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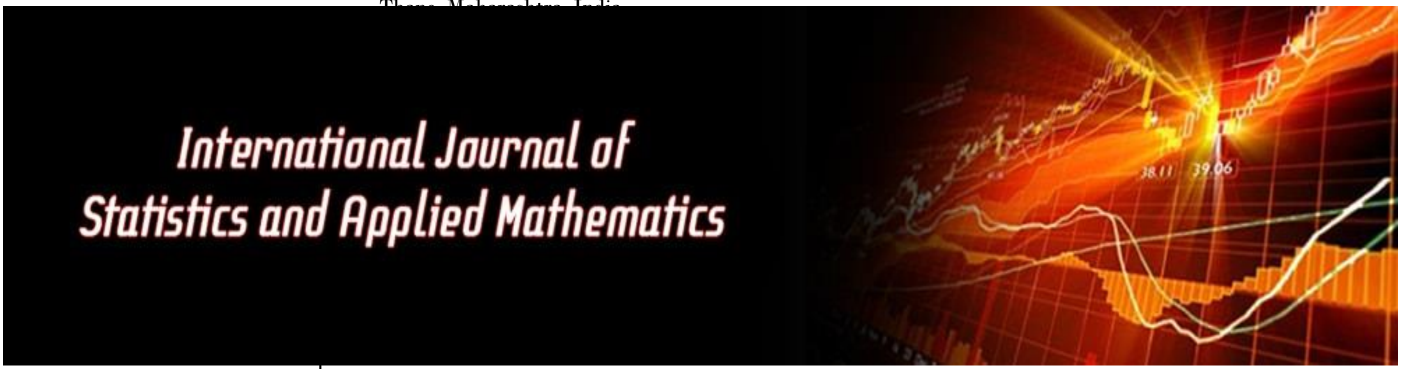

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Ambuja Joshi
Department of Mathematics, Shankar Narayan College of Arts and Commerce, Bhayandar West Thane, Maharashtra, India

Corresponding Author:
Ambuja Joshi
Department of Mathematics, Shankar Narayan College of Arts and Commerce, Bhayandar West

The combined effect of magnetic field and viscous dissipation on the boundary layer flow over a permeable stretching sheet in a casson nanofluid with convective boundary condition

Ambuja Joshi

Abstract
The boundary layer flow formed due to a linearly stretching sheet in a nanofluid is premeditated numerically. The boundary value problem consisting of nonlinear partial differential equations are converted into nonlinear ordinary differential equations, using similarity transformation and are solved numerically using Runge-Kutta Fehlberg method, with shooting technique. The transport equations include the effects of Brownian motion and thermophoresis. Unlike the commonly employed thermal conditions of constant temperature or constant heat flux, the present study uses a convective heating boundary conditions. The solutions for the temperature and nanoparticle concentration distribution depend on the following parameters, namely, Casson fluid parameter β, suction/injection parameter f_w Prandtl number Pr , Lewis number Le , Brownian motion parameter Nb , thermophoresis parameter Nt, Biot number Bi and magnetic field parameter M. Numerical results are presented both in graphical forms, illustrating the effects of these parameters on momentum, thermal and concentration boundary layers. The thermal boundary layer thickness increases, with a rise in the local temperature as the Brownian motion, thermophoresis and convective heating, each intensify. The effect of Lewis number on the temperature distribution is insignificant. With the other parameters unchanging, the local concentration of nanoparticle increases as the convective Biot number increases but decreases as the Lewis number increases.

Keywords: Nanofluid, boundary layer flow, stretching sheet, biot number

1. Introduction
A Nanofluid is a fluid containing nanometer sized particles, called Nanoparticles. These fluids are engineered colloidal suspension of nanoparticles in a base fluid. The nanoparticles used in nanofluids are characteristically made of metals, oxides, carbides, or carbon nanotubes. ordinary base fluids include water, ethylene Glycol and oil. Nanofluids have narrative properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engine, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, in grinding, machining and in boiler gas temperature reduction. They demonstrate enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. Knowledge of the rheological behaviour of nanofluids is found to be very basic in deciding their suitability for convective heat transfer applications.
The fluid flow over a stretching surface has significant applications such as extrusion, wire drawing, metal spinning, hot rolling, etc [1-3]. A wide variety of problems dealing with heat and fluid flow over a stretching sheet have been studied with both Newtonian and non-Newtonian fluids and with the addition of obligatory electric and magnetic fields, different thermal boundary conditions, and power law variation of the stretching velocity. A representative sample of the recent literature on the topic is provided by references [4-12]. After the poineering work by Sakiadis [13], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear

stretching surface. The problem of natural convection in a regular fluid past a vertical plate is a conventional problem first studied theoretically by E. Pohlhausen in giving to an experimental study by Schmidt and Beckmann [14]. In the past few years, convective heat transfer in nanofluids has turn out to be a topic of major existing interest. Recently Khan and Pop [15] used the model of Kuznetsov and Nield [16] to study the boundary layer flow of a nanofluid past a stretching sheet with a constant surface temperature. Makinde, and Aziz [17] investigated the effect of a convective boundary condition on boundary layer flow, heat and mass transfer and nanoparticle fraction over a stretching surface in a nanofluid. The transformed non-linear ordinary differential equations governing the flow are solved numerically by the Runge-Kutta Fourth order method. lately, Pal *et al*. [20] investigated the influence of thermal radiation on mixed convection flow of nanofluid caused by nonlinearly stretching/shrinking sheet. Kandasamy and Periasamy [21] considered study, of free convection flow,heat and mass transfer of Newtonianfluid past nonlinearly stretching sheet with the effect of chemical reaction and magnetic field. The laminar boundary layer flow of electrically conducting fluid towards nonlinearly stretching sheet under the influence of first-order chemical reaction was hypothetically studied by Raptis and Perdikis [22]. On the other hand, the numerical and analytical solutions of steady-state boundary layerflow of micropolar fluid induced due to nonlinearly stretching sheet were established by Damseh *et al*. [23] and Magyari and Chamkha [24], respectively. The joint effects of slip and chemical reaction on electrically conducting fluid over a nonlinearly porous stretching sheet were analyzed by Yazdi *et al*. [25]. Bhattacharyya and Layek [26] considered the velocity slip effects on boundary layer flow of viscous fluid past a permeable stretching sheet with chemical reaction. The steady two-dimensional boundary layer flow of Newtonian fluid due to stretching sheet saturated in nanofluid in the presence of chemical reaction is explored by Kameswaran *et al*. [27]. Aurangzaib *et al*. [28] investigated theoretically the brunt of thermal radiation on unsteady natural convection flow produced by stretching surface in the presence of chemical reaction and magnetic field. Shehzad *et al*. [29] reported the effects of magnetic field on mass transfer flow of Casson fluid past a permeable stretching sheet in the presence of chemical reaction. Pal andMandal [30] investigated the characteristics of mixed convection flow of nanofluid towards a stretching sheet under the influence of chemical reaction and thermal radiation. Similarity solutions for unsteady boundary flow of Casson fluid induced due to stretching sheet embedded in a porous medium in the presence of first order chemical reaction were obtained by Makanda and Shaw [31]. on the other hand, convective boundary condition plays a high-flying role in many engineering processes and industries such as gas turbines, material drying, textile drying, laser pulse heating, nuclear plants,transpiration cooling, and food process. It is because that the convective boundary condition applied at the surface is more practical and realistic. The two-dimensional laminar boundary layer flow of Newtonian fluid caused by porous stretching surface in the presence of Convective boundary condition is investigated numerically by Ishak [32]. Makinde and Aziz [33] analyzed steady incompressible flow of nanofluids towards stretching sheet with convective boundary condition using Boungiornomodel. Moreover, RamReddy *et al*. [34] incorporated the effects of Soret and investigated mixed convection flow due to vertical plate in a nanofluid under convective boundary condition. Das *et al*. [35] discussed the heat and mass transfer flow of hydromagnetic nanofluid past a stretching sheet place in a porous medium with convective boundary condition. The three-dimensional laminar flow of Casson nanofluid due to stretching sheet in the presence of convective boundary condition is developed by Nadeem and Haq [36].Malik *et al*. [37] investigated the effect of convective boundary condition past a stretching sheet in the presence of magnetic field. The two-dimensional electrically conducting flow of Casson nanofluid created by stretching sheet with convective boundary condition is performed by Hussain *et al*. [38].

In the above mentioned appraisal of literature,no work has been reported, and to trounce this void, we have considered the study apprehensive to the effect of a convective boundary condition and magnetic field effect on boundary layer flow, heat transfer and nanoparticle fraction over a permeable stretching sheet in a Casson nanofluid, with viscous dissipation.Further investigated the heat,flow and mass transfer charecteristics of the pertinent parameters numerically and displayed them in graphical form.

2. Convective transport equations

consider steady two-dimensional (x, y) boundary layer flow of a nanofluid past a stretching sheet with a linear velocity variation with the distance x i.e. $U_w = cx$ where c is a real positive number, is stretching rate, and x is the coordinate measured from the location, where the sheet velocity is zero

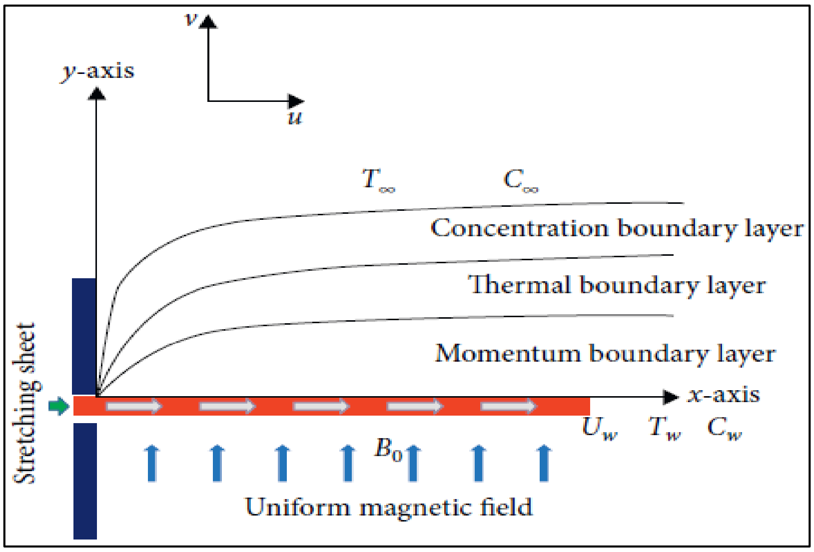

Fig 1: Physical interpretation of the flow model

The sheet surface temperature T_w, to be determined later, is the result of a convective heating process which is characterized by temperature T_f and a heat transfer coefficient h. The nanoparticle volume fraction C at the wall is C_w, while at large values of y,the value is C_∞. The Boungiorno model may be modified for this problem to give the following continuity, momentum, energy and volume fraction equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left\{D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial y}\right)^2\right]\right\}+\frac{\nu}{c_p}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where u and v are the velocity components along the x and y directions, respectively, p is the fluid pressure, ρ_f is the density of base fluid, ν is the kinematic viscosity of the base fluid, α is the thermal diffusivity of the base fluid, $\tau=(\rho c)_p/(\rho c)_f$ is the ratio of nanoparticle heat capacity and the base fluid heat capacity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and T is the local temperature. The subscript ∞ denotes the values of at large values at large values of y where the fluid is quiescent. The boundary condition may b written as

$$y=0, u=cx, v=-v_w, -k\frac{\partial T}{\partial y}=h\left(T_f-T\right), C=C_{w,} \tag{5}$$

$$y\to\infty, u=0, v=0, T=T_\infty, C=C_\infty, \tag{6}$$

We introduce the following dimensionless quantities

$$\eta=(c/\upsilon)^{1/2}y, \psi=(c\upsilon)^{1/2}xf(\eta), \theta=\frac{T-T_\infty}{T_f-T_\infty}, \quad \phi=\frac{C-C_\infty}{C_w-C_\infty}, \tag{7}$$

Where ψ is the stream function with $u=\partial\psi/\partial y, v=-\partial\psi/\partial x$

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-f'^2-Mf'=0, \tag{8}$$

$$\theta''+Prf\theta'+PrNb\phi'\theta'+PrNt\theta'^2+\Pr Ec\left(1+\frac{1}{\beta}\right)f''^2=0, \tag{9}$$

$$\phi''+Lef\phi'+\frac{Nt}{Nb}\theta''=0, \tag{10}$$

subject to the following boundary conditions.

$$f(0)=f_w, f'(0)=1, \theta'(0)=-Bi\left[1-\theta(0)\right], \phi(0)=1, \tag{11}$$

$$f^{'}(\infty)=0,\theta(\infty)=0,\phi(\infty)=0, \quad (12)$$

where primes denote differentiation with respect to η and the five parameters appearing in Eqs. (9-12) are defined as follows.

$$Pr=\frac{\nu}{\alpha}, Le=\frac{\nu}{D_B}, Nb=\frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_{f^\nu}},$$

$$Nt=\frac{(\rho c)_p D_T (T_f - T_\infty)}{(\rho c)_{f^\nu} T_\infty}, Bi=\frac{h(\nu/a)^{1/2}}{k}, f_w = -\frac{v_w}{\sqrt{a\upsilon}}$$

$$EC=\frac{a^2x^2}{c_p(T_w - T_\infty)} \quad \ldots\ldots\ldots\ldots\ldots(13)$$

3. Result and Discussion

Eqs. (8-10) subject to the boundary conditions, Eqs.(11) and (12), were solved numerically using Runge- kutta -Fehlberg fourth order method.

We now turn our attention to the discussion of graphical results that provide additional insights into the problem under investigation. Fig 2 elucidates the fact that increase in suction parameter fw is to enhance velocity profile $f'(\eta)$, this is because, the suction draws more fluid particles in boundary layer region,resulting in decrease of boundary layer thickness.

As noticed in Fig3. velocity profile decrases with increase in Casson fluid parameter β, because enhancement in fluid velocity rises fluid viscosity , thereby one finds deceleration in fluid velocity, which means reduction in yield stress, consequently, thinning of momentum boundary layer thickness is observed.

Where as opposite trend is noticed for the effect of casson fluid parameter β on temperature profile, as depicted in fig4, i.e temperature in the boundary layer increases with increase in Casson fluid parameter, which is consistant with the results obtained By, M. Trivedi1 *et al*. [39], Ullah *et al*. [40], Sudipta Ghosh and Swati Mukhopadhyay[41].

The impact of both Nt and Nb on temperature profile is displayed in fig 5.Here it is observed that temperature increases with increase of Nt as well as Nb. Enhancing Nt and Nb leads to faster random motion of nanoparticles in fluid flow, resulting in increasing thickness of thermal boundary layer, further enlarges temperature of Casson nanofluid abruptly. As in process of thermopheresis, heated nanoparticles close to the sheet move away from heated region to colder region

Fig 6, displays the effect of Lewis number Le, on temperature profile. Lewis number expresses the relative contribution of thermal diffusion rate to species diffusion rate in the boundary layer regime.An increase of Lewis number will reduce thermal boundary layer thickness and will be accompanied with a decrease in temperature. Larger Le will suppress concentration values. I.e inhibit nanoparticle species diffusion. There will be much greater reduction in concentration bopundary layer thickness than thermal boundary layer thickness over an increment in Lewis Number.

Fig. 7 illustrates the effect of Biot number on the thermal boundary layer. As expected, the stronger convection results in higher surface temperatures, causing the thermal effect to penetrate deeper into the quiescent fluid. The temperature profiles depicted in Fig. 8 show that as the Prandtl number increases, the thickness of the thermal boundary layer decreases as the curve become increasingly steeper. Fig 8 shows that the effect of magnetic parameter, on the temperature profiles is noticeable only in region close to the sheet as the curves tend to merge at larger distances from the sheet.

Fig 9, reveal the influence of M on temperature. It is perceptible that increasing values of M enhances temperature in boundary layer region. As it is renowned fact that a resistive-type of force produces the current, that passes through themoving fluid, which is responsible in increasing temperature, resulting in thickening of thermal boundary layer thickness.

Fig 10,reveals the effect made by the viscous dissipation on temperature profile. These profiles are well behaved and very little change occurs in the shapes of profiles with the varying parameter Eckert number Ec. The different values of Ec contribute little to the thickness of the thermal boundary layer.

From Fig 11, effects due to thermophoresis parameter Nb on nanoparticle volume fraction were observed. Due to the increase of thermophoresis parameter Nt, nanoparticle volume fraction increased significantly, and this helped to move the nanoparticles from hot to cold regions. As a result, there is increase in nanoparticle concentration profile.

The effects due to the Brownian motion parameter Nb on nanoparticle concentration profile are exhibited in Fig 12, and here it is noticed that as Nb increases, nanoparticle volume boundary layer thickness decreased.

Nanoparticle concentration profile was found to decrease significantly with the increasing Lewis number Le, as observed from fig 13. With the increase in Le, mass transfer rate increases and as a result concentration boundary layer thickness decreases. For a particular base fluid, an increase in lewis number Le reduces the Brownian diffusion coefficient D_B.Hence the penetration of concentration boundary layer decreases.

It was observed in Fig. 7, that as the convective heating of the sheet is enhanced Bi increases, i.e the thermal penetration depth increases. Because the concentration distribution is driven by the temperature field, one expects that a higher Biot number would endorse a deeper penetration of the concentration. This expectation is certainly realized in Fig. 14, which displays a higher concentrations at higher values of the Biot number, which is consistant with the results of Gbadeyan, J.A et al. [42]

When magnetic M is increased due to rising Lorentz forces, nanoparticle concentration profiles are increased, resulting in thickening of concentration boundary layer , is as shown in fig 15.

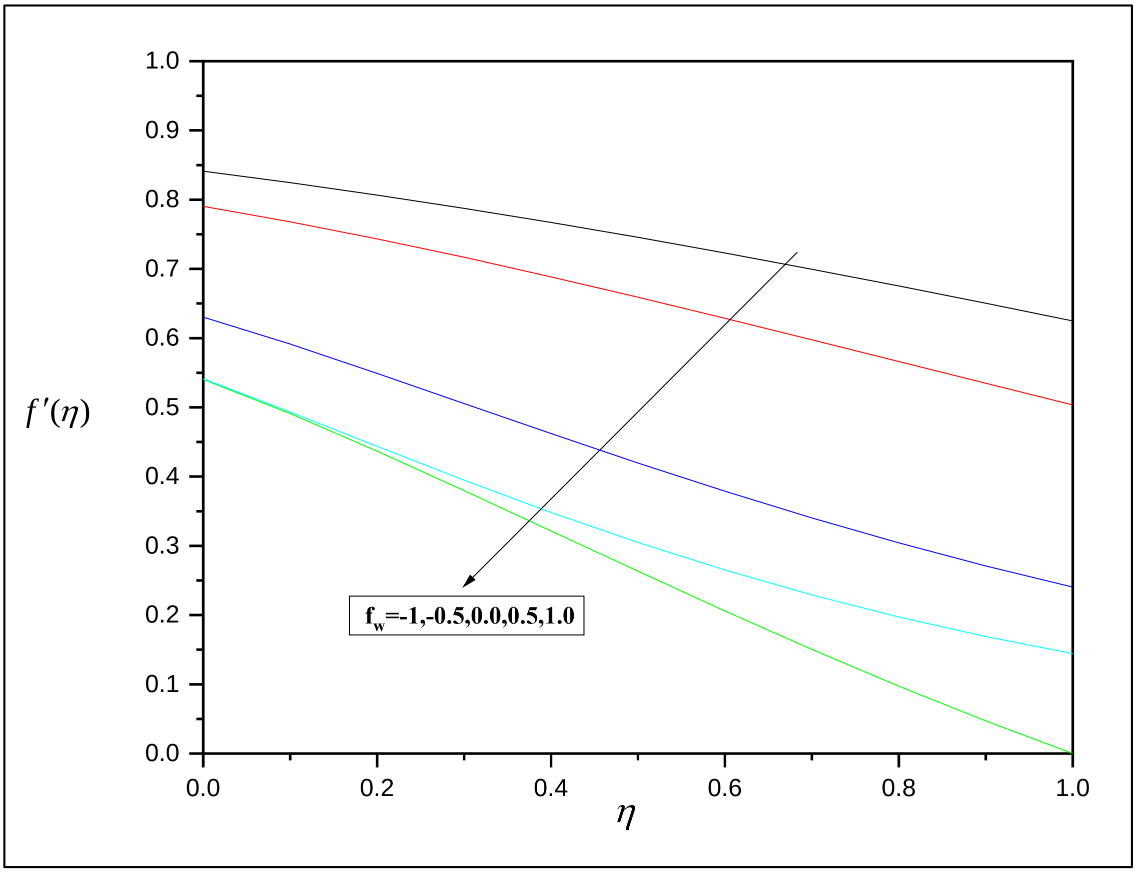

Fig 2: Effect of f_w on velocity profile $f'(\eta)$,when M=2, EC= $Le = 5, Pr = 5, Bi = 0.1$, β=1.0, Nb= Nt= 0.1.

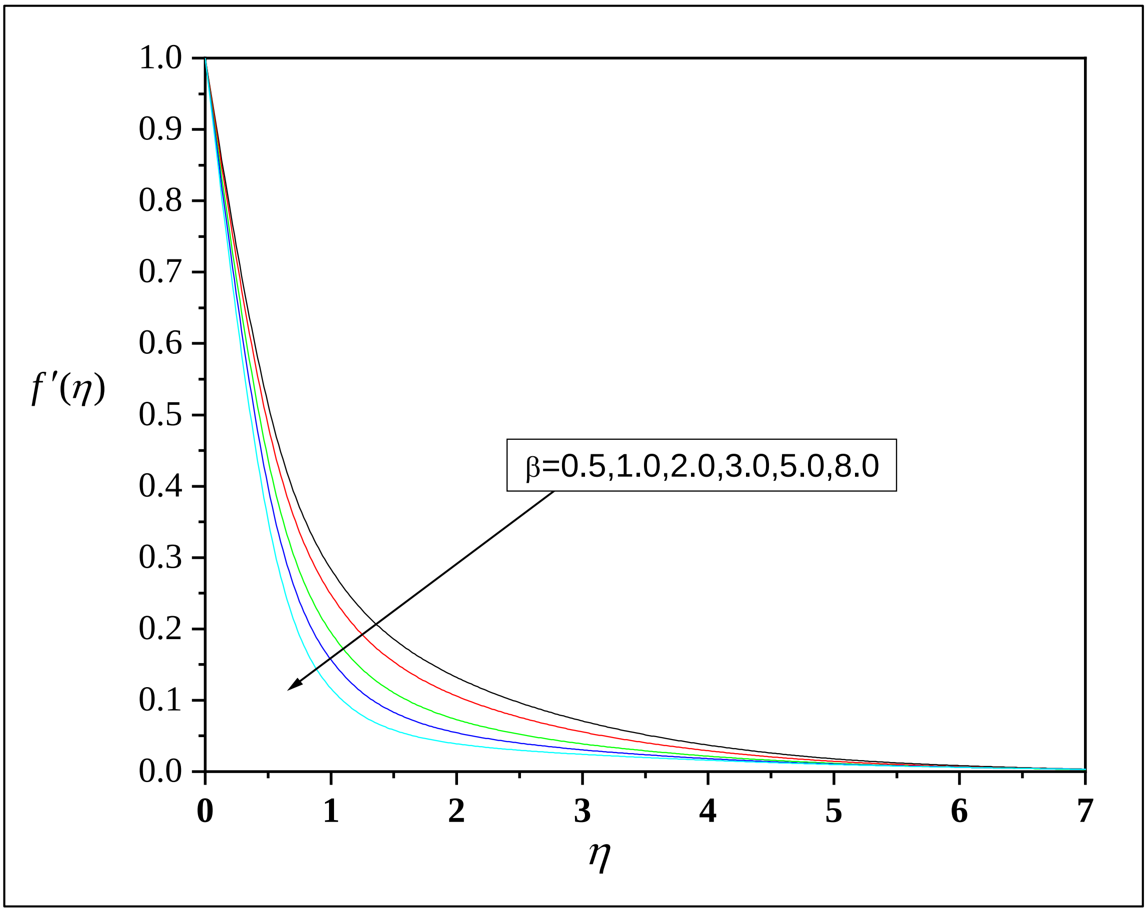

Fig 3: Effect of β on velocity profile $f'(\eta)$,when M=2, EC= $Le = 5, Pr = 5, Bi = 0.1$. Nb= Nt= 0.1, f_w =0.5.

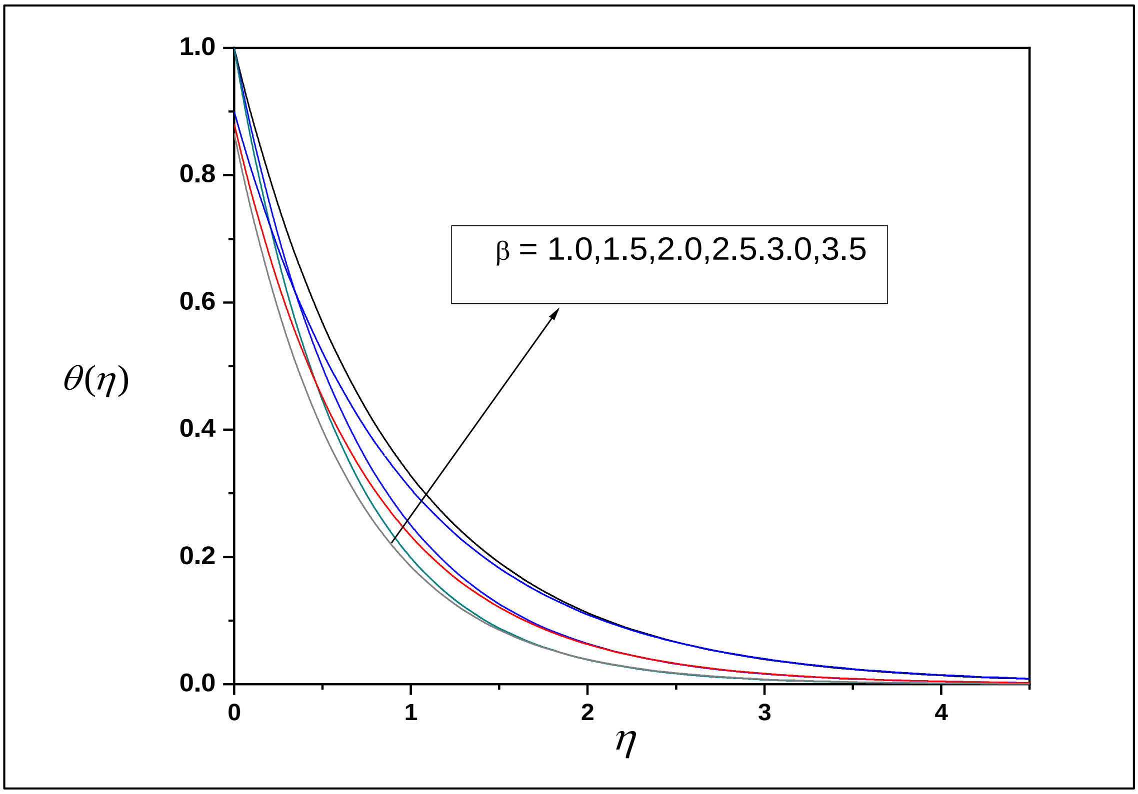

Fig 4: Effect of β on Temperature profile $\theta(\eta)$,when M=2, EC= $Le = 5, Pr = 5, Bi = 0.1$. Nb= Nt= 0.1, f_w =0.5.

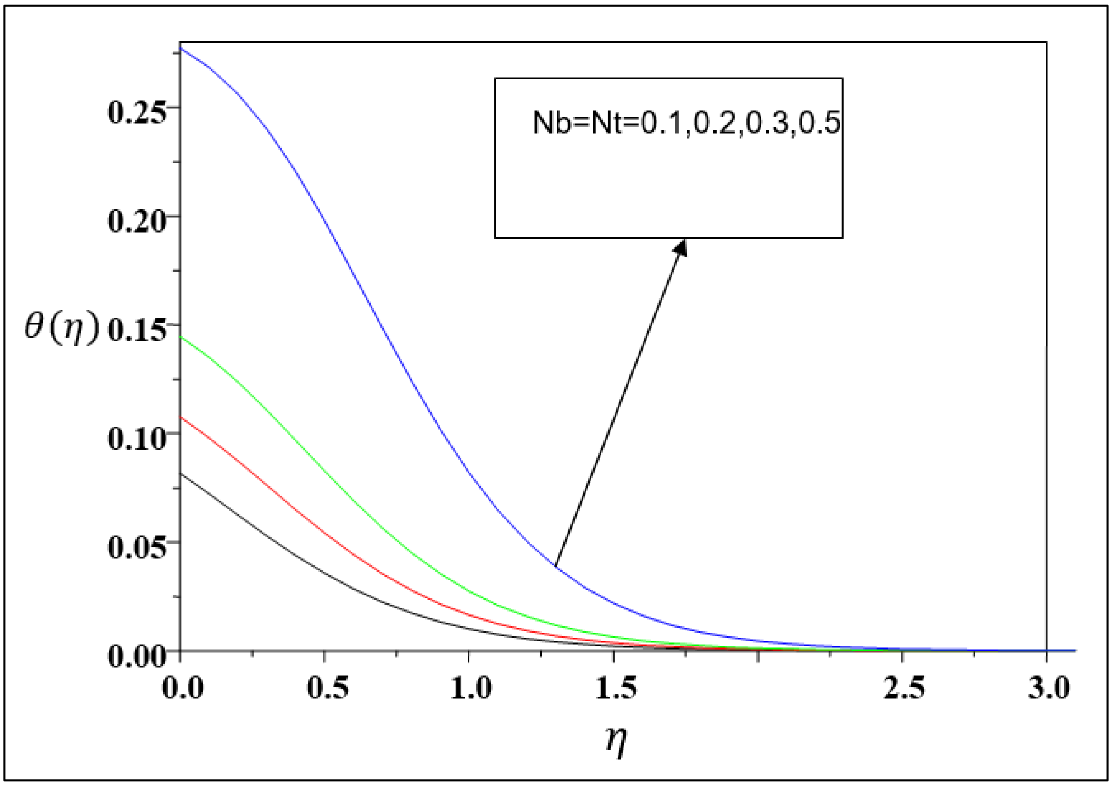

Fig 5: Effect of Nt and Nb on temperature profiles when M=2, EC= $Le = 5, Pr = 5, Bi = 0.1$,β=1.0. f_w =*1.0.*

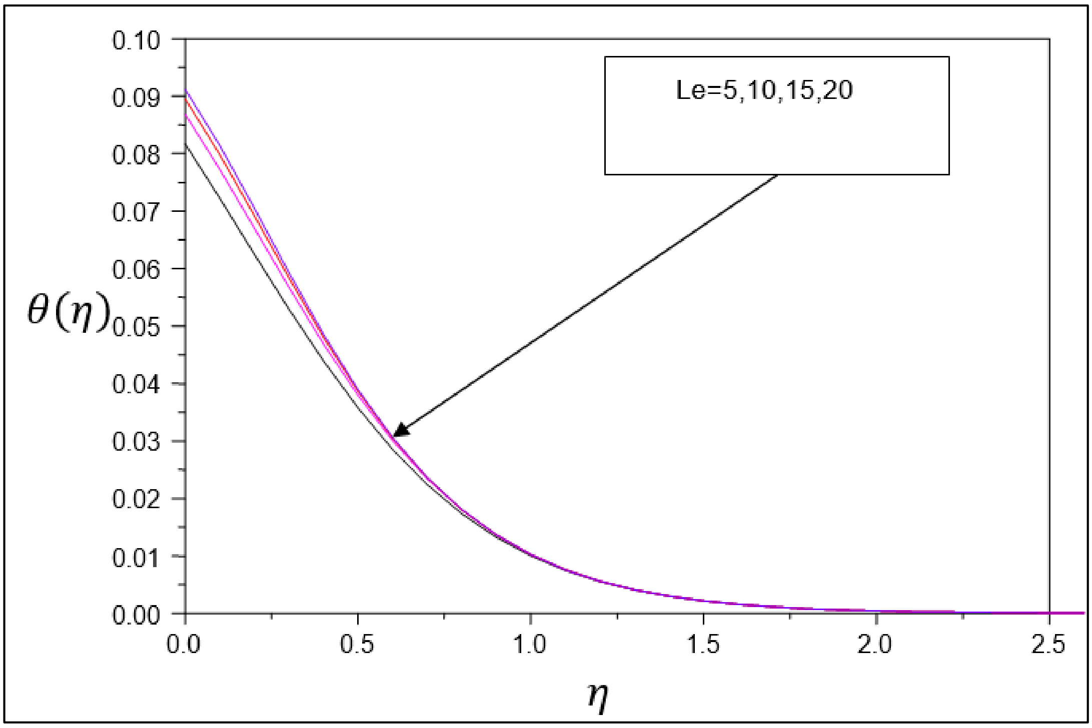

Fig 6: Effect of Le on temperature profiles when M=2, $Nt = Nb = 0.1, Pr = 5, EC = 5, Bi = 0.1, f_w = 0.5, \beta = 1.0.$

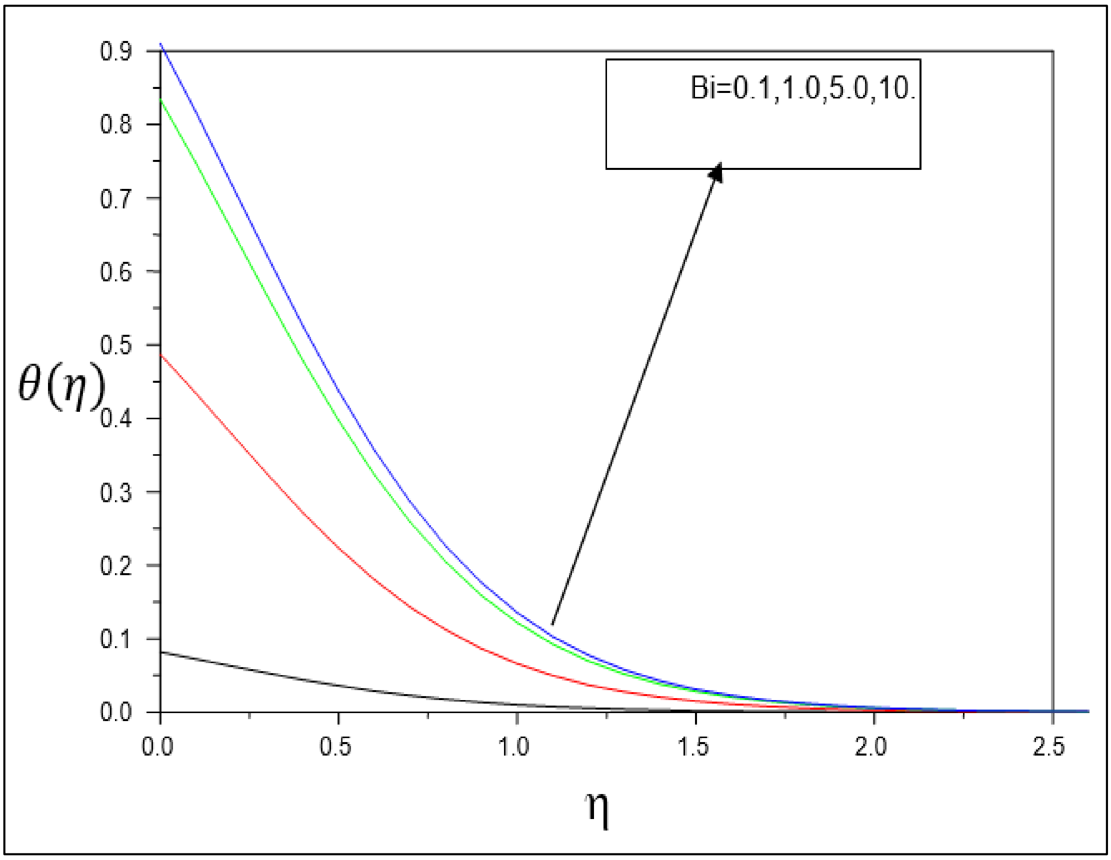

Fig 7: Effect of Bi on temperature profiles when EC=5, M=2, $Nt = Nb = 0.1, Pr = Le = 5, \beta = 2.0, f_w = 0.5.$

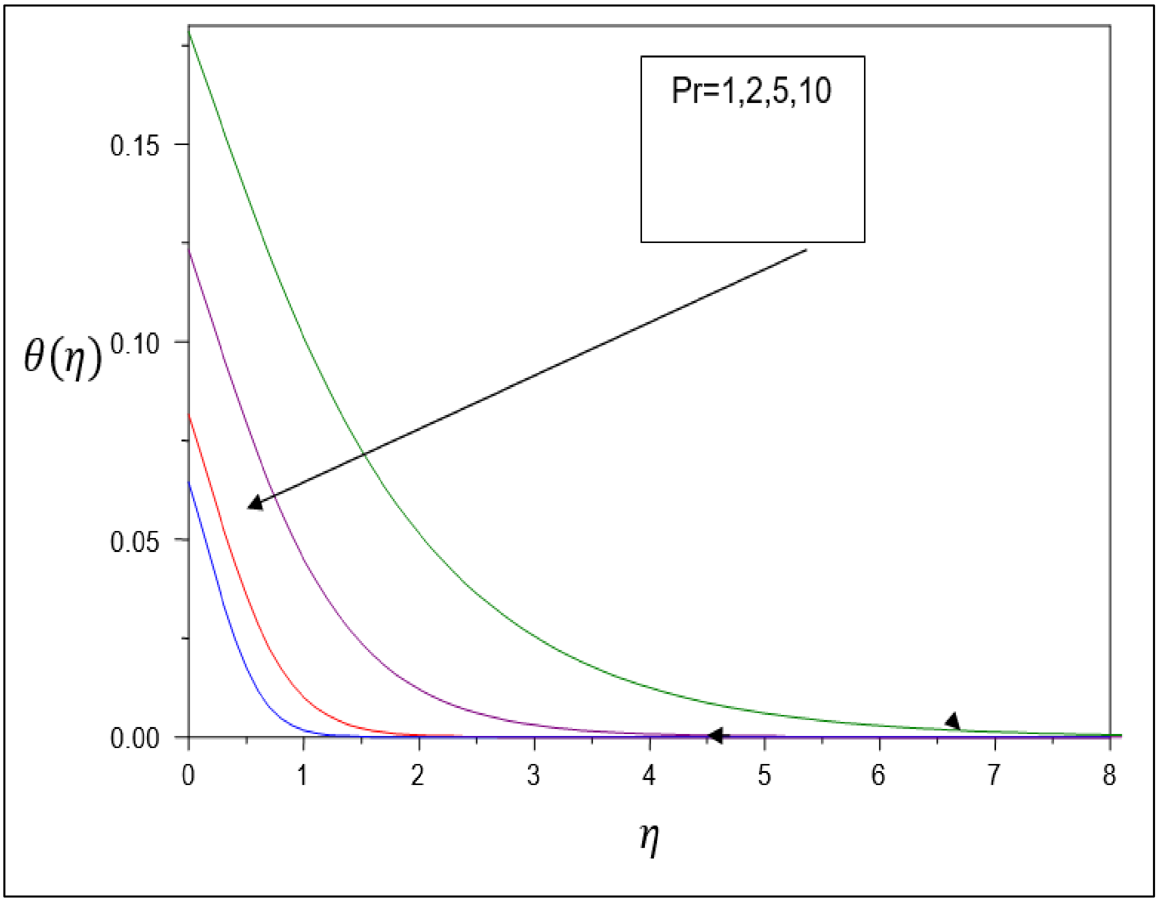

Fig 8: Effect of Pr on temperature profiles when M=2, $Nt = Nb = Bi = 0.1, Le = 5, EC = 5, f_w = 0.5, \beta = 1.0.$

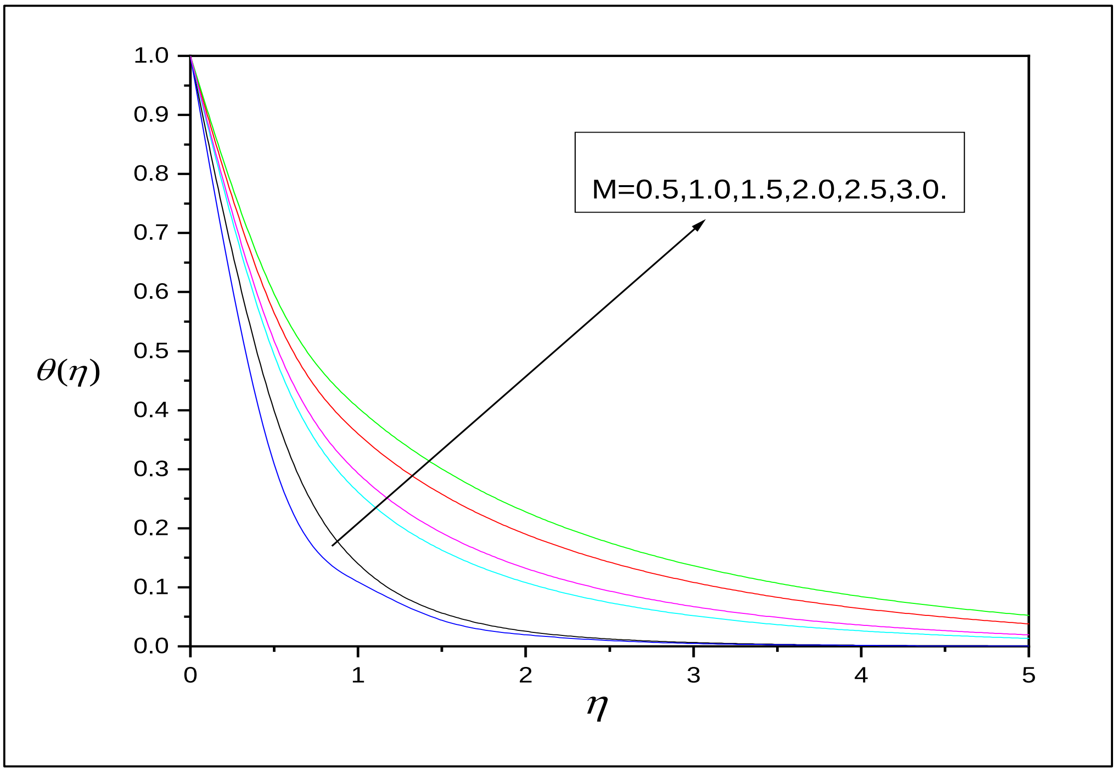

Fig 9: Effect of M on temperature profiles when $Nt = Nb = Bi = 0.1, Le = Pr = 5 = EC = 5, \beta = 2.0, f_w = 0.5.$

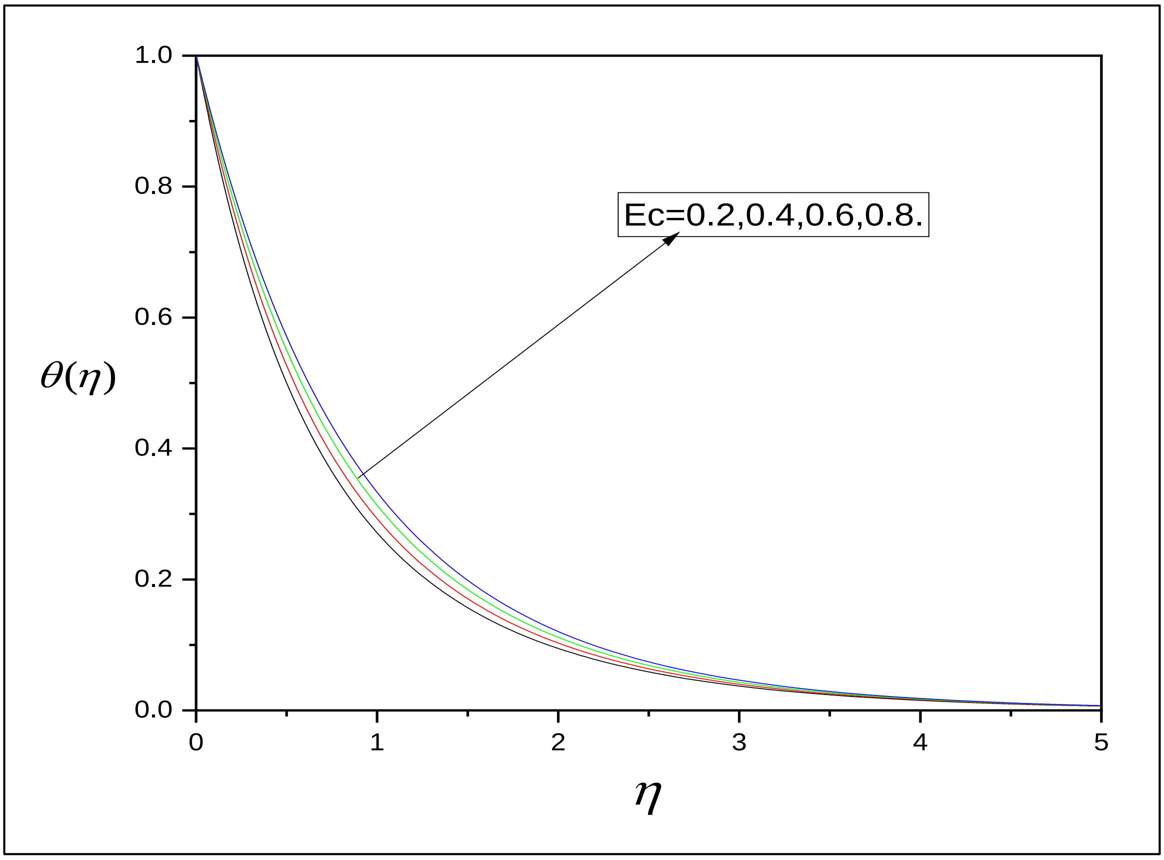

Fig 10: Effect of Eckert Number Ec on Temperature profile θ(η) for different values of Pr=1.0,Nt=Nb=5,Bi=0.5.Le=5.0,Pr=5.0, $f_w = 0.5, \beta = 2.0$.

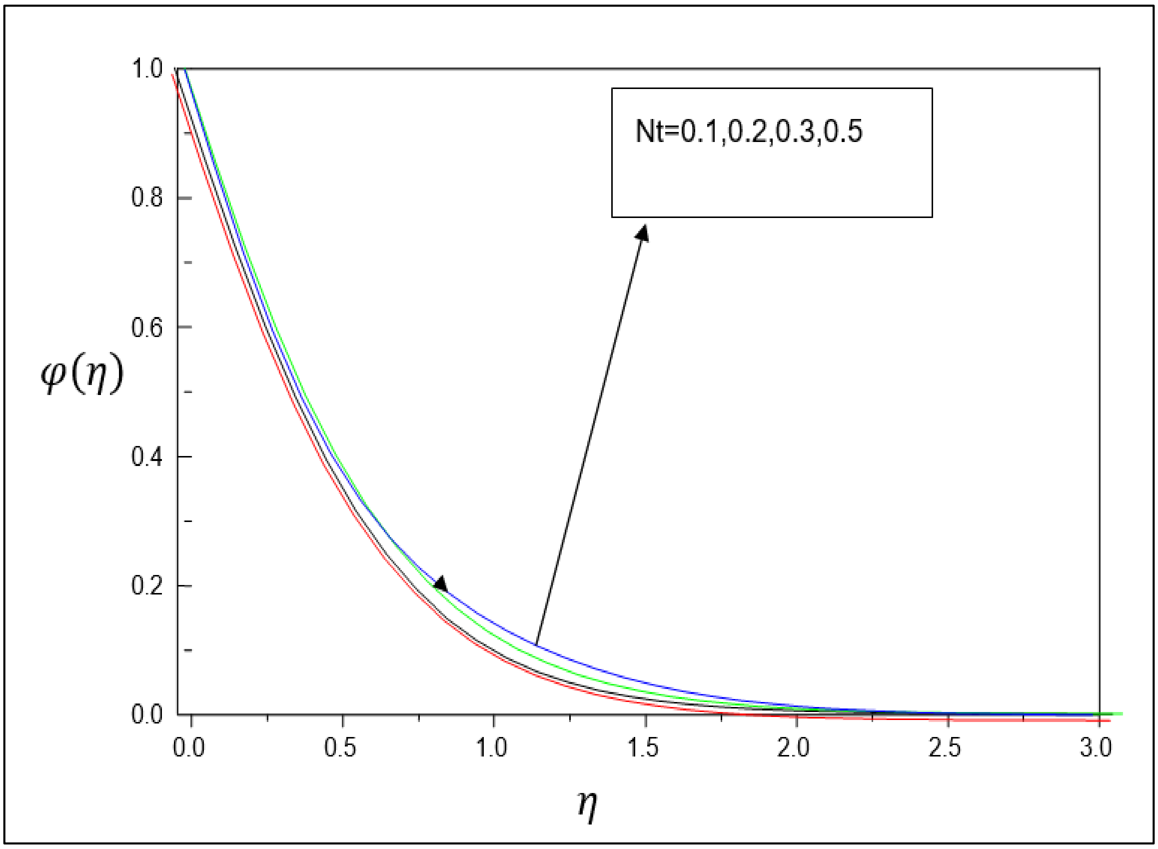

Fig 11: Effect of Nt and Nb on concentration profiles when $Le = 5, Pr = 5, Bi = 0.1$,M=1.0,Ec=0.4, $f_w = 0.5, \beta = 2.0$.

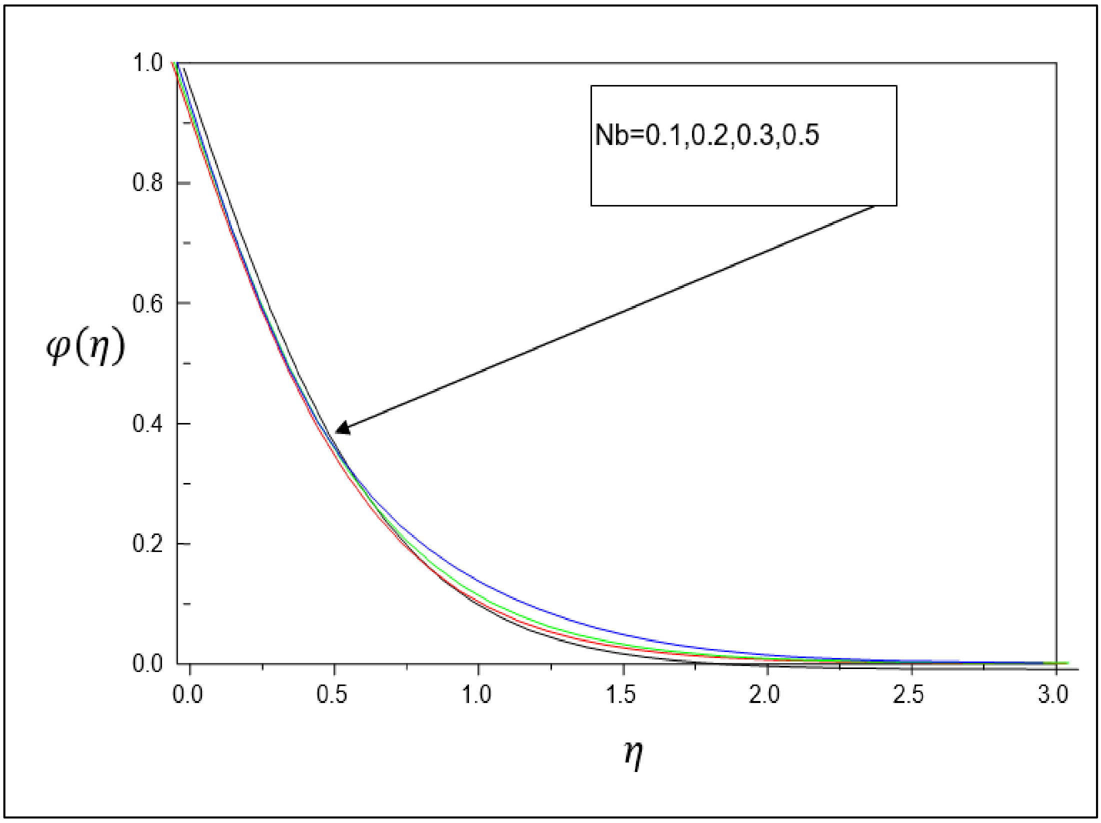

Fig 12: Effect of Nb on concentration profiles when $Nt = 0.1, Pr = 5,\ Le = 5, Bi = 0.1,$***M=1.0,Ec=0.4,*** $f_w = 0.5, \beta = 2.0$.

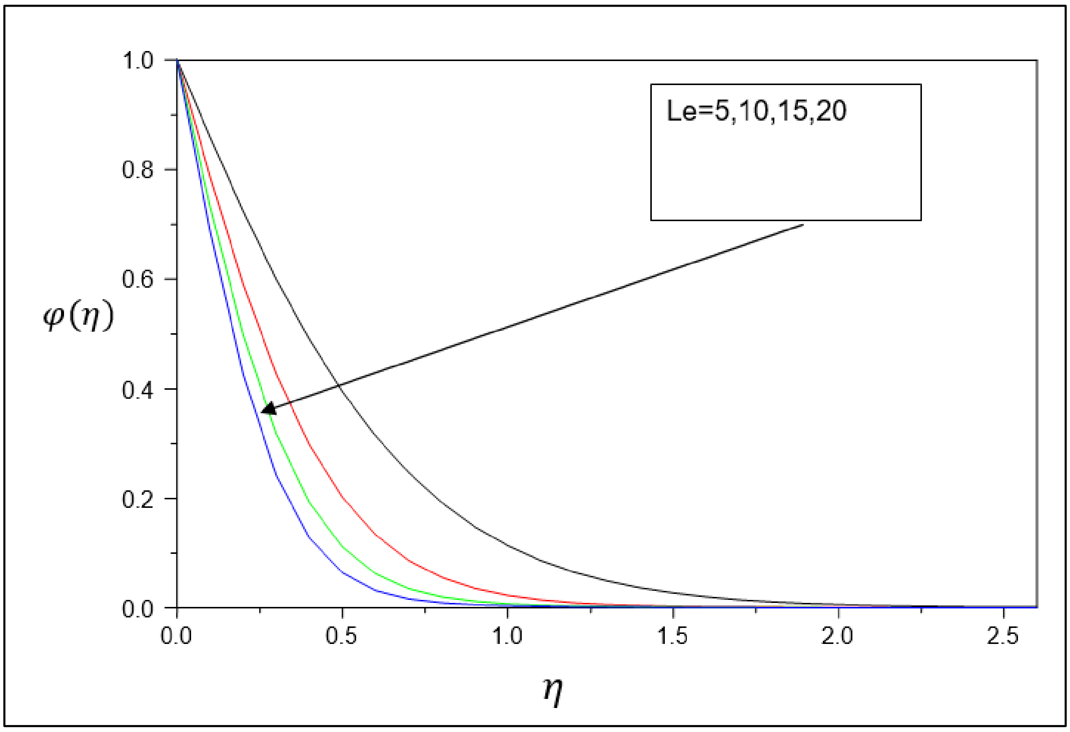

Fig 13: Effect of Le on concentration profiles when $Nt = Nb = 0.1, Pr = 5,\ Le = 5,$ ***,M=1.0,Ec=0.4,*** $f_w = 0.5, \beta = 2.0$.

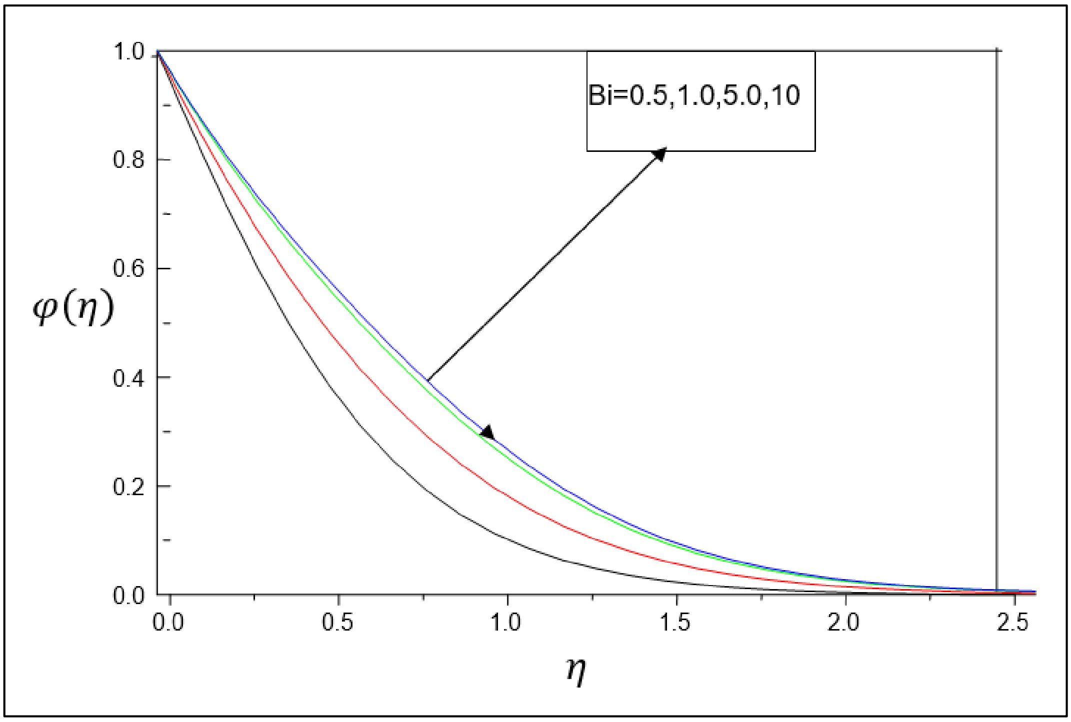

Fig 14: Effect of Bi on concentration profiles when $Nt = Nb = 0.1, Pr = Le = 5.$,M=1.0,Ec=0.4, $f_w = 0.5, \beta = 2.0$.

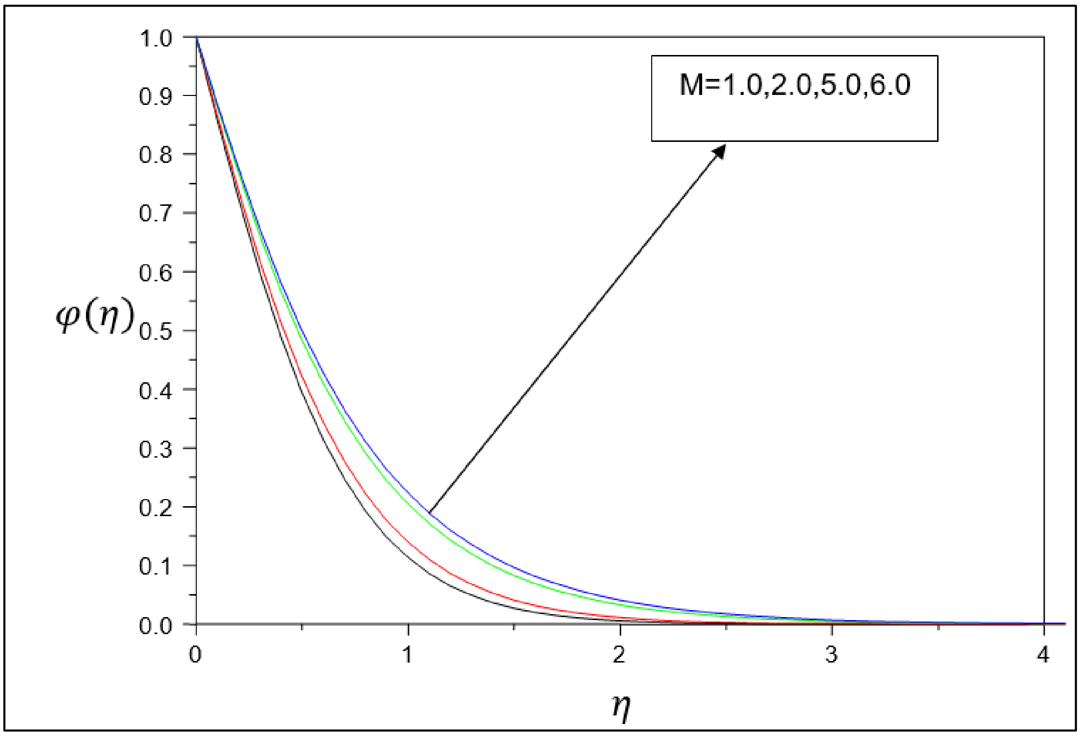

Fig 15: Effect of M on concentration profiles when $Nt = Nb = Bi = 0.1, Le = Pr = 5.$

4. Conclusion

A numerical study of the boundary layer Casson nanofluid flow, induced as as a result of motion of a linearly permeable stretching sheet has been performed. The use of a convective hating boundary condition instead of a constant temperature or a constant heat flux makes this study more general narrative. The following conclusions are derived

1. The transport of momentum, energy and concentration of nanoparticles in the respective boundary layers depends on ,Brownian motion parameter Nb, thermophoresis parameter Nt, Prandtl number Pr, Lewis number Le, Eckert number Ec, Biot number Bi and Magnetic parameter M Casson fluid parameter β,Suction/Injection parameter f_w
2. For a fixed Pr, Le, EC and Bi the thermal boundary thickens and the local temperature rises as the Brownian motion and thermophoresis effects intensify. A similar effect on the thermal boundary is observed when Nb, Nt, Le and Bi are kept fixed and the Prandtl number Pr is increased or when Pr, Nb, Nt and Le are kept fixed and the Biot number is increased.

However, when Pr, Nb, Nt and Bi are kept fixed, and the Lewis number is increased, the temperature distribution is affected only minimally.

3. With the increase in Bi, the concentration boundary layer thickens but the concentration boundary layer becomes thinner as Le increases.
4. Effect of Casson fluid parameter β was to suppress the velocity field, whereas the temperature of
5. nanofluid increase with the increase in Casson fluid parameter β.Fluid velocity was higher for blowing ($f_w > 0$) compared to that of suction. (f_w <0)

Nomenclature

B_i Biot number
a a positive constant associated with linear strecthing
D_B Brownian diffusion coefficient
D_T Thermophoretic diffusion coefficient
$f(\eta)$ Dimensionless steam function
g Gravitational acceleration
h Convective heat transfer coefficient
k Thermal conductivity of the nanofluid
Le Lewis number
Nb Brownian motion parameter
Nt Thermophoresis parameter
Nu Nusselt number
Nur Reduced Nusselt number
$\Pr$ Prandtl number
p pressure
q''_m Wall mass flux
q''_w Wall heat flux
Re_x Local Reynolds number
Sh Sherwood number
Shr Reduced Sherwood number
M Magnetic number
T Local fluid Temperature
T_f Temperature of the hot fluid
T_w Sheet surface (wall) temperature
T_∞ Ambient temperature
u, v Velocity components in x and y directions
C nanoparticle volume fraction
C_w Nanoparticle volume fraction at the wall
C_∞ Nanoparticle volume fraction at large values of y(ambient)
EC Eckert number

Greek symbol

α Thermal diffusivity of the base fluid
η Similarity variable
θ Dimensionless temperature
φ Dimensionless volume fraction
μ Absolute viscosity of the base fluid
υ Kinematic viscosity of the base fluid
ρ_f Density of the base fluid

ρ_p Nanoparticle mass density

$(\rho c)_f$ Heat capacity of the base fluid

$(\rho c)_p$ Heat capacity of the nanoparticle material

$\tau = (\rho c)_p / (\rho c)_f$

ψ Stream function

5. References

1. Altan T, Oh S, Gegel H. Metal foming fundamentals and Applications. American Society of Metals, Metals Park, OH, 1979.
2. Fisher EG. Extrusion of plastics, Wiley, New York, 1976.
3. Tidmore Z, Klein I. Engineering Principles of Plasticating Extrusion, Polymer Science and Engineering Series. Van Norstrand, New York, 1970.
4. Crane IJ. Flow past a stretching plane, J. Appl. Math. Phys. (ZAMP). 1970; 21:645-647.
5. Grupka JL, Bobba KM. Heat transfer characteristics of a continuous stretching surface with variable temperature, J Heat transfer. 1985; 107:248-250.
6. Gupta PS, Gupta AS. Heat and mass transfer on a stretching sheet with suction or blowing, Can. J Chem. Eng. 1977; 55:744-746.
7. Andersson HI. Slip flow past a stretching surface, Acta Mech. 2002; 158:121-125.
8. Dutta BK, Roy P, Gupta AS. Temperature field in flow over a stretching sheet with uniform heat flux, Int. Commun. Heat Mass Transfer. 1985; 12:89-94.
9. Fang T. Flow and heat transfer characteristics of boundary layer over a stretching surface with a uniform-shear free stream, Int. J. Heat Mass Transf. 2008; 51:2199-2213.
10. Magyari E, Keller B. Exact solutions of boundary layer equations for a stretching wall, Eur. J. Mech. B-Fluids 2000; 19:109-122.
11. Labropulu F, Li D, Pop I. Non orthogonal stagnation point flow towards a stretching surface in a non-Newtonian fluid with heat transfer, Int. J Therm. Sci. 2010; 49:1042-1050.
12. Prasad KV, Vajravelu K, Dutta PS. The effects of variable fluid properties on the hydro-magnetic flow and heat transfer over a non-linearly stretching sheet, Int. J Therm. Sci. 2010; 40:603-610.
13. Sakiadas BC. Boundary layer behavior on continuous solid surfaces: I Boundary layer equations for two dimensional and flow, AIChE J. 1961; 7:26-28.
14. Schmidt E, Beckmann W, Das Temperature-und Geschwindikeitsfeld voneiner warme abgebenden senkrechten platte bei naturlicher konvection, II. Die Versuche und ihre Ergibnisse, Forcsh, Ingenieurwes. 1930; 1:391-406.
15. Khan WA, Pop I. Boundary-layer flow of a nanofluid past a stretching sheet, Int. J. Heat Mass Transfer. 2010; 53:2477-2483.
16. Kuznetsov AV, Nield DA. Natural convective boundary-layer flow of a nanofluid past a vertical plate, Int. J Therm. Sci. 2010; 49:243-247.
17. Mankinde OD, Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition, Int. J Therm. Sci. 2011; 501:326-1332.
18. wang CY. Free convection on a vertical stretching surface, J appl. math. Mech. (ZAMM). 1989; 69:418-420.
19. Gorla RSR, Sidawi I. free convection on a vertical stretching surface with suction and blowing, Appl. Sci. Res. 1994; 52:247-257.
20. Pal D, Mandal G, Vajravalu K. Soret and Dufour effects on MHD convective–radiative heat and mass transfer of nanofluids over a vertical non-linear stretching/shrinking sheet. Appl Math Comput. 2016; 287-288:184-200.doi:10.1016/j.amc.2016.04.037
21. Kandasamy R, Periasamy K, KSP Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. Int J Heat Mass Transf. 2005; 48:4557-4561.
22. Raptis A, Perdikis C. Viscous flow over a non-linearly stretching sheet in the presence of a chemical reaction and magnetic field. Int J Non Linear Mech, 2006; 41:527-529.
23. Damseh RA, Al-Odat MQ, Chamkha AJ, Shannak BA, Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. Int J Therm Sci. 2009; 48:1658-1663.
24. Magyari E, Chamkha AJ. Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. Int J Therm Sci, 2010; 49:1821-1828.
25. Yazdi MH, Abdullah S, Hashim I, Sopian K, Slip MHD liquid flow and heat transfer over non-linear permeable stretching surface with chemical reaction. Int J Heat Mass Transf, 2011; 54:3214-3225.
26. Bhattacharyya K, Layek GC. Slip effect on diffusion of chemically reactive species in boundary layer flow over a vertical stretching sheet with suction or blowing. Chem Eng Commun, 2011; 198:1354-1365.
27. Kameswaran PK, Narayana M, Sibanda P, Murthy PVSN, Hydromagnetic nanofluid flow due to a stretching or shrinking sheet with viscous dissipation and chemical reaction effects. Int J Heat Mass Transf. 2012; 55:7587-7595.
28. Aurangzaib, Kasim ARM, Mohammad NF, Shafie S. Effect of thermal stratification on MHD free convection with heat and mass transfer over anunsteady stretching surface with heat source, Hall current and chemical reaction. Int J Adv Eng Sci Appl Math, 2012; 4217-225.
29. Shehzad SA, Hayat T, Qasim M, Asghar S. Effects of mass transfer on MHD flow of Casson fluid with chemical reaction and suction. Brazilian J Chem Eng. 2013; 30:187-195.

30. Pal D, Mandal G, Influence of thermal radiation on mixed convection heat and mass transfer stagnation-point flow in nanofluids over stretching/shrinking sheet in a porous medium with chemical reaction. Nucl Eng Des, 2014; 273:644–652.
31. Makanda G, Shaw S, PS. Diffusion of of chemically reactive species in Casson fluid flow over an unsteady stretching surface in porous medium inthe presence of a magnetic field. Math Probl Eng, 2014; 3:216-221.
32. Ishak A. Similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition. Appl Math Comput. 2010; 217:837-842.
33. Makinde OD, Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Int J Therm Sci, 50(2011)1326–1332.
34. Ram Reddy C, Murthy PVSN, Chamkha AJ, Rashad a M. Soret effect on mixed convection flow in a nanofluid under convective boundary condition. Int J Heat Mass Transf. 2013; 64:384-392.
35. Das K, Duari PR, Kundu PK, Numerical simulation of nanofluid flow with convective boundary condition. J Egypt Math Soc. 2014; 23:435-439.
36. Nadeem S, Ul Haq R, Akbar NS. MHD three-dimensional boundary layer flow of Casson nanofluid past a linearly stretching sheet with convective boundary condition. IEEE Trans Nanotechnol. 2014; 13:109-115.
37. Malik R, Khan M, Munir A, Khan WA. Flow and heat transfer in Siskofluid with convective boundary condition. PLoS One, 2014, 9 http://dx.doi.org/10.1371/journal.pone.0107989.
38. Hussain T, Shehzad SA, Alseidi A, Hayat T, Ramzan M. Flow of Casson nanofluid with viscous dissipation and convective conditions: a mathematical model, J Cent south Univ. 2015; 22:1132-1140.
39. Trivedi M *et al.*, A Paired Quasi-linearization on Magnetohydrodynamic Flow and Heat Transfer of Casson Nanofluid with Hall Effects J Appl. Comput. Mech. 2019; 5(5):849-860.
40. Ullah *et al.*, MHD natural convection flow of Casson nanofluid over nonlinearly stretching Sheet through porous medium withchemical reaction and thermal radiation, Nanoscale Research Letters. 2016; 11:527
41. Sudipta Ghosh, Swati Mukhopadhyay. MHD slip flow and heat transfer of Casson nanofluid over an exponentially stretching permeable sheet. 2017; 14(4):4785-4804.
42. Gbadeyan JA *et al.*, Boundary Layer Flow of a Nanofluid Past a Stretching Sheet with a Convective Boundary Condition in the Presence of Magnetic Field and Thermal Radiation, Australian Journal of Basic and Applied Sciences. 2011; 5(9):1323-1334.