International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2020; 5(3): 37-41

www.mathsjournal.com
Received: 11-03-2020
Accepted: 13-04-2020

Forecasting of Lahi production through fuzzy time invariant series models

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Abstract
This paper comparative study of three models and these models are tested for the forecasting of Lahi crop. The data used for present study is of Pantnagar university agricultural farms. The accuracy of the models is examined on the basis of error estimates. The objective of applying the FTS forecasting models is to develop better models for the prediction of small area based crop yield. This study will help farmers and agro based business industry in their future planning and better post harvesting management of their product.

Keywords: Fuzzy logical relations (FLR), Fuzzy Sets (FS), Fuzzy time series model (FTSM).

1. Introduction

FS theory, introduced by Zadeh [20] has advanced in a number of ways and in many disciplines and it applicable to all most all fields of day to day problems as the real life problems involves high degree of vagueness and uncertainty. Mamdani [9] used this concept and applied to approximate reasoning. The study using possibility distributions with Fuzzy Sets for approximate reasoning was presented by Duboi and Parde [3]. A time series is a sequence of observations taken sequentially in time with an intrinsic feature that the typically adjacent observations are dependent. Future values of time series data of agricultural production process are neither exactly governed by a mathematical function nor by a probability distribution. In such a process non-conventional techniques like FTS and other soft computing techniques are powerful tools. Based upon the FS, Song and Chissom [16-18] presented the definition of FTS and outlined its modeling by means of FRE and approximate reasoning and also applied the FTS model to forecast the enrolments of the University of Alabama using historical data. Chen [1] proposed an alternative simplified method of defuzzification using arithmetic operations. Chen and Wang [2] discussed the forecasting methods in details with the help of fuzzy concepts. Hwang, Chen and Lee [8] discussed forecasting problems through FTS. Huarng [7] proposed heuristic models of fuzzy time series for forecasting by the problem specific heuristic knowledge with Chen's model to improve the forecasting. Yu [19] proposed a refined FTS model to adjust the lengths of the intervals during the formulation of fuzzy relationships. Olmedo [4] studied on the forecasting of Spanish unemployment. Egrioglu et al. [5] presented a probabilistic FTS method by using artificial neural network. Rubio, Bermudez and Vercher [15] discussed stock index forecasting by using a new weighted fuzzy-trend time series method. Based on opinion mining Ravi, Vadlamani and Prasad [14] studied the fuzzy formal concept analysis for CRM in financial services. Axiomatic FS based FTS forecasting is presented by Guo, Pedrycz and Liu [6] and obtained some interesting results. Granular computing and bio-inspired optimization is used by Pritpal and Gaurav [10] to study a hybrid FTS forecasting model. Rana [11] presented a study for FTS models for forecasting rice production. The nested interval based FTS model by Rana [12] was tested for fish production forecasting in India and get better forecasted results than other FTS forecasting models. A comprehensive comparative analysis is presented in a study by Rana [13] for the prediction of agricultural crop yield using FTS models. In the present paper a comparative study of three models based on fuzzy time invariant series is carried out and these models are tested for the forecasting of Lahi crop.

2. Preliminaries

Let U universe of discourse; U= {u_1, $u_{2,\ldots}$ u_n}. A FS A on U is defined as

$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 \ldots\ldots\ldots\ldots\ldots.. f_A(u_n)/u_n$ (1)

where f_A is membership function of A, f:U→[0,1] and $f_A(u_i)$ indicates the grade of membership of u_i in A where $f_A(u_i)\in[0,1]$ and i= 1,2…n

Definition 2.1 Let U(t); t is a non negative integer, on which FS $f_i(t)$; i∊ N are defined and F(t) ={$f_i(t)$, i= 1,2,..},then F(t) is called FTS on U(t).

Definition 2.2 Suppose F (t) is caused by either F(t-1) only or by F(t-2) or F(t-3) or . . .or f(t - m) m>0. This relation can be expressed as the following FRE

F(t) =F(t-1)oR(t,t-1) (2)

Or, F(t) = (F(t-1)∪F(t-2)∪ ... .∪F(t-m))oR_0(t,t-m) (3)

And is called first order model of F(t).

Definition 2.3 Suppose F (t) is caused by either F(t-1), F(t-2), F(t-3) . . .and f(t - m) m>0 simultaneously.
This relation can be expressed as the following FRE
F(t) = (F(t-1)×F(t-2×….F(t-m))o$R_{a,}$ (t,t-m) (4)

And is called m^{th} order model of F(t).

Definition 2.4 If in (2) or (3) or (4) the fuzzy relation R(t,t-1), R_0(t,t-m) or $R_{a,}$ (t,t-m) of F(t) is independent of time t that is for different times t_1 and $t_{2,}$
R(t_1,t_1-1)= R(t_2,t_2-1), orR$_0$(t_1,t_1-1)= R_0(t_2,t_2-1) or R_a(t_1,t_1-1)= R_a(t_2,t_2-1), then F(t) is called a time invariant FTS otherwise it is called a time variant fuzzy time series.

Definition 2.4 Suppose F (t) as a fuzzy time series (t= 0, 1,2, ...). For the first order model R (t, t-1) of F(t) for any $f_i(t)$ ∈F(t) where j ∈ J there exist an f_i(t- 1) ∈ F (t-1) where i ∈ I then the fuzzy relation R_{ij} (t, t-1) is such that
f_j (t) = f_i (t-1) o R_{ij} (t, t-1) (5)

Which is equivalent to IF ….THEN rule as "IF f_i (t-1) THEN f_j (t)", R_i (t, t-1)=∪$_{ij}R_{ij}$ (t, t-1) (6)
R_{ij} (t, t-1)= f_i(t-1)× f_i(t-1) (7)
Here the operator "o" is called Mamdami type max min operator.

3. FTS Algorithm

The used FTS forecasting model and its implementation have the following steps

Step	Action
1	Define the universe of discourse for FS
2	Partitioning the universe of discourse into several even length intervals
3	Define the FS (linguistic variables) on universe of discourse
4	Fuzzification of time series data for fuzzy input
5	Computing the fuzzy relationships
6	Computing the forecasted production (fuzzy output)
7	Defuzzification of fuzzy output for crisp forecasting

4. Application of Algorithm on Historical Data

The implementation of the above algorithm for the production forecasting of the lahi crop is based on the 21 years (1981-82) to (2001-02) time series production data of the University farm

Step 1. Define the universe of discourse to accommodate the time series data. It needs the minimum and maximum production and set as D_{min} and D_{max}. Thus universe of discourse U is defined as [D_{min} –D_1, D_{max} - D_2] here D_1 and D_2 are two proper positive numbers. In the present case of production forecasting universe of discourse is U=[400-1100]

Step 2. Partition the universes of discourse into 7 equal length intervals u_1, u_2,...., u_7 such that
u, =[400,500] u_2 =[500,600] u_3 =[600,700] u_4 =[700,800]
u_5 =[800,900] u_6 =[900,1000] u_7=[1000,1100]

Step 3. Define FS A_1, A_2…….A_7 on U for the linguistic value PRODUCTION as
A_1: poor A_2: below average $A_{3:}$ average
$A_{4:}$ good $A_{5:}$ very good A_6: excellent
$A_{7:}$ bumper

These fuzzy sets in terms of its membership to different intervals are expressed as follows:

A_1:	[1/u_1, .5/u_2, 0/u_3, 0/u_4, 0/u_5, 0/u_6, 0/u_7]
A_2:	[.5/u_1, 1/u_2, 5/u_3, 0/u_4, 0/u_5,0/u_6, 0/u_7]
A_3:	[0/u_1, .5/u_2, l/u_3, .5/u_4, 0/u_5, 0/u_6, 0/u_7]
A_4:	[0/u_1, 0/u_2, .5/u_3, 1/u_4, .5/u_5,0/u_6, 0/u_7]
A_5:	[0/u_1, 0/u_2, 0/u_3, .5/u_4, l/u_5, .5/u_6,0/u_7]
A_6:	[0/u_1, 0/u_2, 0/u_3, 0/u_4, .5/u_5, 1/u_6, .5/u_7]
A_7:	[0/u_1, 0/u_2, 0/u_3, 0/u_4, 0/u_5, .5/u_6, 1/u_7]

Step 4. Fuzzification of the time series data for the fuzzy input to the models are obtained as:

Table 1: Fuzzification of Lahi production

Year	Production Kg/hect.	A_1	A_2	A_3	A_4	A_5	A_6	A_7	Fuzzified production
81-82	1025	0	0	0	0	.2	.7	1	A_7
82-83	512	.9	1	.5	0	0	0	0	A_2
83-84	1005	0	0	0	0	.5	.95	1	A_7
84-85	852	0	0	0	.5	1	.7	.2	A_5
85-86	440	1	.5	0	0	0	0	0	A_1
86-87	502	.95	1	.5	0	0	0	0	A_2
87-88	775	0	0	.5	1	.9	.4	0	A_4
88-89	465	1	.8	.3	0	0	0	0	A_1
89-90	795	0	0	.5	1	.9	.4	0	A_4
90-91	970	0	0	0	0	.5	1	.8	A_6
91-92	742	0	.2	.7	1	.5	0	0	A_4
92-93	635	.3	.8	1	.7	.2	0	0	A_3
93-94	994	0	0	0	0	.5	1	.9	A_6
94-95	759	0	0	.5	1	.7	.2	0	A_4
95-96	883	0	0	0	.5	1	.8	.3	A_5
96-97	599	.5	1	.99	.5	0	0	0	A_1
97-98	499	1	.5	0	0	0	0	0	A_1
98-99	590	.5	1	.8	.3	0	0	0	A_2
99-00	911	0	0	0	.3	.8	1	.7	A_6
00-01	862	0	0	0	.5	1	.8	.5	A_5
01-02	801	-	-	-	-	-	-	-	-

Step 5. The FLR have obtained from the table 1 are in the following table 2

Table 2: FLR of the historical lahi production

1	$A_1 \rightarrow A_2$	$A_1 \rightarrow A_2$	$A_1 \rightarrow A_4$	$A_2 \rightarrow A_1$	$A_2 \rightarrow A_4$
2	$A_2 \rightarrow A_6$	$A_2 \rightarrow A_7$	$A_3 \rightarrow A_6$	$A_4 \rightarrow A_1$	$A_4 \rightarrow A_3$
3	$A_4 \rightarrow A_5$	$A_4 \rightarrow A_6$	$A_5 \rightarrow A_1$	$A_5 \rightarrow A_2$	$A_6 \rightarrow A_7$
4	$A_6 \rightarrow A_4$	$A_6 \rightarrow A_5$	$A_7 \rightarrow A_2$	$A_7 \rightarrow A_5$	-

Further the FLR obtained in table 2 are arranged such that consider the relations only once, i.e. leaving the repeated relations, we thus obtain a total of 17 FLRs which are placed in table 3.

Table 3: FLR groups

1	$A_1 \rightarrow A_2$	$A_1 \rightarrow A_4$	-	-
2	$A_2 \rightarrow A_1$	$A_2 \rightarrow A_4$	$A_2 \rightarrow A_6$	$A_2 \rightarrow A_7$
3	$A_3 \rightarrow A_6$	-	-	-
4	$A_4 \rightarrow A_1$	$A_4 \rightarrow A_3$	$A_4 \rightarrow A_5$	$A_4 \rightarrow A_6$
5	$A_5 \rightarrow A_1$	$A_5 \rightarrow A_2$	-	-
6	$A_6 \rightarrow A_4$	$A_6 \rightarrow A_5$	-	-
7	$A_7 \rightarrow A_2$	$A_7 \rightarrow A_5$	-	-

Thus using the FLR obtained in Table 3 we get the fuzzy time invariant relation R as:

$$R = \cup_{i=1}^{17} R_i \quad \cup \text{ is union operator.}$$

Thus computing all the FLR $R_1, \ldots, R_{15}$ and Taking the union of these computed relations we obtain a fuzzy time invariant relation R as:

$$R = \begin{bmatrix} .5 & 1 & .5 & .5 & 1 & .5 & .5 \\ 1 & .5 & .5 & 1 & .5 & 1 & .5 \\ .5 & .5 & .5 & .5 & .5 & 1 & .5 \\ 1 & .5 & 1 & .5 & 1 & 1 & .5 \\ 1 & 1 & .5 & .5 & .5 & .5 & .5 \\ .5 & .5 & .5 & 1 & 1 & .5 & 0 \\ .5 & 1 & .5 & .5 & 1 & .5 & 0 \end{bmatrix}$$

Step 6. Computation of fuzzy forecast of the crop production have been carried by the three models: Chen's arithmetic model (Model-1), Srivastava and Singh's refined arithmetic model (Model-2) and Song and Chissom model (Model-3)

Model-1

1. If the fuzzified production of the year i is A_j, and there is only one FLR in the FLR groups in Table 3 in which the current state of the production is A_j, then the fuzzy forecasted production of the year i+1 is A_j.

If the fuzzified production of the year i is Aj and there are FLR in the FLRgroup as:

$A_j \rightarrow A_{k1}, A_j \rightarrow A_{k\,2}, \ldots.. A_j \rightarrow A_{kp}$, the A_j forming a relation

With A_{k1}, A_{k2},. A_{kp} is the fuzzy forecasted production for the year i+1

Model-2

Similar to model-1 but it also counts the repeated relations and the frequency is recorded and is used for defuzzification (crisp output).

Model-3

It uses the fuzzy time invariant relation R computed in step 5. If A_{i-1} is the production of the year i-1, the fuzzy forecasted production for the year i will be A_i and will be computed by $A_i = A_{i-1}$ o R. The computed fuzzy output is as

Table 4. Fuzzy output of Lahi Production

Year	A_1	A_2	A_3	A_4	A_5	A_6	A_7
81-82	.5	1	.5	.7	1	.5	.2
82-83	1	.9	.5	1	.9	1	1
83-84	.5	1	.5	.95	1	.5	.5
84-85	1	1	.5	.7	.7	.5	.5
85-86	.5	1	.5	.5	1	.5	.5
86-87	1	.95	.5	1	.95	1	1
87-88	1	.9	1	.5	1	1	.5
88-89	.8	1	.5	.8	1	.8	.8
89-90	1	.9	1	.5	1	1	.5
90-91	.5	.8	.5	1	1	.5	.5
91-92	1	.5	1	.5	1	1	.5
92-93	.8	.5	.7	.8	.7	1	.8
93-94	.5	.9	.5	1	1	.5	.5
94-95	1	.7	1	.5	1	1	.5
95-96	1	1	.5	.8	.8	.5	.5
96-97	1	.5	.5	1	.5	1	1
97-98	.5	1	.5	.5	1	.5	.5
98-99	1	.5	.5	1	.5	1	1
99-00	.8	.8	.5	1	1	.5	.5
00-01	1	1	.5	.8	.8	.5	.5

Step 7. Defuzzification is the process by which fuzzy output of model is transformed to crisp values for getting the forecasted values. The output of each of the three models are defuzzified in the following ways:

Model-1

(1) If the production of the year i is A_j and FLR is $A_j \rightarrow A_k$ and A_k has max membership in interval u_k, then the forecasted production for the year i= j will be midpoint of A_k.

(2) If the fuzzified production of the year i is A_j and there are FLR in the FLR group as: $A_j \rightarrow A_{k1}, A_j \rightarrow A_{k2}, \ldots..$ $A_j \rightarrow A_{kp}$ then A_{k1}, A_{k2} ......, A_{kp} has max membership in the intervals $u_{k1}, u_{k2}, \ldots,$ u_{kp} respectively and m_1, $m_2, \ldots. m_p$ are their respective midpoints, then the forecasted production for the year i+1 will be $(m_1 + m_2 \ldots + m_p)/p$.

(3) If the fuzzified production of a year is A_j, and no FLR is found in logical relationship groups, whose current state of production is A_j, where the maximum membership value of A_j occurs at interval u_j and the midpoint of u_j is m_j then the forecasted production of year i + 1 is m_j.

Model-2

Similar procedures of defuzzification as in model-1 with additional concept of repeated relations and according weighted mean is computed keeping in view of their frequencies.

Model-3

A combined approach is utilized to have the defuzzification of the fuzzy output in table 4 into crisp output. The rules are as follows;

(1) If fuzzy output set has only one max, then choose the midpoint of the interval corresponding to that max as forecasted production.

(2) If the fuzzy output has only one consecutive max, then choose the midpoint of the corresponding conjunct interval as the forecasted value.

(3) Otherwise compute the forecasted value by center of area (COA) or Centroid (COG) method.

5. Computational Results

The forecasted production of lahi obtained by the above three models have been place in the table 5 and is presented in figure 1.

Table 5: Lahi production forecast

Year	Actual production kg/ ha	Forecasted production Method- 1	Forecasted Production Method-2	Forecasted production Method-3
81-82	512	700	700	850
82-83	1005	800	800	850
83-84	852	700	700	850
84-85	440	500	500	624
85-86	502	650	617	700
86-87	775	800	800	850
87-88	465	725	725	900
88-89	795	650	617	850
89-90	970	725	725	900
90-91	742	800	783	800
91-92	635	725	725	900
92-93	994	950	950	950
93-94	759	800	783	800
94-95	883	725	725	900
95-96	599	500	500	633
96-97	499	800	800	850
97-98	590	650	617	700
98-99	911	800	800	850
99-00	862	800	783	667
00-01	801	500	500	663

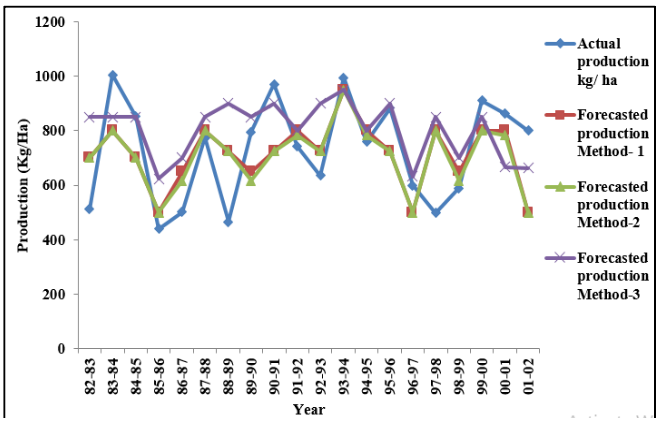

Fig 1: Actual and forecasted Lahi production

6. Conclusion

Three FTS forecasting models applied to this study, the results are almost similar as the average forecasting error of the three models is about 11%. The observation of fuzzy output of the model-3 needs to be tackled by some index based defuzzification procedure for better crisp forecast. The model-3 clearly indicates that defuzzification methods like COA, COG or MOM methods are not suitable for present problem. The interesting features of fuzzy output of the model-3 can be harvested by using some higher order defuzzification method on introducing some indicators. This feature of model-3 gives an advantageous basis in comparison to other two models. The result of this study can be helpful in better farm administration of the crop yield and to agro-based industries for future planning in advance.

7. Acknowledgement

The authors are thankful to University farm administration for providing the time series production data and is duly acknowledged.

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