribution is in the formula:

$$F(t) = \eta_1 F_1(t) + \eta_2 F_2(t) + \eta_3 F_3(t) + \cdots + \eta_n F_n(t) \quad (14)$$

Where:$F(t)$: The cumulative distribution function represents the merged distribution is, $F_1(t), F_2(t), \ldots, F_n(t)$: Represent the cumulative distribution functions of the distribution involved in the merged distribution.
The function of survival or reliability of the merged distribution is in the formula:

$$R(t) = \eta_1 R_1(t) + \eta_2 R_2(t) + \eta_3 R_3(t) + \cdots + \eta_n R_n(t) \quad (15)$$

$R(t)$: Represents reliability function of the merged distribution
$R_1(t), R_2(t), R_3(t), \ldots, R_n(t)$: Represents the function of reliability or survival of the distribution involved in the merged distribution.
Hazard function or failure rate of the merged distribution is in the formula:

$$H(t) = \eta_1 h_1(t) + \eta_2 h_2(t) + \eta_3 h_3(t) + \cdots + \eta_n h_n(t) \quad (16)$$

$H(t)$: Hazard function or failure rate $h_1(t), h_2(t), \ldots, h_n(t)$

The failure function represents the distribution the merged distribution.
In this study, the focus is on the case of combining three distributions; Firstly with one parameter; secondly with two parameters, and thirdly with three parameters. The merged parameter represents the proportion of the contribution of each one in the new distribution by giving different values of the merged parameter. Thus, the probability density function for the merged distribution in this study is in the formula:

$$f(t) = \eta\, f_1(t) + 2(1-\eta)f_2(t) + (\eta - 1)f_3(t) \quad (17)$$

From relationships 1, 5, 9, this study has obtained the probability density function for the distribution of merged Weibull as shown in the formula:

$$f(t) = \eta\, u_1(t) + 2(1-\eta)\, u_2(t) + (\eta - 1)\, u_3(t) \quad (18)$$

Where

$$u_1(t) = \frac{c}{\alpha_1}\left(\frac{t}{\alpha_1}\right)^{c-1} e^{-\left(\frac{t}{\alpha_1}\right)^{c}}$$
$$u_2(t) = \frac{\beta_1}{\alpha_2}\left(\frac{t}{\alpha_2}\right)^{\beta_1-1} e^{-\left(\frac{t}{\alpha_2}\right)^{\beta_1}}$$
$$u_3(t) = \frac{\beta_2}{\alpha_3}\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2-1} e^{\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2}}$$

To verify whether the function in Equation 18 is a probability density function, it must meet the following conditions:

1. $f(t) \geq 0$
2. $\int_0^\infty f(t)dt = 1$

$$\int_0^\infty \eta\, \frac{c}{\alpha_1}\left(\frac{t}{\alpha_1}\right)^{c-1} e^{-\left(\frac{t}{\alpha_1}\right)^{c}} dt + \int_0^\infty 2(1-\eta)\frac{\beta_1}{\alpha_2}\left(\frac{t}{\alpha_2}\right)^{\beta_1-1} e^{-\left(\frac{t}{\alpha_2}\right)^{\beta_1}} dt + \int_\gamma^\infty (\eta - 1)\,\frac{\beta_2}{\alpha_3}\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2-1} e^{-\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2}} dt$$
$$= -\eta\, e^{-\left(\frac{t}{\alpha_1}\right)^{c}}\Big\}_0^\infty - 2(1-\eta)e^{-\left(\frac{t}{\alpha_2}\right)^{\beta_1}}\Big\}_0^\infty - (\eta - 1)e^{-\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2}}\Big\}_\gamma^\infty$$
$$= +\eta + 2 - 2\eta + \eta - 1 = 1$$

The above function represents the probability density function. From relationships 2, 6, 10, this study has obtained the cumulative distribution function of the merged distribution as shown from the formula.

$$F(t) = \eta\left(1 - e^{-\left(\frac{t^c}{\alpha_1}\right)}\right) + 2(1-\eta)\left(1 - e^{-\left(\frac{t^{\beta_1}}{\alpha_2}\right)}\right) + (\eta - 1)\left(1 - e^{\frac{-(t-\gamma)^{\beta_2}}{\alpha_3}}\right) \quad (19)$$

From relationships 3, 7, 11, this study has obtained the function of survival or reliability of the merged distribution as shown from the formula.

$$R(t) = \eta\left(e^{-\left(\frac{t^c}{\alpha_1}\right)}\right) + 2(1-\eta)\left(e^{-\left(\frac{t^{\beta_1}}{\alpha_2}\right)}\right) + (\eta - 1)\left(e^{-\left(\frac{t-\gamma}{\alpha_3}\right)^{\beta_2}}\right) \quad (20)$$

From relationships 4, 8, 12, this study has obtained the Hazard function or failure rate of the merged distribution as shown from the formula.

$$H(t) = \eta\left(\frac{c\, t^{c-1}}{\alpha_1}\right) + 2(1-\eta)\left(\frac{\beta_1\, t^{\beta_1-1}}{\alpha_2{}^{\beta_1}}\right) + (\eta - 1)\left(\frac{\beta_2\,(t-\gamma)^{\beta_2-1}}{\alpha_3{}^{\beta_2}}\right) \quad (21)$$

Methodology

The paper adopted the analytical descriptive method for the distribution of Weibull with one parameter, two parameters, and three parameters based on the scientific references and it's applying and conducting statistical operations. The data has been collected from the hardware malfunctions in the Radiology Department of Muhayil General Hospital. One of these devices was selected as the Radiation Device (Digital X-RAY). This device recorded the greatest number of sudden failures during the period from 1/1/2015 to 31/12/2017, which represents the failure times, shown in Table 1.

Table 1: Data under Study

Month	1	2	3	4	5	6	7	8	9	10	11	12
the number n_i	3	4	3.5	4	5	4	5.5	5	5	6	5	4
Breakdown probability X_i	0.056	0.074	0.064	0.074	0.093	0.074	0.102	0.093	0.093	0.111	0.093	0.074

Results and Discussion

The probability distribution functions with one parameter, two parameters, and three parameters for Weibull distributions were calculated using the equations from 1 to 12 as shown in Table 2. The probability distribution functions for the Weibull distribution of the compound were calculated by selecting values of the parameter (η) equal 0.30, 0.60 and 0.90, respectively by applying equations 18, 19, 20, and 21. The results are shown in Tables 3, 4, and 5. Table 2 has reported the cumulative distribution function, survival or reliability distribution function, and hazard distribution function for one parameter, two parameters, and three parameters.

Table 2: Weibull distribution functions with one parameter, two parameters and three parameters contribution to the Weibull composite distribution $\gamma = 0.015, \beta_1 = 3, \beta_2 = 4, c = 2, \alpha_1 = 5, \alpha_2 = 6, \alpha_3 = 7$

Mo	X_i	$f_1(x)$	$f_2(x)$	$f_3(x)$	$F_1(x)$	$F_2(x)$	$F_3(x)$	$R_1(x)$	$R_2(x)$	$R_3(x)$	$h_1(x)$	$h_2(x)$	$h_3(x)$
1	0.041	.0033	.0233	.0029	.0052	.6953	.0019	.9984	.9998	.0001	.0129	.0001	.0000
2	0.055	.0044	.0420	.0107	.0052	.6953	.0107	.9978	.9997	.0001	.0129	.0001	.0000
3	0.061	.0049	.0517	.0162	.0016	.1175	.0186	.9976	.9997	.0001	.0071	.0003	.0000
4	0.068	.0054	.0642	.0248	.0005	.0234	.0329	.9973	9997.	.0001	.0042	.0004	.0000
5	0.075	.0060	.0781	0360.	.0005	.0234	.0540	.9970	9997.	.0001	.0042	.0004	.0000
6	0.082	.0066	.0934	.0501	.0001	.0010	.0839	.9967	.9996	.0001	.0014	.0000	.0000
7	0.089	.0071	.1100	.0675	.0010	.0597	.1249	.9964	.9996	.0001	.0057	.0001	.0000
8	0.095	.0076	.1253	.0853	.0016	.1175	.1706	.9962	.9996	.0001	.0071	.0003	.0000
9	0.095	.0081	.1310	.0921	.0010	.0597	.1864	.9996	1.000	.0001	.0057	.0001	.0000
10	0.103	.0085	.1473	.1135	.0010	.0597	.2498	.9959	.9995	.0001	.0057	.0001	.0000
11	0.109	.0091	.1650	.1384	.0010	.0597	.3252	.9956	.9995	.0001	.0057	.0001	.0000
12	0.123	.0098	.2101	.2099	.0016	.1175	.5666	.9951	.9994	.0001	.0071	.0004	.0000

Table 3: has shown the cumulative distribution function, survival or reliability distribution function, and hazard distribution function using the parameter 0.30 for 12 months.

Table 3: Weibull distribution functions using the parameter η at η =0.30

Month	X_i	f(x)	F(x)	R(x)	h(x)
1	0.041	.0010	. 0206	.59751	.0010
2	0.055	.0014	. 0374	.59751	.0013
3	0.061	.0015	. 0461	.59941	.0015
4	0.068	.0017	. 0575	.59981	.0016
5	0.075	.0019	.0702	.59981	.0018
6	0.082	.0021	.0843	.60001	.0020
7	0.089	.0022	.0996	.59941	.0021
8	0.095	.0025	. 1138	.59751	.0024
9	0.095	.0028	.1254	.59751	.0025
10	0.103	.0031	. 1344	.59941	.0027
11	0.109	.0032	. 1510	.59961	.0028
12	0.123	.0035	. 1936	.59961	.0021

Table 4 has shown the cumulative distribution function, survival or reliability distribution function, and hazard distribution function using the parameter 0.60 for 12 months.

Table 4: Weibull distribution functions using the parameter η at η =0.60

Month	X_i	f(x)	F(x)	R(x)	h(x)
1	0.041	.0020	.0406	.34221	.0020
2	0.055	.0027	.0732	.34181	.0026
3	0.061	.0030	.0901	.34221	.0029
4	0.068	.0033	. 1121	.34281	.0033
5	0.075	.0037	. 1365	.34251	.0036
6	0.082	.0040	. 1634	.34251	.0039
7	0.089	.0044	. 1927	.34181	.0043
8	0.095	.0047	. 2197	.33921	.0046
9	0.095	.0051	. 2267	.33921	.0047
10	0.103	.0053	0.2586	.34181	.0049
11	0.109	.0055	. 2899	.34221	.0052
12	0.123	.0058	. 3699	.34221	.0059

Table 5 has shown the cumulative distribution function, survival or reliability distribution function, and hazard distribution function using the parameter 0.90 for 12 months.

Table 5: Weibull distribution functions using the parameter η at η =0.90

Month	X_i	f(x)	F(x)	R(x)	h(x)
1	0.041	.0030	.0606	.08481	.0030
2	0.055	.0040	.1090	.08431	.0040
3	0.061	.0044	.1342	.08481	.0044
4	0.068	.0049	.1667	.08571	.0049
5	0.075	.0054	.2029	.08521	.0054
6	0.082	.0059	.2425	.08521	.0059
7	0.089	.0064	.2858	.08431	.0064
8	0.095	.0069	.3256	.08091	.0068
9	0.095	.0072	.3532	.08091	.0071
10	0.103	.0075	.3829	.08431	.0074
11	0.109	.0079	.4288	.08481	.0078
12	0.123	.0089	.5462	.08481	.0089

Conclusion

The study has proposed a new distribution based on the three Weibull distributions, undertaking three different parameters 0.30, 0.60, and 0.90 for 12 months. The results have shown that the probability density functions for the combined Weibull distribution with several different values of the merged (combined) parameter were more flexible at the combination parameter η = 0.60. It is possible to extract several distributions of the Weibull combined distribution and different values for the merged parameter. The present distribution has shown higher distribution flexibility and; therefore, is competent to model lifetime data. it has been observed that the new model offers best fit as compared to the competitive models.

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