International Journal of Statistics and Applied Mathematics 2020; 5(2): 77-79

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2020; 5(2): 77-79

www.mathsjournal.com
Received: 10-01-2020
Accepted: 12-02-2020

Dr. R Murugesan
Associate Professor and Head, Department of Mathematics St, John's College, Palayamkottai, Tirunelveli, Tamil Nadu, India

R Seethalakshmi
Research Scholar, Department of Mathematics, The MDT Hindu College, Pettai, Manonmaniam Sundaranar University Abishekapatti, Tirunelveli, Tamil Nadu, India

V Durai Murugan
Research Scholar, Department of Mathematics, Rani Anna Government College for Women, Gandhi Nagar, Manonmaniam Sundaranar University Abishekapatti, Tirunelveli, Tamil Nadu, India

Corresponding Author:
Dr. R Murugesan
Associate Professor and Head, Department of Mathematics St, John's College, Palayamkottai, Tirunelveli, Tamil Nadu, India

A study on Complementedness in the subgroup lattices of 3 X 3 matrices over Z_2

Dr. R Murugesan, R Seethalakshmi, and V Durai Murugan

Abstract
In this paper we verify the complementedness in the subgroup lattice of the group of 3x3 matrices over Z_2.

Keywords: Matrix group, subgroups, lattice, complementedness

1. Introduction
Let L(G) be the Lattice of Subgroups of G, where G is a group of 3x3 matrices over Z_p having determinant value 1 under matrix multiplication modulo p, where p is a prime number.

$$\text{Let } \mathcal{G} = \left\{ \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} : a,b,c,d,e,f,g,h,i \in Z_p, \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0 \right\}$$

Then $\mathcal{G}$ is a group under matrix multiplication modulo p.

$$\text{Let} G = \left\{ \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in \mathcal{G} : \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1 \right\}$$

Then G is a subgroup of $\mathcal{G}$.

we have, $o(\mathcal{G}) = (p^n - 1)(p^n - p)(p^n - p^2) \ldots \ldots (p^n - p^{n-1})$ [9]
and $o(G) = \frac{(p^n-1)(p^n-p)(p^n-p^2)\ldots\ldots(p^n-p^{n-1})}{p-1}$. [9]

In this paper, we are going to study about the complementedness in the subgroup lattice of the group of 3 x 3 matrices over Z_2.
For ready reference we give the diagram of the lattice of subgroups of G when p=2 and for p=3 we give a split up of the diagram which are found in the thesis of V.Durai Murugan [3].

2.Preliminaries
Definition 2.1: (Poset)
A partial order on a non-empty set P is a binary relation$\leq$ on P that is reflexive, anti-symmetric and transitive. The pair (P, $\leq$) is called a **partially ordered set or poset**. A poset. (P, $\leq$) is totally ordered if every x, y $\in$ P are comparable, that is either x $\leq$ y or y $\leq$ x. A non-empty subset S of P is a chain in P if S is totally ordered by $\leq$.

Definition 2.2
Let (P, $\leq$) be a poset and let S$\subseteq$P. An upper bound of S is an element x$\in$P for which s$\leq$x for all s$\in$S. The least upper bound of S is called the supremum or join of S.A lower bound for S is an

element $x \in P$ for which $x \leq s$ for all $s \in S$. The greatest lower bound of S is called the infimum or meet of S.

Definition 2.3: (Lattice)
Poset (P, ≤) is called a lattice if every pair x, y elements of P has a supremum and an infimum, which are denoted by x Ú y and x **Ù** y respectively.

Definition 2.4:(Covering Relation)
In the poset (P,≤), a covers b or b is covered by a (in notation, a > b or b < a) if and only if b < a and, for no x, b < x < a.

Definition 2.5: (Atom)
An element 'a' is an **atom**, if a > 0 and a dual atom, if a < 1.

Definition 2.6: (Complete Lattice)
A poset is said to be **complete** lattice if all its subsets have both join and meet. In particular, every complete lattice is a **bounded** lattice.

Definition 2.7: (Complemented Lattice)
Let L be a bounded lattice with greatest element 1 and least element 0. Two elements x and y of L are said to be complements of each other if x Ú y = 1 and x **Ù** y = 0. If every element of L has a complement, then L is called a complemented Lattice.
We give below the diagram of L(G) when p=2.

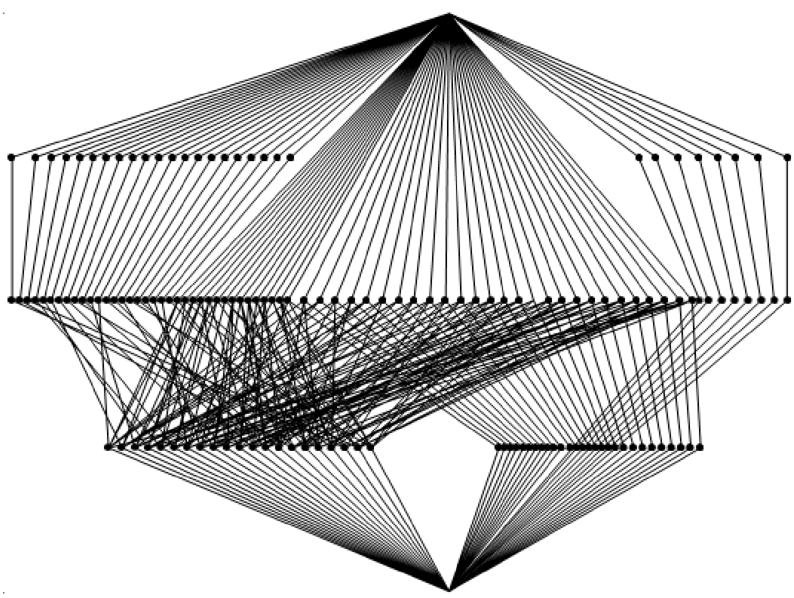

Fig 2.1: L(G) when p=2

I Row (Left to Right): $H_1, H_2, \ldots, H_{21}$ and $K_1, K_2, \ldots, K_{28}$.
II Row (Left to Right): $L_1, L_2, \ldots, L_{35}$ and $M_1, M_2, \ldots, M_{28}$ and $N_1, N_2, \ldots, N_8$.
III Row (Left to Right): $P_1, P_2, \ldots, P_{21}$ and $S_1, S_2, \ldots, S_8$

3. Subgroups of G of different orders in L (G) over Z_2 [3]
Let H be an arbitrary subgroup of G of order 2. Then the number of subgroups of order 2 is 21. Let K be an arbitrary subgroup of G of order 3. Then the number of subgroups of order 3 is 28. Let L be an arbitrary subgroup of G of order 4. Then the number of subgroups of order 4 is 35. Let M be an arbitrary subgroup of G of order 6. Then the number of subgroups of order 6 is 28. Let N be an arbitrary subgroup of G of order 7. Then the number of subgroups of order 7 is 8. Let P be an arbitrary subgroup of G of order 8. Then the number of subgroups of order 8 is 21. Let S be an arbitrary subgroup of G of order 21. Then the number of subgroups of order 21 is 8.
We observe that, the first 21 subgroups of order 4 contain only one subgroups of order 2 and next 14 subgroups of order 4 contains exactly three subgroups of order 2. Each subgroups of order 6 contains exactly three subgroups of order 2 and one subgroup of order 3. Each subgroups of order 8 contains one subgroup of order 4 and four subgroups order 2. Each subgroups of order 21 contains one subgroup of order 7 and seven subgroups of order 3.

4. Complementedness in the lattice of subgroups of the group of 3 x 3 matrices over Z_2
Lemma 4.1
L (G) is complemented lattice if p=2.

Proof:
For L(G) when p=2, the elements of L(G) when p=2 and their respective complements are given in the following table.

Table 4.1: Complements of elements in L(G) when p=2

Subgroup	Complement	Subgroup	Complement	Subgroup	Complement	Subgroup	Complement
H_1	N_1	K_{13}	S_5	L_{18}	K_{18}	M_{16}	N_8
H_2	N_2	K_{14}	S_6	L_{19}	K_{19}	M_{17}	S_1
H_3	N_3	K_{15}	S_7	L_{20}	K_{20}	M_{18}	S_2
H_4	N_4	K_{16}	S_8	L_{21}	K_{21}	M_{19}	S_3
H_5	N_5	K_{17}	S_1	L_{22}	K_{22}	M_{20}	S_4
H_6	N_6	K_{18}	S_2	L_{23}	K_{23}	M_{21}	S_5
H_7	N_7	K_{19}	S_3	L_{24}	K_{24}	M_{22}	S_6
H_8	N_8	K_{20}	S_4	L_{25}	K_{25}	M_{23}	S_7
H_9	N_1	K_{21}	S_5	L_{26}	K_{26}	M_{24}	S_8
H_{10}	N_2	K_{22}	S_6	L_{27}	K_{27}	M_{25}	N_1
H_{11}	N_3	K_{23}	S_7	L_{28}	K_{28}	M_{26}	N_2
H_{12}	N_4	K_{24}	S_8	L_{29}	N_1	M_{27}	N_3
H_{13}	N_5	K_{25}	S_1	L_{30}	N_2	M_{28}	N_4
H_{14}	N_6	K_{26}	S_2	L_{31}	N_3	N_1	H_1
H_{15}	N_7	K_{27}	S_3	L_{32}	N_4	N_2	H_2
H_{16}	N_8	K_{28}	S_4	L_{33}	N_5	N_3	H_3
H_{17}	N_1	L_1	K_1	L_{34}	N_6	N_4	H_4
H_{18}	N_2	L_2	K_2	L_{35}	N_7	N_5	H_5
H_{19}	N_3	L_3	K_3	M_1	S_1	N_6	H_6
H_{20}	N_4	L_4	K_4	M_2	S_2	N_7	H_7
H_{21}	N_5	L_5	K_5	M_3	S_3	N_8	H_8
K_1	S_1	L_6	K_6	M_4	S_4	P_1	K_1
K_2	S_2	L_7	K_7	M_5	S_5	P_2	K_2
K_3	S_3	L_8	K_8	M_6	S_6	P_3	K_3
K_4	S_4	L_9	K_9	M_7	S_7	P_4	K_4
K_5	S_5	L_{10}	K_{10}	M_8	S_8	P_5	K_5
K_6	S_6	L_{11}	K_{11}	M_9	N_1	P_6	K_6
K_7	S_7	L_{12}	K_{12}	M_{10}	N_2	P_7	K_7
K_8	S_8	L_{13}	K_{13}	M_{11}	N_3	P_8	K_8
K_9	S_1	L_{14}	K_{14}	M_{12}	N_4	P_9	K_9
K_{10}	S_2	L_{15}	K_{15}	M_{13}	N_5	P_{10}	K_{10}
K_{11}	S_3	L_{16}	K_{16}	M_{14}	N_6	P_{11}	K_{11}
K_{12}	S_4	L_{17}	K_{17}	M_{15}	N_7	P_{12}	K_{12}
Subgroup	**Complement**	**Subgroup**	**Complement**	**Subgroup**	**Complement**	**Subgroup**	**Complement**
P_{13}	K_{13}	P_{18}	K_{18}	S_2	M_2	S_7	M_7
P_{14}	K_{14}	P_{19}	K_{19}	S_3	M_3	S_8	M_8
P_{15}	K_{15}	P_{20}	K_{20}	S_4	M_4		
P_{16}	K_{16}	P_{21}	K_{21}	S_5	M_5		
P_{17}	K_{17}	S_1	M_1	S_6	M_6		

4. Conclusion

In this paper we proved that the complementedness in the subgroup lattice of the group of 3x3 matrices over Z_2.

References

1. Bourbaki N. Elements of Mathematics, Algebra Chapter Springer Verlag Berlin Heidelberg, New York, London Paris Tokio. 1974; 1:1-3.
2. Fraleigh JB. A first course in Abstract Algebra, Addison – Wesley, London, 1992.
3. Durai Murugan V. A study on recent problems in some specific lattices, Ph.D. thesis, Manonmaniam Sundaranar University, 2020.
4. Gardiner CF. A first course in group theory, Springer-Verlag, Berlin, 1997.
5. Gratzer G. General Lattice theory: Birkhauser Veslag, Basel,1998.
6. Herstien IN. Topics in Algebra, John Wiley and sons, New York, 1975.
7. Jebaraj Thiraviam D. A Study on some special types of lattices, Ph.D thesis, Manonmaniam Sundaranar University,2015.
8. Vethamanickam A, Jebaraj Thiraviam. On Lattices of Subgroups, Int. Journal of Mathematical Archiv. 2015; 6(9):1-11.
9. Vethamanickam A, Durai Murugan V. On the Lattice of Subgroups of 3x3 Matrices over Z_2Int.Journal of Scientific Research and Reviews. 2019; 8(2):4107-4128.