International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2020; 5(3): 122-128

www.mathsjournal.com
Received: 22-01-2020
Accepted: 24-02-2020

Rhonda Magel
Department of Statistics
North Dakota State University
Fargo, USA

Samah Al-Thubaiti
Department of Statistics
North Dakota State University
Fargo, USA

Corresponding Author:
Rhonda Magel
Department of Statistics
North Dakota State University
Fargo, USA

Nonparametric test for the umbrella alternative with known peak in a mixed design

Rhonda Magel and Samah Al-Thubaiti

Abstract
This paper proposes test statistics for detecting umbrella alternative with known peak when the data are a mixture of a randomized complete block and a completely randomized design. Modifications of the Mack-Wolfe and Kim-Kim test are proposed to develop the two statistics for a mixed design. The exact mean and variance of the modified versions of Mack-Wolfe and Kim-Kim test statistic under the null distribution are derived. The proposed test statistics are compared, through a simulation study, to each other and with some of the existing tests in terms of assessing the power and the type I error. Results are given.

Keywords: Randomized complete block design; Completely randomized design; Mixed design; Modified Mack-Wolfe test; Modified Kim-Kim test; Known peak umbrella alternative.

1. Introduction

Many statisticians and scientists prefer the nonparametric approach since it has specific desirable properties. It requires a few assumptions about the underlying populations from which the data are obtained. In particular, most nonparametric tests assume that the underlying distributions are the same type but possibly differ in location. In many cases, the increasing or decreasing nature of the parameters is assumed to be known a priori, such as with drug dosage levels, if the parameters are different. The effect of a drug on the experimental unit might increase to a certain level, and then its effectiveness may decrease with further increasing doses. In this case, an umbrella alternative is an appropriate model. The hypothesis for an umbrella alternative is given in (1)

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \quad vs \quad H_1: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k \qquad (1)$$

With at least one strict inequality, where μ_i is a location parameter for the i^{th} population; $i = 1,2, \ldots, p, \ldots, k$, and p is the peak of the umbrella alternative.

When speaking of any study's design, the first thing that a researcher should start with is thinking of the design that would be used to collect the data. It is possible that the researcher would begin an experiment with a randomized complete block design RCBD. Then for some reason, he could not collect the data for all treatments in each block. One of the reasons could be the experiment is so expensive, and missing observations could occur. At this point, the researcher would decide to change to a completely randomized design CRD, and the consequence of changing the design would be losing all data collected from RCBD. However, not using all available data is a waste of resources and time. As a fact, the more observations that we collect, the more powerful the test will be. For this purpose, we can think of combining the two designs. In this case, we need to introduce a new test statistic for this kind of design, which is called a mixed design. Magel *et al.* (2010) [5] introduced two new test statistics for a mixed design consisting of RCBD and CRD. We suggest adding a modification to the test statistics proposed by Magel *et al.* (2010) [5] in order to attempt to improve the performance of the test's power. The modifier that we use is a square distance between the groups. Recently, researchers have been working on mixed designs to use all the information from their data. Dubnicka *et al.* (2002) [1] used a rank-based procedure to develop a nonparametric test for a mixed paired and two-sample design. They combined the Wilcoxon signed-rank statistic (Wilcoxon (1945) [10] and the Wilcoxon-Mann-Whitney statistic (Mann and Whitney (1947) [8]. Magel and Fu (2014) [7] proposed a similar test to the one developed by Dubnicka *et al*. (2002) [1] for a mixed pair and two-sample design. They combined the standardized versions of the Wilcoxon signed-rank statistic by Wilcoxon (1945) [10] and the Wilcoxon-Mann-Whitney statistic by Mann and Whitney (1947) [8]. Magel *et al*. (2009) [6] developed two variations of test statistics for a mixed design of RCBD and CRD to test for the no decreasing alternative which is a particular case of the umbrella alternative. Magel *et al*. (2010) [5] considered a combination of a Kim-Kim test statistic (Kim and Kim (1992) for RCBD and a Mack-Wolfe test statistic (Mack and Wolfe (1981) [4] for CRD to test for the umbrella alternative with a known peak, p. They considered equal sample sizes for treatments in CRD; and one observation for i^{th} treatment and j^{th} block in RCBD.

2. Proposed Tests for the Mixed Design

We are modifying two nonparametric test statistics for a mixed design of a randomized complete block design and a completely randomized design based on the umbrella hypothesis given in (1) in the case of a known peak. As in Dubnicka *et al*. (2002) [1] and Magel *et al*. (2010) [5], we are considering test statistics in terms of combining modified versions of test statistics weighted by the squared distance between groups for the RCBD and CRD.

2.1. Modified Mack-Wolfe Test Statistic II for (CRD)

We will introduce a modification of the Mack-Wolfe test statistic for the CRD portion, which differs from a previous modification investigated by Gökpinar and Gökpinar (2016) [2]. The modified version of the Mack-Wolfe test statistic MMW_{pII} in (2) gives weight to the Mann-Whitney statistics, which is a squared distance between groups.

$$MMW_{pII} = \sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^2\,U_{ij} + \sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^2\,U_{ji} \quad (2)$$

Where U_{ij} is a Mann-Whitney statistic applied to the observations in i^{th} and j^{th} groups?

2.1.1. The Mean and Variance

Mack and Wolfe (1981) proposed a test statistic that has an asymptotic normal under the null hypothesis, so the modified Mack-Wolfe test statistic MMW_{pII} in (2), when it is standardized, has asymptotically a standard normal distribution under H_0. For this purpose, we need to know the expected value and variance of MMW_{pII} when H_0 is true, and for simplicity, we will use the same sample size for all treatments.

Theorem 1: The null expected value of a modified version of Mack-Wolfe test statistic MMW_{pII}, when the sample sizes are equal is given by

$$E_0(MMW_{pII}) = \frac{n^2}{24}\{p^2(p^2-1) + (k-p+1)^2[(k-p+1)^2-1]\} \quad (3)$$

Proof. See the Appendix.
Theorem 2: The null variance of a modified version of Mack-Wolfe test statistic MMW_{pII}, when the sample sizes are equal is given by (44)

$$Var_0(MMW_{pII}) = \frac{n^2(2n+1)}{12}\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^4 + \sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^4\}$$

$$+\frac{n^3}{12}\{\sum_{i=1}^{p-1}\ \sum_{j=i+1}^{p} sum_{ij} + \sum_{i=p}^{k-1}\ \sum_{j=i+1}^{k} sum_{ji}\} \quad (4)$$

$$+\frac{n^3}{6}\{\frac{p(p-1)(k-p)(k-p+1)[2(k-p)+1][2(p-1)+1]}{36}\}$$

Where,

$$sum_{ij} = (j-i)^2[-(j-i)^2 - \sum_{t=1}^{i-1}t^2 + \sum_{t=1}^{i-1}(j-t)^2 + \sum_{t=1}^{p-i}t^2 + \sum_{t=0}^{j-i-1}t^2 - \sum_{t=1}^{p-j}t^2]$$

$$sum_{ji} = (j-i)^2[-(j-i)^2 - \sum_{t=1}^{i-p}t^2 + \sum_{t=p}^{i-1}(j-t)^2 + \sum_{t=1}^{k-i}t^2 + \sum_{t=0}^{j-i-1}t^2 - \sum_{t=1}^{k-j}t^2]$$

Proof. See the Appendix.
The standardized version of MMW_{pII} is given in (5)

$$MMW_{pII}^{*} = \frac{MMW_{pII} - E_0(MMW_{pII})}{\sqrt{Var_0(MMW_{pII})}} \quad (5)$$

When H_0 is true, MMW_{pII}^* has an asymptotic standard normal distribution. We reject H_0 if $MMW_{pII}^* \geq Z_\alpha$ at α significant level, where Z_α is the upper α quantile of the standard normal distribution.

2.2. Modified Kim-Kim for (RCBD)

The modified Kim-Kim test statistic for the (RCBD) portion is a sum of a modified Mack-Wolfe MMW_{pII} over b blocks in case of known umbrella peak.

$$MKK = \sum_{s=1}^{b} MMW_{spII} \quad (6)$$

where, MMW_{spII} is a modified Mack-Wolfe for s^{th} block.

2.2.1. The Mean and Variance

Kim and Kim (1992) proposed a test statistic which has an asymptotic normal distribution when H_0 is true, so the modified Kim-Kim test statistic MKK in (6), when it is standardized, has an asymptotically standard normal distribution under the null hypothesis. For this purpose, we need to know the expected value and variance of MKK when H_0 is true; for simplicity, we will use the same sample size for all cells (in this case the sample size is one). Kim and Kim (1992) derived the mean and variance for a randomized complete block design by summing the mean and variance of Mack-Wolfe over blocks. As a result, the expected value and variance of MKK are, respectively,

$$E_0(MKK) = \sum_{s=1}^{b} E_0\left(MMW_{spII}\right)$$

$$Var_0(MKK) = \sum_{s=1}^{b} Var_0(MMW_{spII}) \quad (7)$$

The standardized version of MKK is given in (8)

$$MKK^* = \frac{MKK - E_0(MKK)}{\sqrt{Var_0(MKK)}} \quad (8)$$

When H_0 is true, MKK^* has an asymptotic standard normal distribution. We reject H_0 if $MKK^* \geq Z_\alpha$ at α significant level, where Z_α is the upper α quantile of the standard normal distribution.

2.3. Modified Test Statistics for Combining (CRD) & (RCBD)

We will propose a test statistic for a mixed design for a case of a known peak with two different methods of combining the modified test statistics for the (RCBD) and (CRD) portions.

First Method**:** We suggest combining the standardized versions of a modified Mack-Wolfe test statistic and a modified Kim-Kim test statistic as given in (10)

$$MD_I = MMW_{pII}^* + MKK^* \quad (9)$$

Under H_0, the asymptotic distribution of MD_I is a normal distribution with a mean of zero and a variance of 2. The standardized version of MD_I is given in (10)

$$MD_I^* = \frac{MD_I - 0}{\sqrt{2}} \quad (10)$$

Under H_0, MD_I^* has an asymptotic standard normal distribution, and we reject H_0 when $MD_I^* \geq Z_\alpha$ at α significant level, where Z_α is the upper α quantile of the standard normal distribution.

Second Method**:** We suggest combining the two modified versions of the Mack-Wolfe test statistic and the Kim-Kim test statistic as given in (11)

$$MD_{II} = MMW_{pII} + MKK \quad (11)$$

It is noted that the sample sizes will be equal for all treatments for the (CRD) portion and one observation for i^{th} treatment and the s^{th} block in (RCBD) portion. Therefore, when H_0 is true, the mean and the variance of MD_{II} given that we have the same sample size for all treatments in (CRD) portion and one observation for each cell in (RCBD) portion are, respectively,

$$E_0(MD_{II}) = E_0\left(MMW_{pII}\right) + E_0(MKK)$$

$$Var_0(MD_{II}) = Var_0(MMW_{pII}) + Var_0(MKK) (12)$$

Where, $E_0(MMW_{pII})$, $E_0(MKK)$, $Var_0(MMW_{pII})$, and $Var_0(MKK)$ are the expected values and variance of the modified Mack-Wolfe test statistic and the Kim-Kim test statistic, respectively.
The standardized version of MD_{II} is given in (13).

$$MD_{II}^* = \frac{MD_{II} - E_0(MD_{II})}{\sqrt{Var_0(MD_{II})}} (13)$$

Under H_0, MD_{II}^* has an asymptotic standard normal distribution, and we reject H_0 when $MD_{II}^* \geq Z_\alpha$ at α significant level, where Z_α is the upper α quantile of the standard normal distribution.

3. Simulation Study

In this section, we investigate by Monte Carlo simulation the type I error and power of the proposed test statistics in this paper and the proposed tests by Magel *et al*. (2010) [5] based on replications of 5,000 samples. The observations are generated using the SAS, and they are assumed to come from two different types of underlying distributions. The first type is a symmetric distribution, like the standard normal, and student's t with 3 degrees of freedom. The second type is a skewed distribution, like the standard exponential. We assume that the variance of the error terms in (RCBD) and (CRD) are equal for a mixed design. We consider the case of the number of treatments $k = 4$ at $p = 2$, and $k = 5$ at $p = 2, 3$. Moreover, the equal sample sizes n are selected to be $6, 10, 16$ and 20 for the (CRD) portion, and the number of blocks for the (RCBD) portion is considered to be half, equal and twice the sample size for each treatment in the (CRD). Also, we took in account different location parameter configurations. We considered situations in which the peak is distinct or an indistinct.

4. Results from the Simulation Study

In this section, we present the results of the proposed test statistics with a square distance modification as described in section 2.3 for a known peak of the umbrella hypothesis; as well as the results of the test statistics, without modification, introduced by Magel *et al*. (2010) [5] under the umbrella hypothesis once the peak is known for the mixed design of combining a completely randomized design portion (CRD) and a randomized complete block design portion (RCBD). As an illustration, we will present selected results of the estimated type I error ($\alpha = 0.05$), and the estimated power for proposed test statistics at each configuration for the location parameters of the umbrella hypothesis; once we have taken into account the relationship between the sample size for the CRD portion and the numbers of blocks for the RCBD portion. We assume three different underlying distributions, including the standard normal distribution, student's t distribution with 3 degrees of freedom, and the standard exponential distribution. Additionally, the variance for CRD portion is equal to the variance for RCBD portion.

4.1 Four or Five Treatments at Known Peak, $p = 2$.

One can see in Table 1, the estimated type I error is around 0.05 for the proposed test statistics and the test statistics introduced by Magel *et al*. (2010) [5]. When the distance between the first parameter and the 2nd parameter (peak) is less than or equal to the distance between the 2nd parameter (peak) and the third parameter, the modified test statistics generally have higher

Table 1: Estimated Power and Type I Error at Known peak = 2

The Number of Treatments = 4, The sample size =10 for CRD portion										
Location Parameter					Distribution / The Number of Blocks for RCBD portion					
					Normal/b=5		Student's t with 3 df/b=10		Exponential/b=20	
1	2	3	4	Method	N*	M**	N*	M**	N*	M**
0	0	0	0	First	.0570	.0448	.0534	.0532	.0522	.0524
				Second	.0512	.0516	.0504	.0518	.0562	.0480
0	1	.75	.2	First	.7730	.7042	.7326	.6436	.9986	.9928
				Second	.6976	.6284	.5648	.5034	.9584	.9180
.8	1	.75	.2	First	.5040	.5870	.4666	.5348	.9622	.9728
				Second	.4394	.5252	.3494	.4070	.8014	.8336
.75	1	.75	.2	First	.5210	.5962	.4926	.5420	.9658	.9764
				Second	.4610	.5330	.3708	.4246	.8150	.8518
.5	.5	.2	0	First	.2434	.2924	.2314	.2804	.6208	.7426
				Second	.2122	.2554	.1762	.2196	.4212	.5258
.5	.5	0	0	First	.2296	.2876	.2198	.2760	.5904	.7236
				Second	.1998	.2556	.1714	.2144	.3770	.4998
.5	.5	.5	0	First	.2236	.2938	.2250	.2744	.5868	.6986
				Second	.2008	.2634	.1764	.2222	.3930	.4986

The Number of Treatments = 5, The sample size =10 for CRD portion											
Location Parameter						Distribution / The Number of Blocks for RCBD portion					
						Normal/ b=5		Student's t with 3 df / b=10		Exponential / b=20	
1	2	3	4	5	Method	N*	M**	N*	M**	N*	M**
0	0	0	0	0	First	.0512	.0566	.0528	.0540	.0462	.0486
					Second	.0500	.0530	.0520	.0562	.0536	.0468

0	.8	.6	.4	.2	First	.5800	.4970	.5396	.4518	.9884	.9558
					Second	.5134	.4478	.4102	.3472	.8756	.7798
.75	.8	.5	.2	0	First	.5754	.6512	.5220	.6096	.9832	.9878
					Second	.4956	.5814	.3966	.4652	.8544	.8950
.8	.8	.5	.2	0	First	.5488	.6518	.5168	.5924	.9774	.9886
					Second	.4776	.5786	.3994	.4588	.8416	.8892
.5	.5	0	0	0	First	.2466	.3156	.2358	.2842	.6394	.7464
					Second	.2216	.2882	.1718	.2128	.4010	.5120
.5	.5	.5	0	0	First	.3620	.4376	.3476	.4006	.8472	.9142
					Second	.3092	.3794	.2652	.3124	.6164	.6912

* The results for non-modification case. ** The results for modification case.

powers for all distributions, all sample sizes, and all ratios between the RCBD portion and the CRD portion, then the tests proposed by Magel *et al*. (2010) [5]. For example, see the configuration (.5, .5, .5, 0) in Table 1. The powers are all higher for the new modified versions. When the difference between the first parameter and the 2nd parameter (peak) is greater than the distance between the 2nd parameter (peak) and the third parameter, the tests proposed by Magel *et al*. (2010) [5] generally have higher powers. See, for example in Table 1 when k=5, the powers associated with the configuration (0, .8,.6,.4,.2) are higher for the non-modified versions.

Table 2: Estimated Power and Type I Error at Known peak = 3

The Number of Treatments = 5, The sample size = 6 for CRD portion											
Location Parameter						**Distribution / The Number of Blocks for RCBD portion**					
						Normal/ b=3		**Student's t with 3 df / b=6**		**Exponential / b=12**	
1	**2**	**3**	**4**	**5**	**Method**	**N***	**M****	**N***	**M****	**N***	**M****
0	0	0	0	0	First	.0468	.0464	.0510	.0492	.0490	.0466
					Second	.0504	.0498	.0508	.0478	.0518	.0500
0	0	.5	0	0	First	.2916	.2970	.2724	.2622	.7342	.7524
					Second	.2636	.2792	.2204	.2276	.5392	.5540
0	0	.5	.2	.2	First	.2182	.2338	.2084	.2028	.5770	.5786
					Second	.1978	.2160	.1782	.1688	.3964	.4108
0	.4	.7	.2	0	First	.4560	.4750	.4090	.4278	.9426	.9342
					Second	.4036	.4240	.3378	.3432	.8130	.7992
0	.4	.7	.4	.2	First	.3592	.3734	.3336	.3426	.8810	.8566
					Second	.3136	.3374	.2714	.2814	.7252	.6850
.2	.5	.5	.2	0	First	.2252	.2116	.2104	.2068	.6046	.5800
					Second	.1992	.2008	.1730	.1684	.4278	.4032
0	.8	.8	.5	.2	First	.4454	.4574	.4210	.4136	.9160	.9216
					Second	.3924	.4142	.3306	.3268	.7550	.7750
0	.5	.5	.5	0	First	.2870	.2862	.2596	.2790	.7122	.7414
					Second	.2568	.2644	.2130	.2304	.5344	.5672
0	0	.5	.5	.5	First	.1278	.1310	.1264	.1288	.2714	.2750
					Second	.1112	.1260	.1068	.1122	.1808	.1992

***** The results for non-modification case. ****** The results for modification case.

4.2 Five Treatments at Known Peak, $p = 3$.

One can see from the results in Table 2, the estimated type I errors are around 0.05 for the proposed test statistics and the test statistics introduced by Magel *et al*. (2010) [5]. The estimated powers for proposed test statistics with a square distance modification are slightly different from the estimated powers for test statistics, without modification, introduced by Magel *et al*. (2010) [5] in most of the location parameters configurations of the umbrella hypothesis. Namely, many of the estimated powers are within 0.01 of each other, which is the error term in the confidence interval in estimating the power. However, there were some cases when the underlying distribution is symmetric (normal, student's t), and the peak of the umbrella is ***distinct*** in which the estimated powers of the proposed test statistics with modification are higher than the estimated powers of the test statistics without modification. On the other hand, the results for the skewed underlying distribution (exponential) vary from the results for the symmetric underlying distributions. Table 2 shows there is an unclear pattern as to when the modified or unmodified test statistics have the higher estimated powers. It is noted that the results for the estimated power also have an unclear pattern once the sample size is greater than six observations for CRD portion once we change the number of blocks or the type of underlying distribution. In either case of modification or non-modification, the estimated power by using the first method of combining the two test statistics is better than the second method for all situations that considered.

Appendix A

In terms of finding the null expected value and the variance of MMW_{pII} as given in (3) and (4), respectively, we need to use the null expected value, variances, and covariance's for the Mann-Whitney statistic U_{ij} from Tryon and Hettmansperger (1973).

Proof of Theorem 1. Once the sample sizes are all equal to n, then the null expected value of U_{ij} is $\boldsymbol{E_0(U_{ij}) = \frac{n^2}{2}}$. The null expected value of MMW_{pII} is

$$E_0(MMW_{pII}) = \frac{n^2}{2}[\underbrace{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^2}_{1} + \underbrace{\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^2}_{2}](A.1)$$

We can simplify part 1 and part 2 in (A.1) as follows

$$\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^2 = \sum_{i=1}^{p-1}[1^2+2^2+\cdots+(p-i)^2] = \frac{p^2(p^2-1)}{12}$$

$$\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^2 = \sum_{i=p}^{k-1}[1^2+2^2+\cdots+(k-i)^2] = \frac{(k-p+1)^2[(k-p+1)^2-1]}{12}$$

Then, the null expected value of MMW_{pII} is

$$E_0(MMW_p) = \frac{n^2}{24}\{p^2(p^2-1) + (k-p+1)^2[(k-p+1)^2-1]\}(A.2)$$

This completes the proof.

Proof of Theorem 2. The null variance of MMW_{pII} can be found by using the following fact:

$$Var_0(MMW_{pII}) = Var_0(MJT_{upII}) + Var_0(MJT_{downII}) + 2\,Cov_0(MJT_{upII}, MJT_{downII})(A.3)$$

Since we consider an equal sample size, the $Var_0(MJT_{upII})$ is given by

$$Var_0(MJT_{upII}) = \frac{n^2(2n+1)}{12}\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^4 + \frac{n^3}{12}\underbrace{\sum_{A} 2\,(j-i)^2(m-l)^2}_{1}\,(A.4)$$

Where $A = \{(i,j)(l,m)|\{(1,2),(1,3)\},\{(1,2),(1,4)\},\ldots,\{(p-2,p),(p-1,p)\}\}$

For part 1 in (A.4), the null covariance's of the Mann-Whitney statistics are the same with different coefficients for each term, and the sum of the appropriate coefficients is given

$$\sum_{A} 2\,(j-i)^2(m-l)^2 = \{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} sum_{ij}\}(A.5)$$

Where,

$$sum_{ij} = (j-i)^2[-(j-i)^2 - \sum_{t=1}^{i-1}t^2 + \sum_{t=1}^{i-1}(j-t)^2 + \sum_{t=1}^{p-i}t^2 + \sum_{t=0}^{j-i-1}t^2 - \sum_{t=1}^{p-j}t^2]$$

By substituting (A.5) in (A.4), we can find the $Var_0(MJT_{upII})$. In the same way, we can find the $Var_0(MJT_{downII})$.

The $Cov_0(MJT_{upII}, MJT_{downII})$ can be found as follows

$$Cov_0(MJT_{upII}, MJT_{downII}) = \sum_{i=1}^{p-1}\sum_{j=p+1}^{k}(j-p)^2(p-i)^2 Cov_0(U_{ip}, U_{jp})(A.6)$$

Since we consider an equal sample size,

$$Cov_0(MJT_{upII}, MJT_{downII}) = \frac{n^3}{12}\sum_{i=1}^{p-1}\sum_{j=p+1}^{k}(j-p)^2(p-i)^2$$

$$= \frac{n^3}{12}\,\frac{p(p-1)(k-p)(k-p+1)[2(k-p)+1][2(p-1)+1]}{36}(A.7)$$

This completes the proof.

5. Conclusion and Discussion

In this section, we conclude the results of the performance of the estimated powers for the proposed test statistics for the peak known version. The simulation study shows that the first method of combining the two statistics of the CRD and the RCBD is generally better than the second method regardless of the underlying distribution, number of treatments, and the peak. We can distinguish some cases that show the square distance modification results in improvement in an estimated power regardless of the underlying distribution, and the relationship between a sample size of the CRD and block's number of the RCBD.

For the case of having four or five treatments and the peak at second location parameter, the estimated powers of proposed test statistics with a square distance modification for a mixed design in the case of the distinct peak are generally better than the estimated powers of test statistics, without modification, introduced by Magel *et al*. (2010) [5] under the umbrella hypothesis as long as, the difference between the first location parameter and the peak parameter is less than or equal to the difference between the third parameter and the peak parameter, such as (.8, 1, .75, .2), (.75, .8, .6, .4, .2).

For the case of having five treatments and the peak at third location parameter, the results vary from distribution to distribution and among sample sizes as to whether or not the modified versions of the test statistics have higher powers than the non-modified versions of the test statistics. The results show that the estimated powers of proposed test statistics with a square distance modification are slightly different from the estimated powers of test statistics introduced by Magel *et al*. (2010) [5] in most of the location parameters configurations of the umbrella hypothesis. There are a few cases, when the non-modified versions of the test statistics have higher powers, and a few cases when the modified versions of the test statistics have higher powers.

References

1. Dubnicka SR, Blair RC, Hettmansperger TP. Rank-based procedures for mixed paired and two-sample designs. Journal of Modern Applied Statistical Methods. 2002; 1(1):32-41.
2. Gökpinar E, Gökpinar F. A modified mack–wolfe test for the umbrella alternative problem. Communications in Statistics-Theory and Methods. 2016; 45(24):7226-7241.
3. Kim HD, Kim CY. Distribution-free tests for umbrella alternatives in a randomized block design. Journal of Nonparametric Statistics. 1992; 1(4):277-285.
4. Mack GA, Wolfe DA. K-sample rank tests for umbrella alternatives. Journal of the American Statistical Association. 1981; 76(373):175-181.
5. Magel R, Terpstra J, Canonizado K, Park JI. Nonparametric tests for mixed designs. Communications in Statistics-Simulation and Computation. 2010; 39(6):1228-1250.
6. Magel R, Terpstra J, Wen J. Proposed tests for the nondecreasing alternative in a mixed design. Journal of Statistics and Management Systems. 2009; 12(5):963-977.
7. Magel RC, Fu R. Proposed nonparametric test for the mixed two-sample design. Journal of Statistical Theory and Practice. 2014; 8(2):221-237.
8. Mann HB, Whitney DR. On a test of whether one of two random variables is stochastically larger than the other. The annals of mathematical statistics, 1947, 50-60.
9. Tryon PV, Hettmansperger TP. A class of non-parametric tests for homogeneity against ordered alternatives. The Annals of Statistics, 1973, 1061-1070.
10. Wilcoxon F. Individual comparisons by ranking methods. Biometrics bulletin. 1945; 1(6):80-83.