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Some common fixed point results in complete cone metric space with W- distance

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Abstract
It appears from the survey of literatures carried out previously, that there is much scope of carrying out further study on fixed point theory. In the light of this, the present research work proposes to examine the existence, uniqueness and validity of the different properties of fixed point of two self mappings under certain contractive conditions in cone metric space with w-distance. Further, an effort has been made to investigate the existence, uniqueness and application of random fixed point theorems which are stochastic generalizations of classical fixed point theorems, which in other way has also been named as deterministic fixed point theorems.

Keywords: Fixed point, cone, Cone metric space, w-distance, self mappings, contractive condition.

1. Introduction

Fixed point theory is a basic tool in application of various branches of mathematics from linear algebra, elementary calculus to topology and analysis. This theory is very closed to other disciplines like Game theory, Optimization theory, Physics, Economics etc. Fixed point theory has been developed in various spaces in last century. In modern day this theory is very favourite for researchers. The concept of fixed point theory was firstly propounded by Poincare [9] in 1886. After that in 1912, Brower [2] proved fixed point theorem for solution of equation $f(x) = x$. In 1922, Polish mathematician Stefan Banach [1] who was one of the founders of functional analysis gave a principle which was one of the fundamental principles in the field of functional analysis. In that principle S. Banach stated as follow-*"Every contraction mapping of a complete metric space X into itself has a unique fixed point."* (Bonsall 1962).

In 2007, concept of a metric space was generalized by Huang and Zhang [5] by replacing the set of real numbers by an ordered Banach space and obtained some fixed point theorems for mappings satisfying different contractive conditions. On the other hand during last decade of 20th century Osama kada [6] and Naoki Shioji [11] introduced the concept of metric space with w- distance, gave some examples, properties of w- distance and improved some known results also. By using this concept proved a complete metric space which was generalized the fixed point theorems of Subrahmanyan [12], Kannan [7] and Ciric [3]. In consecutive years various mathematician established some fixed point theorems.

In this sequel in 2011, Dhanorkar and Salunke [4] and in 2012 Manish Sharma and Rajesh Shrivastva [10] obtained some fixed point results for cone metric space with w-distance.

In 2013, S. k. Tiwari, R. P. Dubey [15] proved some fixed point theorems for generalized contractive mapping in cone metric space and with A. K. Dubey [16] they gave common fixed point results in cone metric spaces. Recently in (July 2017), S. K. Tiwari, and Kaushik Das [14] extended some common fixed point results for contractive mappings in cone metric spaces.

In this research paper we will define w-distance, cone metric space, normality of cone and proof some new common fixed point theorems for contractive mapping on cone metric space with w-distance. This result improved and modified some well known results in literatures also.

1.2. Preliminaries

We recall some standard notations and definitions

Definition 1.2.1: Let E be a real Banach space and P be a subset of E. Then P is called **cone** iff,

a) P is closed and nonempty subset of E and P≠{0};

b) $a,b \in R^{+}, x, y \in P \Rightarrow ax+by \in P$;

c) $x \in P$ and $-x \in P \Rightarrow x=0$.

Given a cone $P \subseteq E.$ We define a partial ordering $\leq$ on E with respect to P by $x \leq y \Leftrightarrow y-x \in P.$ We shall write $x << y$ if $y-x \in \operatorname{int} P.$ If $\operatorname{int} P \neq \phi$, then cone P will be solid.

Definition 1.2.2: The cone P is said to be **normal** if there is a number K>0 such that for all $x, y \in E, 0 \leq x \leq y$ implies $\|x\| \leq k\|y\|.$

Definition 1.2.3: The cone P is said to be **regular** if every upper bounded increasing sequence is convergent i.e. if $\{x_n\}_{n\geq 1}$ is a sequence such that $x_1 \leq x_2 \leq x_3 \leq \ldots \leq y$ for some $y \in E$, then $\exists x \in E$ such that $\lim_{n\to\infty}\|x_n - x\| = 0$.
In similar manner the cone P is regular if and only if every sequence which is decreasing and bounded below is convergent.

Definition 1.2.4: Let X is a non empty set. Let $d : X \times X \to E$ be a mapping satisfies

d_1: $0 < d(x,y)$ And $d(x,y)=0 \Leftrightarrow x=y$ for all $x, y \in X$

d_2: $d(x,y)=d(y,x)$ For all $x, y \in X$

d_3: $d(x,y) \leq d(x,z)+d(z,x)$ For all $x, y, z \in X$

Then d is called cone metric on X and (X, d) is called **cone metric space.**

Example: Let E=R^2, $P=\{(x,y)\in E : x, y \geq 0\} \subset R^2$ X=R and $d : X \times X \to R$ such that $d(x,y)=(|x-y|, \alpha|x-y|)$, where α≥0 is a constant. Then (X, d) is cone metric space.

Definition 1.2.5. Let *X* be cone metric space with *d*. Suppose the mapping $p: X \times X \to E$ is called w-distance on *X* if

(w_1) $0 \leq p(x,y)$ for all $x, y \in X$,

(w_2) $p(x,z) \leq p(x,y)+p(y,z)$ for all $x, y, z \in X$,

(w_3) $p(x,z) \to E$ is a lower semi- continuous for all $x, z \in X$

(w_4) for any $\alpha > 0$ there exists $\beta > 0$ such that $p(z,x) < \beta$ and $p(z,y) < \beta$ imply $d(x,y) < \alpha$, where $\alpha, \beta \in E.$

Example: Let *X* be norm linear space with Euclidian norm. Then a mapping $p: X \times X \to [0,\infty)$ defined by $p(x,y)=\|x\|+\|y\|$, for all $x, y \in X$ is a *w*-distance on *X*.

Definition 1.2.6: Let (X, d) be a cone metric space with *w*-distance *P*, $x \in X$ and $\{x_n\}_{n\geq 1}$ be a sequence in *X*. Then,

(1) $\{x_n\}$ is called a *p*-Cauchy sequence whenever for every $\{x_n\}$ there exists a positive integer *N* such that, for all $m, n \geq N, p(x_n, x) < \alpha.$

(2) $\{x_n\}$ sequence is called a *p*-convergent to a point $x \in X$ whenever for every $\alpha \in E, 0 < \alpha,$ there is a positive integer *N* such that, for all $n \geq N, p(x, x_n) < \alpha.$ We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x.$

(3) *(X, d)* is a complete cone metric space with *w*-distance if every Cauchy sequence is p-convergent.

Lemma 1.2.7: There is not normal cone with normal constant *M*<1.

1.3. Main Result

In this section we will establish following three theorems which improved various existed fixed point results.

Theorem 1.3.1. Let (X, d) be a complete cone metric space with w-distance p and let *P* be a normal cone with normal constant *K*. Suppose $T_1, T_2 : X \to X$ be any two continuous and surjective mappings satisfying the contractive condition.

$p(T_1x, T_2y) \leq \alpha p(x,y) + \beta[p(x,T_1x) + p(y,T_2y)]$…………1.3.1.1

for all $x, y \in X$, where $\alpha > 0, \frac{1}{2} \leq \beta \leq 1$ are constants, with $\alpha + 2\beta < 1$. Then T_1 and T_2 have a common fixed point in X.

Proof: Let x_o be an arbitrary point in X. Since T_1 and T_2 be onto (surjective), there exist $x_o \in X$ and $x_1 \in X$ such that $T_1(x_1) = x_0, T_2(x_2) = x_1$.

In this way, we define the sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ by
$x_{2k} = T_1 x_{2k+1}$ for $k = 0,1,2,3$ …….
And $x_{2k+1} = T_2 x_{2k+2}$ for $k = 0,1,2,3$ … ….

Note that, If $x_{2k} = x_{2k+1}$ some $k \geq 1$, then it is fixed point of T_1 and T_2.
Now putting $x = x_{2k+1}$ and $y = x_{2k+2}$, we have

$$p(x_{2k}, x_{2k+1}) = p(T_1 x_{2k+1}, T_2 x_{2k+2})$$
$$p\, x_{2k}, x_{2k+1} \leq \alpha p(x_{2k+1}, x_{2k+2}) + \beta[p(x_{2k+1}, T_1 x_{2k+1}) + p(x_{2k+2}, T_2 x_{2k+2})]$$
$$\leq p(x_{2k+1}, x_{2k+2}) + \beta[p(x_{2k+1}, x_{2k}) + p(x_{2k+2}, x_{2k+1})]$$
$$= (\alpha + \beta)\, p(x_{2k+1}, x_{2k+2}) + \beta p(x_{2k+1}, x_{2k})$$
$$\Rightarrow (1 - \beta)\, p(x_{2k}, x_{2k+1}) \leq (\alpha + \beta)\, p(x_{2k+1}, x_{2k+2})$$
$$\Rightarrow p(x_{2k+1}, x_{2k+2}) \leq \frac{\alpha+\beta}{1-\beta}\, p(x_{2k}, x_{2k+1})$$
$$\Rightarrow \quad p(x_{2k+1}, x_{2k+2}) \leq h\, p(x_{2k}, x_{2k+1}), \text{ where } h = \frac{\alpha+\beta}{1-\beta}, 0 \leq h \leq 1.$$

In general,
$$p(x_{2k}, x_{2k+1}) \leq h\, p(x_{2k-1}, x_{2k}) \leq \dots \dots \dots \leq h^{2k} p(x_0, x_1)$$
So, for $k < m$, we have
$$p(x_{2k}, x_{2m}) \leq p(x_{2k}, x_{2k+2}) + \dots \dots \dots + p(x_{2m-1}, x_{2m})$$
$$\leq (h^{2k} + h^{2k+1} + \cdots \cdots \cdots + h^{2m-1}) p(x_0, x_1)$$
$$\leq \frac{h^{2k}}{1-h}\, p(x_0, x_1)$$
Implies that $p(x_{2k}, x_{2m}) \leq \frac{h^{2k}}{1-h} K p(x_0, x_1)$ (since P is normal).

Thus, $\lim_{k,m \to \infty} p(x_{2k}, x_{2m}) = 0$. $\{\because 0 \leq k < 1, k \to \infty, \frac{h^{2k}}{1-h} \to 0\}$
Therefore $\{x_{2k}\}$ is a Cauchy sequence in X.
Since (X, d) is a complete cone metric space. Therefore, there exist $x^* \in X$ such that $x_{2k} \to x^*$ as $k \to \infty$.
Now,
$$p(T_1 x^*, x^*) \leq p(T_1 x_{2k+1}, T_1 x^*) + p((T_1 x_{2k+1}, x^*) \to 0 \text{ as } k \to \infty.$$
Since $x_{2k} \to x^*$ *and* $T_1 x_{2k+1} \to T_1 x^*$ *as* $k \to \infty$. Therefore $p(T_1 x^*, x^*) = 0$. This implies that, $T_1 x^* = x^*$. Hence x^* is a fixed point of T_1.
Similarly, it can be established that $T_2 x^* = x^*$.
Therefore, $T_1 x^* = x^* =, T_1 x^*$.
Thus x^* is the common fixed point of pair of maps T_1 and T_2.This completes the proof of this given theorem.

Theorem 1.3.2: Let (X, d) be a complete cone metric space with w-distance p and let P be a normal cone with normal constant K. Suppose $T_1, T_2 : X \to X$ be any two continuous and surjective mappings satisfying the contractive condition

$$p(T_1 x, T_2 y) \leq \alpha[p(x, y) + p(x, T_1 x) + p(y, T_2 y)] + \beta[p(x, T_2 y) + p(y, T_1 x)] \quad \dots 1.3.2.1$$

for all $x, y \in X$, where $\alpha \geq 0, \beta < 1$ are constant, with $3\alpha + 2\beta < 1$. Then T_1 and T_2 have a common fixed point in X.

Proof: Let x_o be an arbitrary point in X. Since T_1 and T_2 be onto mappings. Then, there exist $x_o \in X$ and $x_1 \in X$ such that,

$$T_1(x_1) = x_0, T_2(x_2) = x_1$$

In this way, we define the sequences $\{x_{2k}\}$ and $\{x_{2k+1}\}$ by
$x_{2k} = T_1 x_{2k+1}$ for $k = 0,1,2,3$ …….
and $x_{2k+1} = T_2 x_{2k+2}$ for $k = 0,1,2,3$ … ….
Note that, If $x_{2k} = x_{2k+1}$ some $k \geq 1$, then it is fixed point of T_1 and T_2.
Now putting $x = x_{2k+1}$ and $y = x_{2k+2}$, we have

$$p(x_{2k}, x_{2k+1}) = p(T_1 x_{2k+1}, T_2 x_{2k+2})$$
$$\leq \alpha[p(x_{2k+1}, x_{2k+2}) + p(x_{2k+1}, T_1 x_{2k+1}) + p(x_{2k+2}, T_2 x_{2k+2})]$$
$$+ \beta[p(x_{2k+1}, T_2 x_{2k+2}) + p(x_{2k+2}, T_1 x_{2k+1})]$$
$$\leq \alpha[p(x_{2k+1}, x_{2k+2}) + p(x_{2k+1}, x_{2k}) + p(x_{2k+2}, x_{2k+1})]$$
$$+ \beta[p(x_{2k+1}, x_{2k+1}) + p(x_{2k+2}, x_{2k})]$$
$$\leq \alpha\, [2p(x_{2k+1}, x_{2k+2}) + p(x_{2k+1}, x_{2k})]$$

$+\beta[\, p(x_{2k+1}, x_{2k+2}) + \, p(x_{2k+2}, x_{2k+1}) + \, p(x_{2k+1}, x_{2k})]$
$= \, (2\alpha + 2\beta)p(x_{2k+1}, x_{2k+2}) + (\alpha + \beta)p(x_{2k}, x_{2k+1})$
$\Rightarrow [1 - \, (\alpha + \beta)] \, p(\, x_{2k}, x_{2k+1}) \leq (2\alpha + 2\beta) \, p(x_{2k+1}, x_{2k+2})$
$\Rightarrow \quad p(\, x_{2k}, x_{2k+1}) \, \leq \, \frac{(2\alpha+ \, 2\beta)}{1-(\alpha+\beta)} p(x_{2k+1}, x_{2k+2}$
Therefore, $p(x_{2k+1}, x_{2k+2} \,) \leq hp(\, x_{2k}, x_{2k+1}), \, where \; h \, = \, \frac{(2\alpha+ \, 2\beta)}{1-(\alpha+\beta)}, 0 \, \leq \, h \, \leq \, 1.$

In general
$p(\, x_{2k}, x_{2k+1}) \leq h \, p(x_{2-1}, x_{2k} \,) \leq \dots \, \dots\dots \leq \, h^{2k} p(x_{0,} x_1)$
So for $k \, < \, m$, we have
$p(x_{2k}, x_{2m}) \, \leq \, p(x_{2k}, x_{2k+2}) + \dots \, \dots \, \dots + d(x_{2m-1}, x_{2m})$
$\leq \, (h^{2k} \, + h^{2k+1} + \cdots \cdots \cdots + h^{2m-1}) p(x_0, x_1)$
$\leq \frac{h^{2k}}{1-h} \, p(x_0, x_1)$

implies $\lim_{k,m\to\infty} p(x_{2k}, x_{2m}) \leq \dfrac{h^{2k}}{1-h} p(x_0, x_1)$ (Since P is normal)

Thus for $\lim_{k,m\to\infty} p(x_{2k}, x_{2m}) = 0$, $k < \, m.$

Therefore $\{x_{2k}\}$ is a Cauchy sequence in X.
Since (X, d) is a complete cone metric space.
Therefore, there exist $x^* \in X$ such that $x_{2k} \to \, x^*$ as $k \, \to \, \infty$.

Now,
$p(T_1 \, x^*, x^*) \, \leq \, p(T_1 x_{2k+1}, T_1 \, x^*) \, + \, p((T_1 x_{2k+1}, x^*) \to 0$ as $k \, \to \, \infty$.

Since $x_{2k} \to x^*$ *and* $T_1 x_{2k+1} \to \, T_1 \, x^*$ *as* $k \, \to \, \infty$. Therefore $p(T_1 \, x^*, x^*) = 0$.
This implies that, $T_1 \, x^* = x^*$. Hence x^* is a fixed point of T_1.
Similarly, it can be establish that $T_2 \, x^* = \, x^*$.
Therefore, $T_1 \, x^* = \, x^* =, T_1 \, x^*$.
Thus x^* is the common fixed point of pair of maps T_1 and T_2 . This completes the proof.

Theorem 1.3.3: Let (X, d) be a complete cone metric space with w-distance p and consider P be a normal cone with normal constant K. Suppose $T_1, T_2 : X \to X$ be any two continuous and onto mappings satisfying the contractive condition.

$p(T_1 x, T_2 y) \leq \alpha p(x, y) + \beta p(x, T_1 x) + \gamma p(y, T_2 y) + \eta[\, p(x, T_1 x) + \, p(y, T_2 y)]$………1.3.3.1

for all x, $y \in$ X, where $\alpha \geq 0, \beta > 0, \gamma \, \leq \frac{1}{2}$ *and* $\frac{1}{2} < \, \eta \, \leq \, 1$ are constants with $\alpha \, + \, \beta + \gamma \, + \, 2\eta < 1$. Then T_1 and T_2 have a common fixed point in X.

Proof: Let x_o be an arbitrary point in X. Since T_1 and T_2 be onto mappings, then there exist $x_o \in X$ and $x_1 \in X$ such that

$$T_1(x_1) = x_0, \; T_2(x_2) = x_1$$

In this way, we define the sequences $\{x_{2k}\}$ and $\{x_{2k+1}\}$ by
$x_{2k} \, = \, T_1 x_{2k+1}$ for $k \, = \, 0,1,2,3$ ….…and
$x_{2k+1} \, = T_2 x_{2k+2}$ for $k \, = \, 0,1,2,3$ … ….
Note that, If $x_{2k} \, = \, x_{2k+1}$ some $k \geq 1$, then it is fixed point of T_1 and T_2.
Now putting $x \, = x_{2k+1}$ and $y \, = x_{2k+2}$, Then we have
$p(\, x_{2k}, x_{2k+1}) = \, p(T_1 x_{2k+1}, T_2 x_{2k+2})$
$p(\, x_{2k}, x_{2k+1}) \leq \, \alpha p(x_{2k+1}, x_{2k+2}) + \, \beta p(x_{2k+1}, T_1 x_{2k+1}) + \, \gamma p(x_{2k+2}, T_2 x_{2k+2}) + \, \eta[p(x_{2k+1}, T_1 x_{2k+1}) + p(x_{2k+2}, T_2 x_{2k+2})$
$\leq \, \alpha p(x_{2k+1}, x_{2k+2}) + \, \beta p(x_{2k+1}, x_{2k}) + \, \gamma \, p(x_{2k+2}, x_{2k+1})$
$+\eta[p(x_{2k+1}, x_{2k}) + p(x_{2k+2}, x_{2k+1}) \,]$
$= (\alpha + \gamma + \eta)p(x_{2k+1}, x_{2k+2}) + (\beta + \eta) \, p(x_{2k+1}, x_{2k})$
$\Rightarrow [1 - \, (\beta \, + \, \eta)] \, p(\, x_{2k}, x_{2k+1}) \leq (\alpha + \gamma + \eta) \, p(x_{2k+1}, x_{2k+2})$
$\Rightarrow \quad p(\, x_{2k}, x_{2k+1}) \, \leq \, \frac{\alpha+\gamma+\eta}{1-(\beta+\eta)} p(x_{2k+1}, x_{2k+2})$
$\Rightarrow \quad p(x_{2k+1}, x_{2k+2} \,) \leq hp(\, x_{2k}, x_{2k+1}), \, where \; h \, = \, \frac{\alpha+\gamma+\eta}{1-(\beta+\eta)}, 0 \, \leq \, h \, \leq \, 1$
In general
$p(\, x_{2k}, x_{2k+1}) \leq h \, p(x_{2k-1}, x_{2k} \,) \leq \dots \, \dots\dots \leq h^{2k} p(x_{0,} x_1)$
So, for $k \, < \, m$, we have
$p(x_{2k}, x_{2m}) \, \leq \, p(x_{2k}, x_{2k+2}) + \dots\dots\dots + p(x_{2m-1}, x_{2m})$
$\leq \, (h^{2k} \, + h^{2k+1} + \cdots \cdots \cdots + h^{2m-1}) p(x_0, x_1)$
$\leq \frac{h^{2k}}{1-h} \, p(x_0, x_1)$

implies $\lim_{k,m\to\infty} p(x_{2k}, x_{2m}) \le \frac{h^{2k}}{1-h} Kp(x_0, x_1)$ (Since P is normal)

Thus for $\lim_{k,m\to\infty} p(x_{2k}, x_{2m}) = 0$, $k < m$.

Therefore $\{x_{2k}\}$ is a Cauchy sequence in X.
Since (X, d) is a complete cone metric space.
Therefore, there exist $x^* \in X$ such that $x_{2k} \to x^*$ as $k \to \infty$.
Now,

$p(T_1 x^*, x^*) \le p(T_1 x_{2k+1}, T_1 x^*) + p((T_1 x_{2k+1}, x^*) \to 0$ $as\ k \to \infty$.

$Since\ x_{2k} \to x^*\ and\ T_1 x_{2k+1} \to T_1 x^*\ as\ n \to \infty$. Therefore $p(T_1 x^*, x^*) = 0$.
This implies that, $T_1 x^* = x^*$. Hence x^* is a fixed point of T_1
Similarly, it can be establish that $T_2 x^* = x^*$.
Therefore, $T_1 x^* = x^* =, T_1 x^*$. Thus x^* is the common fixed point of pair of maps T_1 and T_2.
This completes the proof.

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