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An integrated supply chain model with inflation, lead time and capital constraint

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Abstract
This paper considers an integrated supply chain model which comprises of a supplier, a retailer and an end customer. The inventory deteriorates and has a certain life time, after this time item has no demand. For the consideration of real world practices the supplier's lead time has been considered in the study. This study also examines the effect of inflation. In this model shortages are allowed during lead time and are completely backlogged. The derived model is also illustrated numerically.

Keywords: Supply Chain, Inflation, Lead Time, Deterioration

Introduction

Inventory Management preserve the information about activities within firms that make sure the delivery of products to consumers. The subsystems that operate these functions contain sales, production, warehousing, ordering and delivery. In different companies the activities related with each of these areas may not be strictly controlled by separate subsystems, but these functions should be performed in a series in order to have a well-developed inventory control system.

In the inventory theory, it is considered that all the costs associated with the inventory control system remains constant over the passing of time, which is not practical in real life phenomenon. The effect of inflation is the costs of everything rising up over the time. Most of articles published so far do not assumed the effect of inflation and time value of money as parameters of the system. But due to increasing rate of inflation in all the countries the market situation has been changed to an extent during the last three decades. Nowadays inflation becomes an eternal feature in the inventory models. When little amount of inflation is often observed as having a positive effect on the market, hence it is not possible to ignore the effects of inflation the inventory control system. Inflation comes into the picture of the inventory due to having an impact on the present worth of the future inventory cost. Thus the inflation plays an important role in the inventory management and production system. The initiated regarding the approach of inflation was taken by Buzacott (1975). He developed the model with the effect of inflation by assuming a constant inflation rate. Misra (1979) developed an inventory model assuming the inflation. Furthermore Brahmbatt (1982), Hwong and Sohn (1983) proposed their models in which the time value of money and different inflation rates were measured.

On the other hand the effects of time, storage facilities, climate conditions etc. are those factors which affect the quality of the merchandise stored in the warehouse. Each product has its own characteristic. Several of the products are influenced by the environmental conditions such as humidity, moisture, temperature and some others are affected by the storage facilities or the time of storage. For example tires, mettles, radioactive and chemical substances are affected by temperature, moisture and humidity while food items, vegetables, milk, milk products are some items which are deteriorate with the passage of time and also affect by the method of storage. Iron and lead are decomposed due to moisture and air. The analysis of decaying inventory was initially instigated by the Ghare and Schrader (1963), who have presented a classical economic order quantity model with a constant rate of deterioration. The study has incorporated the inventory control problems related to decaying products without lifetime.

Covert and Philip (1973) extended Ghare and Schrader's (1963) deduce an EOQ model with a variable rate of deterioration by considering the two-parametric Weibull distribution. Order level inventory systems with ramp type demand rate for deteriorating items were discussed by Mandal and Pal (1998) and Panda *et al*. (2007). A note on the inventory models for deteriorating items with ramp type demand was developed by Deng *et al*. (2007). They have proposed an extended inventory model with ramp type demand rate and its optimal feasible solution.

Notation

A_D :Deteriorated amount

θ : Detoriatrion rate

I(t) : The inventory levelat time t

D(t): Demand rate

t_1 : Replenishment cycle time

L : Lead time

Pc :Purchase cost

Sc : Shortage cost

c : The unit cost per item

A : The ordering cost of inventory per order

Dc: Total deterioration cost per cycle

Q : Maximum inventory level

Hc : Holding cost per cycle

h :The inventory holding cost per unit item per unit time

I_0 : Ordering Quantity

Assumptions

1. The demand rate is stock - dependent and is of the form D(t) = a + bI(t), a > 0, b > 0
2. Shortages are allowed.
3. Replenishment rate is infinite
4. Lead time is constant
5. Holding cost is a function of time.
6. Deterioration rate is constant.
7. During lead time shortages are allowed.
8. There is no repair or replenishment of the deteriorated items during the inventory cycle

Mathematical Model

The behavior of inventory level depicted in Figure 1, in the starting period the system does not have any item to serve to the customer, because of the lead time and hence the level of inventory undergo the backlog due to the absence of the serviceable items. At time $t = L$ the distributer deliver the inventory at the supplier's place and it starts to decrease due to the demand and deterioration.

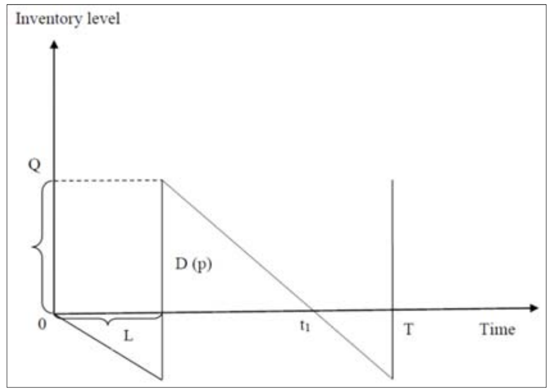

Fig 1: behavior of stock level in the production system

The governing differential equations of the system are given by:

$$\frac{dI(t)}{dt} = -\theta I(t) - [a + bI(t)] \qquad L \le t \le t_1$$

$$\frac{dI(t)}{dt} = -B[a + bI(t)] \qquad t_1 \le t \le T$$

with boundary conditions

$$I(t_1) = 0, \quad I(T) = -S$$

From equation (1) we get,

$$I(t) = \frac{a}{b+\theta}\left\lfloor e^{(b+\theta)(t_1-t)} - 1 \right\rfloor \qquad for\ L \le t \le t_1$$

$$\because I(L) = Q$$

$$Q = \frac{a}{b+\theta}\left\lfloor e^{(b+\theta)(t_1-L)} - 1 \right\rfloor$$

From equation (2) we have,

$$I(t) = \frac{a}{b}\left\lfloor e^{Bb(t_1-t)} - 1 \right\rfloor \qquad for\ t_1 \le t \le T$$

$$\because I(T) = -S$$

$$S = -\frac{a}{b}\left\lfloor e^{Bb(t_1-T)} - 1 \right\rfloor$$

The cost components are as follows

Holding Cost

$$H.C. = \int^{t_1} h.I(t).e^{-rt}\, dt$$

$$= \frac{ah}{r(b+\theta)(b+\theta+r)}\left[(b+\theta)e^{-rt_1} + \{re^{(b+\theta)(t_{1[}-L)} - (b+\theta+r)e^{-rL}\right]$$

Ordering quantity

$$I_0. = \int_L^{T_1} D(t)\, dt$$

$$= \int_L^{\substack{T \\ t_1}} D(t)\, dt + \int_{t_1}^{T} D(t)\, dt$$

$$= \frac{a\theta}{(b+\theta)}(t_1 - L) + \frac{ab}{(b+\theta)^2} e^{(b+\theta}\{e^{-(b+\theta)L} - e^{-(b+\theta)t_1}\} + \frac{a}{bB}\{1 - e^{bB(t_1-L)}\}$$

Purchasing cost

$$P.C = cI_0$$

$$= c\left[\frac{a\theta}{(b+\theta)}(t_1 - L) + \frac{ab}{(b+\theta)^2}e^{(b+\theta)}\{e^{-(b+\theta)L} - e^{-(b+\theta)t_1}\} + \frac{a}{bB}\{1 - e^{bB(t_1-L)}\}\right]$$

Shortage cost

$$S.C = -S\int_{t_1}^{T} I(t).e^{-rt}\,dt$$

$$= \frac{Sa}{b}\left[\frac{e^{bBt_1}}{bB+r}\left[e^{-(bB+r)T} - e^{-(bB+r)t_1}\right] + \frac{1}{r}\left[e^{-rt_1} - e^{-rT}\right]\right]$$

Deteriorated amount

$$A_D = Q - \int_{L}^{t_1} D(t)\,dt$$

$$= \frac{a}{b+\theta}\{e^{(b+\theta)(t_1-L)} - 1\} - \frac{a}{b+\theta}\{t_1 - L\} + \frac{a}{b+\theta}e^{(b+\theta)t_1}\{e^{-(b+\theta)} - e^{(b+\theta)L}\}$$

Deteriorated cost

$$D.C = c.A_D$$

$$= c\left[\frac{a}{b+\theta}\{e^{(b+\theta)(t_1-L)} - 1\} - \frac{a}{b+\theta}\{t_1 - L\} + \frac{a}{b+\theta}e^{(b+\theta)t_1}\{e^{-(b+\theta)} - e^{(b+\theta)L}\}\right]$$

Total cost variable function for one cycle is

$$T.C = O.C + P.C + H.C + D.C + S.C$$

$$= A + c\left[\frac{a\theta}{(b+\theta)}(t_1 - L) + \frac{ab}{(b+\theta)^2}e^{(b+\theta)}\{e^{-(b+\theta)L} - e^{-(b+\theta)t_1}\} + \frac{a}{bB}\{1 - e^{bB(t_1-L)}\}\right] +$$

$$+ \frac{ah}{r(b+\theta)(b+\theta+r)}\left[(b+\theta)e^{-rt_1} + \{re^{(b+\theta)(t_1-L)} - (b+\theta+r)e^{-rL}\right] +$$

$$+ c\left[\frac{a}{b+\theta}\{e^{(b+\theta)(t_1-L)} - 1\} - \frac{a}{b+\theta}\{t_1 - L\} + \frac{a}{b+\theta}e^{(b+\theta)t_1}\{e^{-(b+\theta)} - e^{(b+\theta)L}\}\right] +$$

$$+ \frac{Sa}{b}\left[\frac{e^{bBt_1}}{bB+r}\left[e^{-(bB+r)T} - e^{-(bB+r)t_1}\right] + \frac{1}{r}\left[e^{-rt_1} - e^{-rT}\right]\right]$$

Where TC is the function of $L,\ t_1$ and T only. Now our objective function is to minimize TC w. r. t. $L,\ t_1$ and T.

Numerical example

Example: The above theoretical results are illustrated through the numerical verification; we have considered the following input parameters in appropriate units.

Table 1: Parametric values of the input parameters in appropriate units.

A	a	b	c	h	S	θ	P	r
100	10	0.11	4	5	7	0.005	1.05	0.0006

Putting the values of given parameters and solve the problem, we get the optimal replenishment policy for the problem, provide in the following table

Table 2: Optimal results for the replenishment policy based the input parameters given in the table 1.

T.C	t_1	L	T
85.2399	8.35132	8	10

Sensitivity Analysis

The effect of parameters on the cost functions is shown in the following tables.

Table 3: Optimal results for the same set of values as in the given example for the parameter A.

Increase in A	New value of A	T.C	t_1
original	100	85.2399	8.35132
+50	150	90.2399	8.35132
+20	120	87.2399	8.35132
-20	80	83.2399	8.35132
-50	50	80.2399	8.35132

Table 4: Optimal results for the same set of values as in the given example for the parameter a

Increase in a	New value of a	T.C	t_1
original	10	85.2399	8.35132
+50	15	122.86	8.35132
+20	12	100.288	8.35132
-20	8	70.1919	8.35132
-50	5	47.62	8.35132

Table 5: Optimal results for the same set of values as in the given example for parameter b.

Increase in b	New value of b	T.C	t_1
original	0.11	85.2399	8.35132
+50	0.165	61.084	8.2441
+20	0.132	73.2859	8.3098
-20	0.088	102.874	8.39175
-50	0.055	154.309	8.45248

Table 6: Optimal results for the same set of values as in the given example for parameter c.

Increase in c	New value of c	T.C	t_1
original	4	85.2399	8.35132
+50	6	116.184	7.93901
+20	4.8	97.8359	8.18934
-20	3.2	72.3528	8.5098
-50	2	52.4963	8.74164

Table 7: Optimal results for the same set of values as in the given example for parameter L.

Increase in L	New value of L	T.C	t_1
Original	8	85.2399	8.35132
+50	12	83.5557	10.1264
+20	9.6	79.3299	9.11897
-20	6.4	98.1406	7.50392
-50	4	130.196	6.08003

Table 8: Optimal results for the same set of values as in the given example for parameter h.

Increase in h	New value of h	T.C	t_1
Original	5	85.2399	8.35132
+50	7.5	86.2969	8.27859
+20	6	85.6707	8.3186
-20	4	84.7948	8.39056
-50	2.5	84.0889	8.46666

Table 9: Optimal results for the same set of values as in the given example for parameter S.

Increase in S	New value of S	T.C	t_1
original	7	85.2399	8.35132
+50	10.5	88.864	8.72804
+20	8.4	86.9136	8.52618
-20	5.6	83.1305	8.12863
-50	3.5	78.6784	7.6494

Table 10: Optimal results for the same set of values as in the given example for deterioration rate.

Increase in θ	New value of θ	T.C	t_1
Original	0.005	85.2399	8.35132
+50	0.0075	83.6773	8.35259
+20	0.006	84.6066	8.35183
-20	0.004	85.8846	8.3508
-50	0.0025	86.8736	8.34999

Table 11: Optimal results for the same set of values as in the given example for parameter P.

Increase in P	New value of P	T.C	t_1
Original	1.05	85.2399	8.35132
+50	1.575	88.4309	8.66975
+20	1.26	86.696	8.49698
-20	0.84	83.4402	8.17009
-50	0.525	79.7397	7.79171

Table 12: Optimal results for the same set of values as in the given example for parameter r.

Increase in r	New value of r	T.C	t_1
Original	0.0006	85.2399	8.35132
+50	0.0009	86.139	8.33949
+20	0.00072	85.6006	8.34658
-20	0.00048	84.8779	8.35606
-50	0.0003	84.3323	8.36318

Conclusion

Here in this article our EPQ inventory model deals with the realistic assumptions such as perishable items, stock out situation, lead time consideration. Lead time is variable and the deteriorating rate is assumed to be constant while the holding cost is taken as time dependent which is much better than the constant rate. Demand rate is assumed as stock dependent and the shortages are allowed which is partially backlogged. Backlogging rate is considered to be constant. Numerical example has been given to illustrate the theoretical results and the effect of various parameters has been analyzed in the sensitivity analysis. Our model can be so much beneficial for the practitioners because of its flexibility. This model helps the policy makers to control the inventory.

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