International Journal of Statistics and Applied Mathematics 2020; 5(3): 47-54

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2020; 5(3): 47-54

www.mathsjournal.com
Received: 22-03-2020
Accepted: 24-04-2020

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Confidence intervals for difference of signal-to-noise ratios of two-parameter exponential distributions

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Abstract
Coefficient of variation is the standard deviation divided by the mean. It is a measure of relative variability in different units. Signal-to-noise ratio (SNR) is the reciprocal of coefficient of variation. It is the ratio of mean to standard deviation of measurement. The SNR is extensively used in many application fields. Exponential distribution describes the lengths of the inter-arrival times in a homogeneous Poisson process. For positively skewed data, the exponential distribution is as important as the normal distribution in sampling theory and statistics. Two-parameter exponential distribution has an important role in medical sciences and life testing. For two populations, the difference of SNRs of two-parameter exponential distributions is interesting. In this paper, the problem of constructing confidence intervals for difference of SNRs of two-parameter exponential distributions is considered. Generalized confidence interval (GCI) approach, large sample approach, method of variance estimates recovery (MOVER) approach, and parametric bootstrap (PB) approach are applied to construct the confidence intervals. Based on the simulation study, the results indicated that the coverage probability of the GCI approach does not depend on difference of SNRs, but the coverage probability is around the nominal confidence level of 0.95. The large sample and MOVER approaches provide the conservative confidence intervals when the value of θ is large. The PB approach provides better in term of coverage probability. As a result, the PB approach is recommended to construct the confidence intervals for the difference of SNRs. The proposed approaches are illustrated using a real data set from medical science.

Keywords: Average length, coverage probability, signal-to-noise ratio, simulation, two-parameter exponential distribution

Introduction

Two-parameter exponential distribution is used in applied statistics. For instance, Lawless JF (1977) [1] analyzed lifetime data to predict the confidence intervals for the two-parameter exponential distribution. Baten A & Kamil A (2009) [2] considered inventory management systems with hazardous items of two-parameter exponential distribution. Petropoulos C (2011) [3] proposed new classes of improved confidence intervals for the scale parameter of two-parameter exponential distribution. Jiang L & Wong ACM (2012) [4] presented interval estimations for the scale parameter of two-parameter exponential distribution. Thangjai W & Niwitpong S (2017) [5] proposed new confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions. Saothayanun L & Thangjai W (2018) [6] constructed the confidence intervals for the signal-to-noise ratio (SNR) of two-parameter exponential distribution. Thangjai W *et al.* (2018) [7] suggested the simultaneous confidence intervals for the differences of means of several two-parameter exponential distributions. Thangjai W & Niwitpong S (2020) [8] proposed the simultaneous confidence intervals for the differences of coefficients of variation of two-parameter exponential distributions.

Standard deviation is used dispersion measures in statistics. The standard deviation is not the most appropriate indicator when the dispersion of distributions of several variables is compared. Coefficient of variation is widely used rather than the standard deviation. Inverse of the coefficient of variation is called the SNR. The SNR is commonly used in finance and image processing. In the literature, many studies had discussed about the SNR. For more detailed inference of the SNR, see Sharma KK & Krishna H (1994) [9], George F & Kibria BMG (2012) [10], Albatineh AN *et al.* (2014) [11], Albatineh AN *et al.* (2017) [12], Saothayanun

L & Thangjai W (2018) [6], Niwitpong S (2018) [13], Thangjai W & Niwitpong S (2019) [14], Thangjai W & Niwitpong S (2019) [15], and Thangjai W & Niwitpong S (2020) [16].

The literature on the SNR of two-parameter exponential distribution is limited. Saothayanun L & Thangjai W (2018) [6] studied the confidence intervals for the SNR in one population. The objective of this paper is extend the confidence intervals to two populations. The confidence intervals for the difference of SNRs of two-parameter exponential distributions are considered using the generalized confidence interval (GCI) approach, the large sample approach, the method of variance estimates recovery (MOVER) approach, and the parametric bootstrap (PB) approach.

2. Methods

Suppose that two independent random variables $X = (X_1, X_2, \ldots, X_n)$ and $Y = (Y_1, Y_2, \ldots, Y_m)$ follow two-parameter exponential distributions. The means of X and Y are $E(X) = \lambda_X + \beta_X$ and $E(Y) = \lambda_Y + \beta_Y$, respectively. Also, the variances of X and Y are $Var(X) = \lambda_X^2$ and $Var(Y) = \lambda_Y^2$, respectively.

The difference of SNRs of two-parameter exponential distributions can be written as

$$\theta = \theta_X - \theta_Y = \frac{E(X)}{\sqrt{Var(X)}} - \frac{E(Y)}{\sqrt{Var(Y)}} = \frac{\lambda_X + \beta_X}{\lambda_X} - \frac{\lambda_Y + \beta_Y}{\lambda_Y} . \quad (1)$$

An estimator of θ has the following form

$$\hat{\theta} = \hat{\theta}_X - \hat{\theta}_Y = \frac{\hat{\lambda}_X + \hat{\beta}_X}{\hat{\lambda}_X} - \frac{\hat{\lambda}_Y + \hat{\beta}_Y}{\hat{\lambda}_Y} , \quad (2)$$

Where

$$\hat{\beta}_X = X_{(1)} = \min(X_1, X_2, \ldots, X_n) ,\ \hat{\beta}_Y = Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_m) ,\ \hat{\lambda}_X = \bar{X} - X_{(1)} , \text{ and } \hat{\lambda}_Y = \bar{Y} - Y_{(1)} .$$

Therefore, solving yields

$$\hat{\theta} = \frac{\bar{X}}{\bar{X} - X_{(1)}} - \frac{\bar{Y}}{\bar{Y} - Y_{(1)}} . \quad (3)$$

According to Saothayanun L & Thangjai W (2018) [6], the variances of $\hat{\theta}_X$ and $\hat{\theta}_Y$ defined as

$$Var(\hat{\theta}_X) = \frac{2n^2\lambda_X^2 - n\lambda_X^2 + 2n^2\lambda_X\beta_X + n^2\beta_X^2}{(n-1)^3\lambda_X^2} \quad (4)$$

and

$$Var(\hat{\theta}_Y) = \frac{2m^2\lambda_Y^2 - m\lambda_Y^2 + 2m^2\lambda_Y\beta_Y + m^2\beta_Y^2}{(m-1)^3\lambda_Y^2} . \quad (5)$$

Since $\hat{\theta}_X$ and $\hat{\theta}_Y$ are independent, the variance of difference of SNRs has the following form

$$Var(\hat{\theta}) = \frac{2n^2\lambda_X^2 - n\lambda_X^2 + 2n^2\lambda_X\beta_X + n^2\beta_X^2}{(n-1)^3\lambda_X^2} + \frac{2m^2\lambda_Y^2 - m\lambda_Y^2 + 2m^2\lambda_Y\beta_Y + m^2\beta_Y^2}{(m-1)^3\lambda_Y^2} . \quad (6)$$

2.1 Generalized Confidence Interval

Roy A & Mathew T (2005) [17] proposed the following pivots of β_X and λ_X

$$W_{1X} = \frac{2n(\hat{\beta}_X - \beta_X)}{\lambda_X} \sim \chi_2^2 \quad (7)$$

and

$$W_{2X} = \frac{2n\hat{\lambda}_X}{\lambda_X} \sim \chi^2_{2n-2}, \tag{8}$$

where χ^2_2 and χ^2_{2n-2} denote chi-squared distributions with 2 and $2n-2$ degrees of freedom, respectively. Similarly, the pivots of β_Y and λ_Y defined as follows

$$W_{1Y} = \frac{2m(\hat{\beta}_Y - \beta_Y)}{\lambda_Y} \sim \chi^2_2 \tag{9}$$

and

$$W_{2Y} = \frac{2m\hat{\lambda}_Y}{\lambda_Y} \sim \chi^2_{2m-2}, \tag{10}$$

Where

χ^2_2 and χ^2_{2m-2} denote chi-squared distributions with 2 and $2m-2$ degrees of freedom, respectively. The generalized pivotal quantity for θ based on Equation (7) - Equation (10) is given by

$$R_\theta = \left[1 + \frac{1}{2n}\left(\frac{\hat{\beta}_X W_{2X}}{\hat{\lambda}_X} - W_{1X}\right)\right] - \left[1 + \frac{1}{2m}\left(\frac{\hat{\beta}_Y W_{2Y}}{\hat{\lambda}_Y} - W_{1Y}\right)\right]. \tag{11}$$

Therefore, the $100(1-\alpha)\%$ generalized confidence interval for difference of SNRs is obtained by

$$CI_{\theta.GCI} = [L_{\theta.GCI}, U_{\theta.GCI}] = [R_\theta(\alpha/2), R_\theta(1-\alpha/2)], \tag{12}$$

Where

$R_\theta(\alpha/2)$ and $R_\theta(1-\alpha/2)$ denote the $100(\alpha/2)$-th and the $100(1-\alpha/2)$-th percentiles of R_θ, respectively.

2.2 Large Sample Confidence Interval

Again, let $\hat{\theta}$ be the difference of SNRs of two-parameter exponential distributions. Also, let $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_m)$ be observed values of $X = (X_1, X_2, \ldots, X_n)$ and $Y = (Y_1, Y_2, \ldots, Y_m)$, respectively. Also, let $\bar{x}$, $\bar{y}$, $x_{(1)}$, and $y_{(1)}$ be observed values of $\bar{X}$, $\bar{Y}$, $X_{(1)}$, and $Y_{(1)}$, respectively. Hence,

$$\hat{\theta} = \frac{\bar{x}}{\bar{x} - x_{(1)}} - \frac{\bar{y}}{\bar{y} - y_{(1)}} \tag{13}$$

and

$$V\hat{a}r(\hat{\theta}) = \frac{2n^2(\bar{x} - x_{(1)})^2 - n(\bar{x} - x_{(1)})^2 + 2n^2(\bar{x} - x_{(1)})x_{(1)} + n^2(x_{(1)})^2}{(n-1)^3(\bar{x} - x_{(1)})^2} + \frac{2m^2(\bar{y} - y_{(1)})^2 - m(\bar{y} - y_{(1)})^2 + 2m^2(\bar{y} - y_{(1)})y_{(1)} + m^2(y_{(1)})^2}{(m-1)^3(\bar{y} - y_{(1)})^2}. \tag{14}$$

Therefore, the $100(1-\alpha)\%$ large sample confidence interval for difference of SNRs is obtained by

$$CI_{\theta.LS} = [L_{\theta.LS}, U_{\theta.LS}] = [\hat{\theta} - z_{1-\alpha/2}\sqrt{V\hat{a}r(\hat{\theta})}, \hat{\theta} + z_{1-\alpha/2}\sqrt{V\hat{a}r(\hat{\theta})}], \tag{15}$$

Where

$z_{1-\alpha/2}$ denotes the $100(1-\alpha/2)$ -th percentile of the standard normal distribution.

2.3 MOVER Confidence Interval

Using the concept of Saothayanun L & Thangjai W (2018) [6], lower limit and upper limit for θ_X are

$$l_X = \frac{(\hat{\lambda}_X + \hat{\beta}_X)\hat{\lambda}_X - \sqrt{((\hat{\lambda}_X + \hat{\beta}_X)\hat{\lambda}_X)^2 - l_{1X}u_{2X}(2(\hat{\lambda}_X + \hat{\beta}_X) - l_{1X})(2\hat{\lambda}_X - u_{2X})}}{u_{2X}(2\hat{\lambda}_X - u_{2X})} \tag{16}$$

and

$$u_X = \frac{(\hat{\lambda}_X + \hat{\beta}_X)\hat{\lambda}_X + \sqrt{((\hat{\lambda}_X + \hat{\beta}_X)\hat{\lambda}_X)^2 - u_{1X}l_{2X}(2(\hat{\lambda}_X + \hat{\beta}_X) - u_{1X})(2\hat{\lambda}_X - l_{2X})}}{l_{2X}(2\hat{\lambda}_X - l_{2X})}, \tag{17}$$

Where

$$l_{1X} = \hat{\lambda}_X + \hat{\beta}_X - \sqrt{\left[\hat{\lambda}_X - \frac{n\hat{\lambda}_X}{z_{\alpha/2}\sqrt{n-1} + (n-1)}\right]^2 + \left[\frac{\hat{\lambda}_X}{n}\ln(\alpha/2)\right]^2},$$

$$u_{1X} = \hat{\lambda}_X + \hat{\beta}_X + \sqrt{\left[\frac{n\hat{\lambda}_X}{-z_{\alpha/2}\sqrt{n-1} + (n-1)} - \hat{\lambda}_X\right]^2 + \left[\frac{\hat{\lambda}_X}{n}\ln(1-\alpha/2)\right]^2},$$

$$l_{2X} = \frac{n\hat{\lambda}_X}{\sqrt{n-1}(z_{\alpha/2} + \sqrt{n-1})}$$

and

$$u_{2X} = \frac{n\hat{\lambda}_X}{\sqrt{n-1}(-z_{\alpha/2} + \sqrt{n-1})}.$$

Similarly, the lower limit and the upper limit for θ_Y are

$$l_Y = \frac{(\hat{\lambda}_Y + \hat{\beta}_Y)\hat{\lambda}_Y - \sqrt{((\hat{\lambda}_Y + \hat{\beta}_Y)\hat{\lambda}_Y)^2 - l_{1Y}u_{2Y}(2(\hat{\lambda}_Y + \hat{\beta}_Y) - l_{1Y})(2\hat{\lambda}_Y - u_{2Y})}}{u_{2Y}(2\hat{\lambda}_Y - u_{2Y})} \tag{18}$$

and

$$u_Y = \frac{(\hat{\lambda}_Y + \hat{\beta}_Y)\hat{\lambda}_Y + \sqrt{((\hat{\lambda}_Y + \hat{\beta}_Y)\hat{\lambda}_Y)^2 - u_{1Y}l_{2Y}(2(\hat{\lambda}_Y + \hat{\beta}_Y) - u_{1Y})(2\hat{\lambda}_Y - l_{2Y})}}{l_{2Y}(2\hat{\lambda}_Y - l_{2Y})}, \tag{19}$$

where

$$l_{1Y} = \hat{\lambda}_Y + \hat{\beta}_Y - \sqrt{\left[\hat{\lambda}_Y - \frac{m\hat{\lambda}_Y}{z_{\alpha/2}\sqrt{m-1} + (m-1)}\right]^2 + \left[\frac{\hat{\lambda}_Y}{m}\ln(\alpha/2)\right]^2},$$

$u_{1Y} = \hat{\lambda}_Y + \hat{\beta}_Y + \sqrt{\left[\frac{m\hat{\lambda}_Y}{-z_{\alpha/2}\sqrt{m-1}+(m-1)} - \hat{\lambda}_Y\right]^2 + \left[\frac{\hat{\lambda}_Y}{m}\ln(1-\alpha/2)\right]^2}$, $l_{2Y} = \frac{m\hat{\lambda}_Y}{\sqrt{m-1}(z_{\alpha/2}+\sqrt{m-1})}$,

and

$$u_{2Y} = \frac{m\hat{\lambda}_Y}{\sqrt{m-1}(-z_{\alpha/2}+\sqrt{m-1})}.$$

According to the research paper of Donner A & Zou GY (2010) [18], the lower limit and the upper limit for θ are

$$L_{\theta.MOVER} = \hat{\theta} - \sqrt{(\hat{\theta}_X - l_X)^2 + (u_Y - \hat{\theta}_Y)^2} \tag{20}$$

and

$$U_{\theta.MOVER} = \hat{\theta} + \sqrt{(u_X - \hat{\theta}_X)^2 + (\hat{\theta}_Y - l_Y)^2}. \tag{21}$$

Therefore, the $100(1-\alpha)\%$ MOVER confidence interval for difference of SNRs is obtained by

$$CI_{\theta.MOVER} = [L_{\theta.MOVER}, U_{\theta.MOVER}], \tag{22}$$

where $L_{\theta.MOVER}$ and $U_{\theta.MOVER}$ are defined in Equation (20) and Equation (21), respectively.

2.4 Parametric Bootstrap Confidence Interval

The PB approach is a resampling approach based on independently sampling with replacement from an existing sample data with same sample size.

Let $X^* = (X_1^*, X_2^*, \ldots, X_n^*)$ be sample with replacement from $X = (X_1, X_2, \ldots, X_n)$ with sample size n and let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ be the observed values of $X^* = (X_1^*, X_2^*, \ldots, X_n^*)$. Similarly, let $Y^* = (Y_1^*, Y_2^*, \ldots, Y_m^*)$ be the sample from $Y = (Y_1, Y_2, \ldots, Y_m)$ with replacement sample size m and let $y^* = (y_1^*, y_2^*, \ldots, y_m^*)$ be also the observed values of $Y^* = (Y_1^*, Y_2^*, \ldots, Y_m^*)$.

The re-sampled sample is called a bootstrap sample. The difference of SNRs from bootstrap sample is obtained by

$$\theta^* = \theta_X^* - \theta_Y^* = \frac{\lambda_X^* + \beta_X^*}{\lambda_X^*} - \frac{\lambda_Y^* + \beta_Y^*}{\lambda_Y^*}. \tag{23}$$

An estimator of the difference is

$$\hat{\theta}^* = \hat{\theta}_X^* - \hat{\theta}_Y^* = \frac{\hat{\lambda}_X^* + \hat{\beta}_X^*}{\hat{\lambda}_X^*} - \frac{\hat{\lambda}_Y^* + \hat{\beta}_Y^*}{\hat{\lambda}_Y^*}, \tag{24}$$

Where

$\hat{\beta}_X^* = X_{(1)}^* = \min(X_1^*, X_2^*, \ldots, X_n^*)$, $\hat{\beta}_Y^* = Y_{(1)}^* = \min(Y_1^*, Y_2^*, \ldots, Y_m^*)$, $\hat{\lambda}_X^* = \bar{X}^* - X_{(1)}^*$, and $\hat{\lambda}_Y^* = \bar{Y}^* - Y_{(1)}^*$.

That is

$$\hat{\theta}^* = \frac{\bar{X}^*}{\bar{X}^* - X_{(1)}^*} - \frac{\bar{Y}^*}{\bar{Y}^* - Y_{(1)}^*} \tag{25}$$

For replicate B times, there are totally B estimates of the difference of SNRs θ^* from B bootstrap sample.

The sampling distribution is constructed with these B bootstrap statistics. The distribution is used to make the confidence interval for the difference of SNRs. Therefore, the $100(1-\alpha)\%$ PB confidence interval for difference of SNRs is obtained by

$$CI_{\theta.PB} = [L_{\theta.PB}, U_{\theta.PB}] = [\hat{\theta} - z_{1-\alpha/2}S^*, \hat{\theta} + z_{1-\alpha/2}S^*], \quad (26)$$

Where

$\hat{\theta}$ is defined in Equation (3), $z_{1-\alpha/2}$ denotes the $100(1-\alpha/2)$-th percentile of the standard normal distribution, and S^* denotes the standard deviation of $\hat{\theta}^*$ defined in Equation (25).

3. Simulation Study

In this section, a simulation is performed to compare the proposed approaches for constructing confidence intervals of difference of signal-to-noise ratios (SNRs).

Monte Carlo simulation study is performed using the R statistical package to evaluate the performance of the confidence intervals for difference of SNRs. The 95% confidence intervals based on the generalized confidence interval (GCI) approach, the large sample approach, the method of variance estimates recovery (MOVER) approach, and the parametric bootstrap (PB) approach are compared. The data are generated from two-parameter exponential distributions, denoted as $X \sim Exp(\lambda_X, \beta_X)$ and $Y \sim Exp(\lambda_Y, \beta_Y)$. The scale parameters λ_X and λ_Y are fixed equal to be 1. The location parameters β_X and β_Y are computed to get the SNRs $(\theta_X, \theta_Y) =$ (2,-5), (2,-2), (2,2), and (2,5). The 5,000 simulation runs and 2,500 generalized pivotal quantities are used. The coverage probabilities and average lengths of the confidence intervals for difference of coefficients of variation can be calculated by the following algorithm.

Algorithm 1.

For a given n, m, λ_X, λ_Y, β_X, β_Y, θ_X, θ_Y, and θ

Step 1: Generate x from two-parameter exponential distribution with λ_X and β_X

Step 2: Generate y from two-parameter exponential distribution with λ_Y and β_Y

Step 3: Calculate $\bar{x}$, $x_{(1)}$, $\bar{y}$, and $y_{(1)}$

Step 4: Compute $L_{\theta.GCI(h)}$ and $U_{\theta.GCI(h)}$

Step 5: Compute $L_{\theta.LS(h)}$ and $U_{\theta.LS(h)}$

Step 6: Compute $L_{\theta.MOVER(h)}$ and $U_{\theta.MOVER(h)}$

Step 7: Compute $L_{\theta.PB(h)}$ and $U_{\theta.PB(h)}$

Step 8: If $L_{(h)} \leq \theta \leq U_{(h)}$ set $p_{(h)} =$ 1, else $p_{(h)} =$ 0

Step 9: Calculate $U_{(h)} - L_{(h)}$

Step 10: Repeat the step 1 - step 9 for a large number of times (say, M times) and calculate coverage probability and average length

4. Results and Discussion

The coverage probabilities and average lengths of the confidence intervals for difference of SNRs are shown in Table 1 and Figures 1 - 4. The results indicate that the coverage probabilities of the GCI approach are less than and greater than the nominal confidence level of 0.95 in some cases. The coverage probability of the GCI approach does not depend on difference of SNRs $\theta = \theta_X - \theta_Y$. For $(\theta_X, \theta_Y) =$ (2,-5), the large sample approach has coverage probabilities less than the nominal confidence level of 0.95, whereas the MOVER approach has coverage probabilities greater than the nominal confidence level. Nevertheless, the coverage probabilities of the large sample approach and the MOVER approach appear to be conservative for large value of θ. Moreover, the coverage probabilities of the PB approach are greater than the nominal confidence level for all cases and they also are stable. The average lengths indicate that the average lengths of the GCI approach are narrower than those of the large sample, MOVER, and PB approaches

Table 1: Coverage probabilities (CP) and average lengths (AL) of 95% two-sided confidence intervals for difference of SNRs of two-parameter exponential distributions

(n, m)	(λ_X, λ_Y)	(θ_X, θ_Y)	GCI		LS		MOVER		PB	
			CP	AL	CP	AL	CP	AL	CP	AL
(30,30)	(1,1)	(2,-5)	0.9498	4.5534	0.9488	4.4923	0.9710	6.2348	0.9650	5.1755
		(2,-2)	0.9516	2.3786	0.9596	2.5355	0.9480	4.8337	0.9648	2.6937
		(2,2)	0.9520	1.1298	1.0000	2.4813	1.0000	7.2545	0.9802	1.3372
		(2,5)	0.9462	3.1318	0.9932	4.4317	0.9982	12.6779	0.9638	3.6066
(30,50)	(1,1)	(2,-5)	0.9530	3.5060	0.9554	3.5188	0.9684	5.5444	0.9628	3.7772
		(2,-2)	0.9426	1.8848	0.9692	2.2096	0.9546	4.5954	0.9584	2.0603
		(2,2)	0.9484	0.9969	1.0000	2.1924	1.0000	5.2700	0.9736	1.1625
		(2,5)	0.9546	2.4415	0.9938	3.4996	1.0000	7.4809	0.9646	2.6656
(50,50)	(1,1)	(2,-5)	0.9462	3.4565	0.9350	3.3080	0.9590	3.6984	0.9530	3.7051
		(2,-2)	0.9506	1.8013	0.9552	1.8714	0.9530	2.4963	0.9552	1.9249
		(2,2)	0.9510	0.8357	1.0000	1.8491	1.0000	3.1769	0.9656	0.9200
		(2,5)	0.9448	2.3757	0.9936	3.2947	0.9904	5.5459	0.9560	2.5718
(50,100)	(1,1)	(2,-5)	0.9486	2.4561	0.9496	2.4546	0.9638	2.9826	0.9560	2.5451
		(2,-2)	0.9478	1.3314	0.9752	1.5967	0.9622	2.2653	0.9508	1.3947
		(2,2)	0.9492	0.7177	1.0000	1.5893	1.0000	2.4455	0.9652	0.7771
		(2,5)	0.9428	1.7053	0.9948	2.4477	0.9990	3.5267	0.9518	1.7823
(100,100)	(1,1)	(2,-5)	0.9498	2.4155	0.9326	2.2595	0.9468	2.3473	0.9516	2.4825
		(2,-2)	0.9486	1.2586	0.9558	1.2823	0.9518	1.4480	0.9566	1.2993
		(2,2)	0.9488	0.5737	1.0000	1.2740	1.0000	1.6334	0.9570	0.6007
		(2,5)	0.9490	1.6465	0.9920	2.2531	0.9876	2.8479	0.9550	1.7050
(100,200)	(1,1)	(2,-5)	0.9490	1.7247	0.9454	1.7006	0.9536	1.8312	0.9492	1.7532
		(2,-2)	0.9442	0.9304	0.9736	1.1001	0.9638	1.2790	0.9466	0.9486
		(2,2)	0.9474	0.4930	1.0000	1.0975	0.9998	1.3371	0.9556	0.5120
		(2,5)	0.9504	1.1909	0.9934	1.6968	0.9954	2.0025	0.9532	1.2145
(200,200)	(1,1)	(2,-5)	0.9418	1.7018	0.9202	1.5740	0.9256	1.5990	0.9416	1.7267
		(2,-2)	0.9482	0.8841	0.9502	0.8917	0.9544	0.9436	0.9500	0.8964
		(2,2)	0.9522	0.3985	0.9998	0.8882	1.0000	1.0017	0.9558	0.4078
		(2,5)	0.9502	1.1552	0.9932	1.5693	0.9878	1.7550	0.9548	1.1738

5. Application

Data from the medical science reported by the Freireich TR *et al*. (1963) [19] is used for analysis. The data presents time to relapse of the patients with acute leukemia treated by drug 6-mercaptopurine and placebo. The generalized confidence interval (GCI) approach, the large sample approach, the method of variance estimates recovery (MOVER) approach, and the parametric bootstrap (PB) approach are demonstrated using the data. The summary statistics of the drug 6-mercaptopurine are $n =$ 21, $\bar{x} =$ 17.10, and $x_{(1)} =$ 6. The summary statistics of the placebo are $m =$ 21, $\bar{y} =$ 8.67, and $y_{(1)} =$ 1. The difference of signal-to-noise ratio (SNRs) is $\hat{\theta} =$ 0.4102. The 95% generalized confidence interval is [0.1452, 0.7297] with interval length 0.5845. The 95% large sample confidence interval is [-0.6739, 1.5314] with interval length 2.2053. The 95% MOVER confidence interval is [-7.3289, 11.0102] with interval length 18.3391. The 95% PB confidence interval is [-0.0411, 0.8986] with interval length 0.9397. From the results, all the proposed confidence intervals contain the true difference of SNRs. However, the interval length of the generalized confidence interval is smaller than the interval lengths of other confidence intervals. It is clear that these results confirm our simulation study in term of interval length.

6. Conclusion

The signal-to-noise ratio (SNR) is applied in many applications. Extending the research paper of Saothayanun L & Thangjai W (2018) [6], this paper is interested in difference of SNRs of two-parameter exponential distributions. The confidence intervals for the difference of SNRs are proposed based on the generalized confidence interval (GCI) approach, the large sample approach, the method of variance estimates recovery (MOVER) approach, and the parametric bootstrap (PB) approach. The performance of these approaches is investigated using Monte Carlo simulations. The GCI approach uses software package to construct the generalized confidence interval. The large sample approach, the MOVER approach, and the PB approach use formula to estimate the confidence intervals. The results indicated that the coverage probabilities of the PB approach are greater than the nominal confidence level of 0.95 and the performance of the PB approach is satisfactory as compared with other approaches. As a result, the PB approach is recommended for constructing the confidence interval for the difference of SNRs of two-parameter exponential distributions.

7. Acknowledgments

This research was funded by Faculty of Applied Science, King Mongkut’s University of Technology North Bangkok. Contract No. 6345101.

8. References

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