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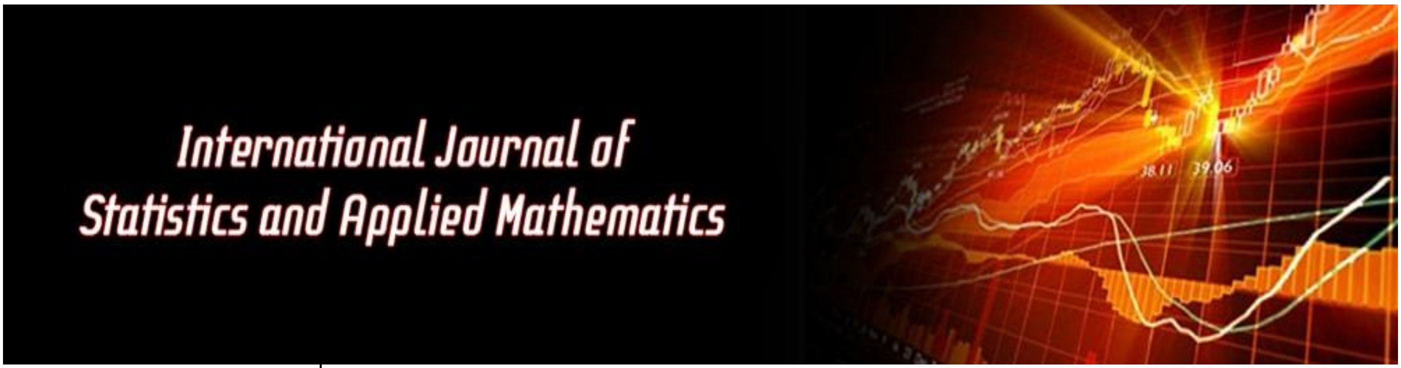

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About conditions of controllability of ensamble trajectories of differential inclusion with delay

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Abstract
In this paper we consider the control problem of an ensemble of trajectories of differential inclusion with a delayed argument. The controllability conditions for the ensemble of trajectories from the initial state $\varphi^0(\cdot)$ to a given terminal set $Y = Y(t)$ are studied. The results are obtained by methods of the theory of differential inclusions and multi-valued analysis. Necessary and sufficient conditions for (φ^0, Y)-controllability and complete Y-controllability are given. The significance of these results in studies of control problems under conditions of limited information is discussed.

Keywords: Differential inclusion, control system, delay, ensemble of trajectories, controllability conditions

Introduction
In real situations, effective management of complex technical objects and processes is impossible without taking into account factors such as incomplete information on external parameters and inaccuracy of the initial a priori data on the initial state of the system. The control problems in conditions of limited information of various types lead to the so-called information models of control systems. Information models for dynamical systems described by ordinary differential equations can be represented in a general form in the form

$$\dot{x} = f(t, x, u, v), t \geq t_0, x(t_0) = x^0,\ u \in U\ , \qquad (1)$$

Where $\dot{x} = \frac{dx}{dt}$, $u = u(t)$ is the control parameter, $w = w(t)$ is the parameter of unknown external influences.
Known methods for studying a model of the form (1) are developed taking into account the degree of limited information regarding external parameters and the initial state of the system. When the probability distribution functions of the parameter $w = w(t)$ and the initial data are known, system (1) is studied by methods of the theory of stochastic control. However, this assumption cannot always be considered admissible in the control process, and in addition, the use of probabilistic-statistical characteristics does not always guarantee the desired quality of system control.
In applied problems, a more common situation is when a priori data on the initial state of the system and the parameters of external influences are minimal, i.e. there are no statistical descriptions of them, and information is limited to setting only the set of possible values of unknown parameters. In such cases, we can say that the model under consideration (1) represents a control system in conditions of uncertainty.
It is well known from the theory of differential inclusions and multi-valued analysis [3, 15] that under fairly general assumptions, control system (1) with indefinite parameters $w = w(t) \in W$ and initial data $x^0 \in D$ is equivalent to differential inclusion

$$\dot{x} \in f(t,x,u,W)) \equiv \{ f(t,x,u,w) : w \in W) \}, \ t \ge t_0, \ x(t_0) \in D, u \in U.$$

The resulting mathematical model belongs to the class of so-called controlled differential inclusions [6, 9, 16]

$$\dot{x} \in F(t,x,u), t \ge t_0, \ x(t_0) \in D, u \in U, \tag{2}$$

Where $F(t,x,u)$ is a multi-valued display, $u = u(t)$ is a control parameter.
For control systems under conditions of uncertainty, the properties of the ensemble of trajectories, methods for estimating the reachability set and forecasting the phase state of the system and others are of great interest [2, 4, 5, 16]. Depending on the criterion for assessing the state of the system, various problems of optimal control of the ensemble of trajectories are studied: control by speed, minimax, and other criteria [4, 5, 7, 8, 13, 16].
The control problem of ensemble of trajectories of differential inclusions can have various statements. The control problem for such systems can be posed, for example, as the problem of the complete or partial immersion of the end points of all possible trajectories of a system on a given terminal set. Some statements of controllability problems for differential inclusions, understood as controllability of an ensemble of trajectories with respect to given initial and terminal states, were considered in [6, 9, 16]. For such systems, the controllability conditions and some properties of the set of controllability points with respect to the terminal set are studied, in particular, its topological structure is studied.
The delay factor has a significant impact on the dynamics of the system and, therefore, the final result of research on control and optimization problems. This is also confirmed by studies on controlled differential inclusions with delays. Some properties of controlled differential inclusions with delays were studied in [10, 11]. In particular, the conditions of compactness, convexity of the set of absolutely continuous solutions, as well as their dependence on the control parameter and the initial state are revealed.
The control problems for differential inclusions with delays are characterized by a feature related to the specifics of the problem of controlling the ensemble of trajectories and the delay factor. Separate results on the control of the ensemble of trajectories for differential inclusions with delay were obtained in [10]. In the case of linearity of such systems, the some controllability conditions of the ensemble of trajectories are studied. In [12, 14, 17], for such systems, the minimax optimal control problem for the ensemble of trajectories, and in [17] the nonsmooth problem with terminal constraints was considered.

2. Statement of the problem
Consider a differential inclusion with delay of the form

$$\dot{x} \in F(t, x(t), x(t-h), u(t)), \tag{3}$$

Where $x \in R^n, \ F(t,y,u) \subset R^n, \ u \in R^m, \ t \ge t_0.$ Here, the parameter *u* plays the role of control actions.

By an admissible control for system (3) we mean a measurable bounded *m* - vector function $u = u(t)$ defined on a certain interval $T = [t_0, t_1]$ of time.

Denote $U_T(L)$ - the set of all admissible controls $u(t), t \in T$, with values from a closed ball $S_L = \{v \in R^m : \|v\| \le L\}$.
We denote by $H_T(u, \varphi^0)$ the set of absolutely continuous solutions $x = x(t), \ t \in T$, differential inclusion (3) corresponding to the control $u \in U_T(L)$ and the initial condition

$$x(t) = \varphi^0(t), \quad t \in T_0, \tag{4}$$

Where $\varphi^0(\cdot) \in C^n(T_0)$ ($C^n(T_0)$ is the space of continuous on $T_0 = [t_0 - h, t_0]$ n -vector-functions).
We consider the controllability problem for controlled differential inclusion with delay (3) in the sense of complete immersion of the ensemble of trajectories on a given terminal set. More precisely, assuming given a convex closed set of terminal states $Y = Y(t) \subset R^n$, $t \ge t_0$, we consider following.

Problem: it is required to find out the conditions under which there is an admissible control $u(t), \ t \in T$ such that all the solutions corresponding to it $x(\cdot) \in H_T(u, \varphi^0)$ satisfy the condition $x(t_1) \in Y(t_1)$.
Consider the set $X_T(t, u, \varphi^0) = \{\xi \in R^n : \xi = x(t), x(\cdot) \in H_T(u, \varphi^0)\}$ consisting of all points of the space of phase states lying on the trajectories $x(\cdot) \in H_T(u, \varphi^0)$ for a given admissible control $u = u(t)$, the initial function $\varphi^0 \in C^n(T_0)$, and time $t \in T = [t_0, t_1]$. The multi-valued mapping $t \to X_T(t, u, \Phi^0)$ is called the ensemble of trajectories of the system (3) - (4).

Definition 1. *We say that the ensemble of trajectories of system (3) is controllable from the initial state* $\varphi^0(\cdot)\in C^n(T_0)$ *to the set of terminal states* $Y=Y(t)$ *(system (3)* (φ^0,Y) *-controlled) if there is an admissible control* $u=u(t)$, $t\in T=[t_0,t_1]\subset T_\infty=[t_0,\infty]$ *such that the corresponding ensemble of trajectories satisfies the boundary condition.*

$$X_T(t_1,u,\varphi^0)\subset Y(t_1) \quad (5)$$

Definition 2. *An ensemble of trajectories of system (3) is called completely controllable into a terminal set* $Y=Y(t)$ *(system (3) is completely Y-controllable) if the system is* (φ^0,Y)*-controlled for each initial function* $\varphi^0(\cdot)\in C^n(T_0)$ *for which* $\varphi^0(t_0)\notin Y(t_0)$.

From Definition 1 it is clear that (φ^0,Y) - the controllability of the system means the solvability of the boundary value problem (3) - (4) in the class of admissible controls $u=u(t)$ defined on a certain time interval $T=[t_0,t_1]$. Clarification of the conditions for the solvability of this problem is the main goal of the study provided for in this paper. In addition, we will study the conditions for the complete *Y*-controllability of the ensemble of trajectories of the system under consideration.

3. Research methods
Auxiliary Results

It is clear from the above definitions that for the controllability of the ensemble of trajectories of system (3), the conditions for the compactness and convexity of the set $X_T(t,u,\varphi^0)$, as well as the continuous dependence of $X_T(t,u,\varphi^0)$ on (t,u) are essential. Therefore, with respect to the right-hand side $F(t,x,y,u)$ of differential inclusion (3), we impose some conditions that must ensure that these requirements are met.

Assumption 1.

1) For any $(t,x,y,u)\in T_\infty\times R^n\times R^n\times R^m$ set $F(t,x,y,u)$ convex compact from R^n

2) The multi-valued map $(t,x,y,u)\to F(t,x,y,u)$ is measurable in $t\in T_\infty$ for $\forall(x,y,u)\in R^n\times R^n\times R^m$ and continuous in (x,y,u) for almost all $t\in T_\infty$.

3) The multi-valued mapping $(x,y)\to F(t,x,y,u)$ satisfies the Lipschitz condition: $h(F(t,x',y',u),F(t,x'',y'',u))\le l(t,u)[\|x'-x''\|+\|y'-y''\|]$, $\forall x',x'',y'',y''\in R^n$, where the function $l(t,u)$ is such that $l(t,u(t))$ is summable on $T=[t_0,t_1]$ for any admissible control $u=u(t)$, $t\in T=[t_0,t_1]$ ($h(F_1,F_2)$ – Hausdorf metric).

4) There are functions $g_i(t,u),\ i=1,2,$ such that $g_i(t,u(t)),\ i=1,2,$ summable on *T* functions for any admissible controls $u=u(t)$, $t\in T=[t_0,t_1]$, $T\subset T_\infty$, and it is true

$$\|\xi\|\le g_1(t,u)(\|x\|+\|y\|)+g_2(t,u),\forall\xi\in F(t,x,y,u),\quad (t,x,y,u)\in T_\infty\times R^n\times R^n\times R^m.$$

5) The support function $C(F(t,x,y,u),\psi)=\max\{(\xi,\psi):\xi\in F(t,x,y,u)\}$ is concave in (x,y) for almost all $t\in T_\infty$ and all $(u,\psi)\in R^m\times R^n$.

From the results of [11] it easily follows

Lemma 1. *Let Assumption 1 hold Then:*

A) For any $u\in U_T(L),T\in T_\infty$, $\varphi^0(\cdot)\in C^n(T_0)$*, and* $t\in T=[t_0,t_1]$*, the set* $X_T(t,u,\varphi^0)$ *is a non-empty convex compact set from* R^n*;*

B) The multi-valued map $(t,u)\to X_T(t,u,\varphi^0)$ *is continuous on* $T\times U_T(L)$ *in the metric* $R^1\times L_2(T)$*, where* $L_2(T)$ *is the space* of square summable functions.

Let the right-hand side of differential inclusion (3) have the form:

$$F(t,x,y,u)=A(t)x+A_1(t)y+b(t,u), \quad (6)$$

Those consider the following linear model:

$$\dot{x}\in A(t)x(t)+A_1(t)x(t-h)+b(t,u(t)), \quad (7)$$

Where $A(t)$ and $A_1(t)$ are square matrices of size *n*, $b(t,u)$ is a nonempty subset of R^n. The following conditions will be imposed on the right-hand side of differential inclusion (7):

Assumption 2.

1) Elements of $n \times n$ -matrices $A(t)$ and $A_1(t)$ are summable on any $T=[t_0,t_1]\subset T_\infty$;

2) For any $(t,u)\in T_\infty \times R^m$, the set $b(t,u)$ is a convex compact from R^n;

3) The multi-valued mapping $(t,u)\to b(t,u)$ is measurable in $t\in T_\infty$ and continuous in $u\in R^m$, moreover, $\|\xi\|\le \beta_1(t)\|u\|+\beta_2(t),\ \forall \xi\in b(t,u),\ (t,u)\in T_\infty\times R^m$, where $\beta_i(\cdot),\ i=1,2,$, are functions summable on any interval $T\subset T_\infty$.

4) The support function $C(b(t,u),\psi)=\max\{(\xi,\psi):\xi\in b(t,u)\}$ is convex in $u\in V$ for almost all $t\in T_\infty$.

Under the conditions of Assumption 2, the following representation of the ensemble of trajectories of system (7) is valid through its parameters:

$$X_T(t,u,\varphi^0)=F(t,t_0)\varphi^0(t_0)+\int_{t_0}^{t_0+h}F(t,\tau)A_1(\tau)\varphi^0(\tau-h)d\tau+$$
$$+\int_{t_0}^{t}F(t,\tau)b(\tau,u(\tau))d\tau, \tag{8}$$

Where $F(t,\tau)-n\times n$ is a matrix function satisfying the equation

$$\frac{\partial F(t,\tau)}{\partial \tau}=-F(t,\tau)A(\tau)-F(t,\tau+h)A_1(\tau+h),\ \tau\le t,$$
$$F(t,t-0)=E,\ \ F(t,\tau)\equiv 0,\ \ \tau>t+0,$$

E– identity $n\times n$ - matrix.

Lemma 2. *Let Assumption 2 be satisfied, and for any* $(t,u)\in T_\infty\times R^m$*, the set* $b(t,u)$ *is a compact from* R^n *(not necessarily convex). Then all the statements of Lemma 1 are true and, moreover, the support function* $C(X_T(t,u,\varphi^0),\psi)=\max\{(\xi,\psi):\xi\in X_T(t,u,\varphi^0)\}$ *is convex in* $u\in U_T(L)$ *for all* $t\in T$ *and all* $\psi\in R^n$.

Indeed, if the conditions of Assumption 2 are satisfied, then the multi-valued mapping $(t,x,y,u)\to F(t,x,y,u)$ of the form (6) satisfies the conditions of Assumption 1. Therefore, for the control system (7), all the statements of Lemma 1 remain valid. Using the representation of the ensemble of trajectories of the linear system (7), and taking into account the properties of the integral of multi-valued mappings, we verify that the convexity property of each set $X_T(t,u,\varphi^0)$, $t\in T=[t_0,t_1]$ is preserved without requiring the convexity of the values of the multi-valued mapping $(t,u)\to b(t,u)$.

Further, using formula (8) and the properties of the support functions, we have:

$$C(X_T(t,u,\varphi^0),\psi)=(F(t,t_0)\varphi^0(t_0)+\int_{t_0}^{t_0+h}F(t,\tau)A_1(\tau)\varphi^0(\tau-h)d\tau,\psi)+$$
$$+\int_{t_0}^{t}C(F(t,\tau)b(\tau,u(\tau)),\psi)d\tau \tag{9}$$

From this formula for the support function of the set $X_T(t,u,\varphi^0)$ it easily follows that under condition 4) of Assumption 2, the support function $C(X_T(t,u,\varphi^0),\psi)$ is convex in $u\in U_V(T)$ for all $t\in T$ and all $\psi\in R^n$.

4. The main results

According to Definition 1, system (3) (φ^0,Y) -controllable if and only if relation (5) holds for some admissible control $u\in U_T(L)$, i.e. inclusion system: $X_T(t_1,u,\varphi^0)\subset Y(t_1),\ u\in U_T(L)$ compatible. Therefore, by virtue of Lemma 2 and the results of [6], the following statement is true.

Theorem 1. *For* (φ^0,Y) *- controllability of system (3) it is necessary, and for linear system (7) it is necessary and sufficient that there exists* $t_1>t_0$ *such that*

$$\sup_{\|\psi\|=1} \inf_{u\in U_T(L)} \overline{conc}\left[C\left(X_T\left(t_1,u,\varphi^0\right),\psi\right)-C\left(Y(t_1),\psi\right)\right]\le 0 \quad (10)$$

Where $\overline{conc} f$ is the concave closure [2] of the function f

The above theorem gives an implicit criterion for controllability of the ensemble of trajectories in the form of relation (10). Although it is complicated by the sequence of three operations: $\sup_{\|\psi\|=1} \inf_{u\in U_V(T)} \overline{conc} f$, Theorem 1 will be the theoretical basis for the subsequent results we obtained on controllability conditions.

Using formula (9) for the support function of the set $X_T\left(t_1,u,\varphi^0\right)$ and using the properties of the operation of transition to concave closures, from Theorem 1 we obtain the following result.

Theorem 2. *For* $\left(\varphi^0,Y\right)$ *- controllability of system (7), it is necessary and sufficient that the relation*

$$\sup_{\|\psi\|=1}\{\left(F(t_1,t_0)\varphi^0(t_0),\psi\right)+\int_{t_0}^{t_0+h}\left(F(t,\tau)A_1(\tau)\varphi^0(\tau-h),\psi\right)+$$
$$+\inf_{u\in U_T(L)} \overline{conc}_{\psi}\left[\int_{t_0}^{t_1} C\left(F(t_1,t)b(t,u(t)),\psi\right)-C\left(Y(t_1),\psi\right)\right]\le 0$$

for some $t_1>t_0$.

Consider another special case of a system with delay (3).

Let in (7) $b(t,u)=B(t)u+Q(t)$, where $B(t)$ is a $n\times m$ -matrix, $Q(t)$ is a nonempty subset of R^n, i.e. we consider a linear controlled differential inclusion

$$\dot{x}\in A(t)x(t)+A_1(t)x(t-h)+B(t)u(t)+Q(t) \quad (11)$$

Assumption 3.

1) Elements of $n\times m$ -matrix $B(t)$ are summable on any $T=[t_0,t_1]\subset T_\infty$;

2) $Q(t)$ – convex closed and bounded subsets R^n;

3) The multi-valued mapping $t\to Q(t)$, $t\ge t_0$ is measurable.

Under Assumption 3, all conditions of Assumption 2 are satisfied. Therefore, since

$$\inf_{u\in U_T(L)} \overline{conc}_{\psi}\left[\int_{t_0}^{t_1} C\left(F(t_1,t)b(t,u(t)),\psi\right)-C\left(Y(t_1),\psi\right)\right]=$$
$$=-L\int_{t_0}^{t_1}\|B'F'(t_1,t)\psi\|dt+\overline{conc}_{\psi}\left[\int_{t_0}^{t_1} C\left(F(t_1,t)Q(t),\psi\right)dt-C\left(Y(t_1),\psi\right)\right],$$

Then from Theorem 2 it follows that controllability criterion for the ensemble of trajectories of system (11) have the form:

$$\sup_{\|\psi\|=1}\{\left(F(t_1,t_0)\varphi^0(t_0),\psi\right)+\int_{t_0}^{t_0+h}\left(F(t_1,\tau)A_1(\tau)\varphi^0(\tau-h),\psi\right)d\tau+$$
$$-L\int_{t_0}^{t_1}\|B'F'(t_1,t)\psi\|dt+\overline{conc}_{\psi}\left[\int_{t_0}^{t_1} C\left(F(t_1,t)Q(t),\psi\right)dt-C\left(Y(t_1),\psi\right)\right]\le 0. \quad (12)$$

We introduce the notation:

$$S(t,\varphi^0)=F(t,t_0)\varphi^0(t_0)+\int_{t_0}^{t_0+h}F(t,\tau)A_1(\tau)\varphi^0(\tau-h)d\tau+\int_{t_0}^{t}F(t,\tau)Q(\tau)d\tau,\ t>t_0.$$

Then we have:

$$(F(t_1,t_0)\varphi^0(t_0),\psi)+\int_{t_0}^{t_0+h}(F(t_1,\tau)A_1(\tau)\varphi^0(\tau-h),\psi)d\tau+$$

$$+\overline{conc}_{\psi}\left[\int_{t_0}^{t_1} C(F(t_1,t)Q(t),\psi)dt - C(Y(t_1),\psi)\right] =$$

$$= -\overline{co}_{\psi}[C(Y(t_1),\psi) - C(S(t_1,\varphi^0),\psi)] = -C(Y(t_1) \underline{*} S(t_1,\varphi^0),\psi),$$

Where $Y(t_1) \underline{*} S(t_1,\varphi^0)$ is the geometric difference of the sets $Y(t_1)$ and $S(t_1,\varphi^0)$, i.e. $Y(t_1) \underline{*} S(t_1,\varphi^0) = \{\xi \in R^n : \xi + S(t_1,\varphi^0) \subset Y(t_1)\}$. Therefore, relation (12) takes the form:

$$\inf_{\|\psi\|=1}\{L\int_{t_0}^{t_1}\|B'F'(t_1,t)\psi\|dt + C(Y(t_1)\underline{*}S(t_1,\varphi^0),\psi)\} \geq 0 \tag{13}$$

Put:

$$P(t) = \int_{t_0}^{t} F(t,\tau)Q(\tau)d\tau,\ t > t_0 \quad p^0 = F(t,t_0)\varphi^0(t_0) + \int_{t_0}^{t_0+h} F(t,\tau)A_1(\tau)\varphi^0(\tau-h)d\tau.$$

Then $S(t,\varphi^0) = p^0 + P(t), t > t_0$, $C(Y(t_1)\underline{*}S(t_1,\varphi^0),\psi) = C(Y(t_1)\underline{*}P(t_1),\psi) - (p^0,\psi)$

Therefore, condition (13) can be written as follows:

$$\inf_{\|\psi\|=1}\{L\int_{t_0}^{t_1}\|B'F'(t_1,t)\psi\|dt + C(Y(t_1)\underline{*}P(t_1),\psi) - (p^0,\psi)\} \geq 0 \tag{14}$$

Thus, we have obtained the following controllability criterion for the ensemble of trajectories of system (11) into the terminal set $Y = Y(t)$.

Theorem 3. *The ensemble of trajectories of system (11) is* (φ^0, Y) *controllable if and only if relation (14) holds, where* $t_1 > t_0$ *and* $L > 0$.

Now, using this result, we will find out the conditions for the complete Y – controllability of system (11).

Theorem 4. *Let there exist* $t_1 > t_0$ *such that* $Y(t_1)\underline{*}P(t_1) \neq \varnothing$ *and*

$$\lambda \equiv \inf_{\|\psi\|=1}\int_{t_0}^{t_1}\|B'(t)F'(t_1,t)\psi\|dt > 0. \tag{15}$$

Then system (11) is completely Y – controllable.

Proof. According to Theorem 3, it is enough for us to show that relations (14) holds for each initial function $\varphi^0(\cdot) \in C^n(T_0)$, $\varphi^0(t_0) \notin Y(t_0)$, and for some $t_1 > t_0$. We have:

$$\inf_{\|\psi\|=1}\{L\int_{t_0}^{t_1}\|B'F'(t_1,t)\psi\|dt + C(Y(t_1)\underline{*}P(t_1),\psi) + (p^0,\psi)\} \geq$$

$$\geq L\inf_{\|\psi\|=1}\int_{t_0}^{t_1}\|B'F'(t_1,t)\psi\|dt + \inf_{\|\psi\|=1} C(Y(t_1)\underline{*}P(t_1),\psi) + \inf_{\|\psi\|=1}(p^0,\psi) \geq$$

$$= L\lambda + \inf_{\|\psi\|=1} C(Y(t_1)\underline{*}P(t_1),\psi) - \|p^0\| \tag{16}$$

Since, by condition (15) $\lambda > 0$, then for

$$L \geq \frac{\|p^0\| - \inf_{\|\psi\|=1} C(Y(t_1)\underline{*}P(t_1),\psi)}{\lambda}$$

From (16) we obtain (14). And this completes the proof of the theorem.

5. Discussion of results and conclusion

The problem of controllability is one of the important problems in the theory of optimal control. Here, for a controlled differential inclusion with delay, the controllability problem was considered as the problem of the complete immersion of an ensemble of trajectories on a given terminal set. The goal of the study was set: clarification of the controllability conditions expressed in terms of the parameters of the system under consideration.

When studying the controllability ensemble trajectory control problem, the main conditions on the right-hand side of the considered differential inclusions are given in the form of assumptions 1-3. They cover a wide class of differential inclusions with delay (3), as well as their linear models of the form (7) and (11).

Of the results obtained, the most general is Theorem 1. Although it is an implicit controllability criterion for the systems under consideration, the remaining results were obtained based on this theorem. In this case, the properties of the ensemble of trajectories of linear systems were taken into account. Among the important characteristics of the ensemble of trajectories of differential inclusions used in the subject under study, it is worth noting the properties given in the auxiliary lemmas.

Theorem 2 gives necessary and sufficient conditions for (φ^0, Y) - controllability of the ensemble of trajectories linear in state of system (7). And in Theorem 3 they are refined for a linear system, both in state and in control. Here, formula (8) of the representation of the ensemble of trajectories and formula (9) of its support function are essentially used. We note that a feature of the linear structure of systems found their expression in the study of the controllability problem, as evidenced by the results of Theorem 4.

The studied controllability property of the ensemble of trajectories of differential inclusions is of immediate interest for systems of the form.

$$\dot{x} = Ax(t) + A_1 x(t-h) + Bu(t) + q(t) \qquad (17)$$

When $q(t)$ is the parameter of indefinite external influences with values from $Q(t) \subset R^n$. Theorems 4 give the sufficient conditions of controllability for the ensemble of trajectories for the linear model (17) of the control object with delay under conditions of inaccuracy of external influence forces.

In particular, when the control system is stationary, i.e. $A(t) \equiv A, \quad A_1(t) = A, \quad B(t) \equiv B,$ then the condition of non-degeneracy of the governing equations [1] of the system $\dot{x} = Ax(t) + A_1 x(t-h) + Bu(t)$ is sufficient to satisfy condition (15). Therefore, this condition will be sufficient for controllability of the linear system (17) with a variable terminal state $Y = Y(t)$.

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