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Optimal identification of ARIMA model for Predicting CPI in Nigeria using output based criterion

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Abstract
The sum of squares deviation forecast criterion (SSDFC) is as output based model selection criterion employed to identify the best fitted ARIMA (p, d, q) models for predicting consumer price index (CPI) in Nigeria. The data set covering the period of 1995M1 to 2018M12 was used in the analysis. Unit root test used indicates that CPI is integrated order zero (I (0)) after first difference. The correlogram plots indicated that CPI follows an ARIMA (p, 1, 0) process. And out of the five subclasses of ARIMA (p, 1, 0) models compared, SSDFC chose ARIMA(5,1,0) as the best fitted model. The diagnostic test on the model residuals showed that ARIMA (5, 1, 0) is adequate. Hence, the model can be recommended for predicting the dynamics of CPI in Nigeria.

Keywords: ARIMA, SSDFC, CPI, Unit root test

1. Introduction

Consumer price index is a good and conventional measure of inflation and one of the key indicators of economic performance of every monetary policy of any country. The Consumer Price Index (CPI) measures the changes over time in the price level of consumer goods and services generally purchased by households. The year-on-year rate of change in the CPI is widely used as an indicator of the inflation affecting consumers. So any model that patterns the behaviour of CPI to its closest precision is very important to economic policy makers in forecasting future CPI.

Habimana *et al.* (2016) [6] in their study employed Box and Jenkins methodology to model the dynamic of CPI and to forecast its future values in the short term in Rwanda. Applying main steps; model identification, parameter estimation and diagnostic checking, ARIMA (4,1,6) was selected as a potential model which can fits well data, as well as to make also accurate forecast. Hence, the forecast was made for 12 months ahead of the year 2016, and the findings have shown that the CPI was likely to continue rising up with time.

Jere *et al.* (2016) [7] used monthly CPI data which were collected from January 2003 to December 2017 to model CPI in Zambia. The models that were compared are the Autoregressive Integrated Moving average (ARIMA) model and Multi-cointegration (ECM) model. Results showed that the ECM was the best fit model of CPI in Zambia since it showed smallest errors measures. Lastly, a forecast was done using the ECM and results show an average growth rate for food CPI at 6.63% and an average growth rate for non-food CPI at 7.41%. Nyoni (2019) [10] adopted Box and Jenkins approach in forecasting Australian CPI using ARIMA models. The result presented ARIMA (1, 1, 0) for forecasting CPI in Australia. The result of the study showed that CPI in Australia is likely to continue in an upward trend in the next decade.

Kharimah et al (2015) [8] analyzed the CPI in Malaysia using ARIMA models with a data set ranging over the period January 2009 to December 2013 and revealed that the ARIMA(1, 1, 0) was the best model to forecast CPI in Malaysia. Sarangi et al (2018) [12] analyzed the consumer price index using Neural Network models with 159 data points and revealed that ANNs are better methods of forecasting CPI in India. Nyoni and Nathaniel (2019) [11], based on ARMA, ARIMA and GARCH models; studied inflation in Nigeria using time series data on inflation

rates from 1960 to 2016 and found out that the ARMA (1, 0, 2) model is the best model for forecasting inflation rates in Nigeria.
Not understanding averagely the variability pattern of inflation in any economy can lead to miss targeted monetary policy. The effect of price instability due to inflationary pressure is inimical to the purchasing power of the national currency which culminates to high standards of living. Inflation reduces investment volume whether domestic and foreign investment in an economy. Forecasting CPI is an important factor for any economy because it is essential in economic planning for the future. Therefore, identifying a more accurate forecasting model is a major contribution to the economic growth of Nigeria.

2. Materials and Methods

This section presents methods and sources of data collection, variable measurement, and method of unit root test, ARIMA model specification, and model identification, method of data analysis, model comparison techniques and diagnostic checks.

2.1 Source of Data and variable measurement

The monthly consumer price index (CPI) data was obtained from published central bank of Nigeria (CBN, 2018) statistical bulletin. The univariate time series data collected covered the period of 1995M1-2018M12 (288 observations of monthly CPI data). CPI is usually a conventional measured inflation.

2.2. Moving Average (MA) Process

Moving average model is a sub-class of ARIMA models that uses lagged error term as predictor variable. Let $u_t (t = 1, 2, 3, ...)$ be a white noise process, a sequence of independently and identically distributed (iid) random variables with E$(u_t) = 0$ and $Var(u_t) = \sigma^2$. Then the q^{th} order MA model is given as:

$$x_t = u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q} \quad (1)$$

4This model is expressed in terms of past errors and estimated coefficients $\theta_k (k = 1,2,3,\cdots,q)$ of the model are used for forecasting. Therefore only q errors predictors will affect the current level X_t but higher order errors predictors do not affect x_t. Hence it is a short memory model.

2.3. Auto-Regression (AR) Process

An autoregressive model of order *p*, That is, AR (*p*) can be expressed as;

$$x_t = \delta + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \alpha_3 x_{t-3} + \cdots + \alpha_p x_{t-p} + u_t \quad (2)$$

In this case, all previous values will have cumulative effects on the current level x_t and thus, it is a long-run memory model. $u_t \sim N(0, \sigma^2)$

2.4. ARMA (p, q) Model Specification

An integrated order one time series $\{X_t\}$ is said to follow an autoregressive moving average model of orders p and q denoted by ARMA(p, q) if it satisfies the difference equation

$$x_t = \delta + \alpha_1 x_{t-1} + \cdots + \alpha_p x_{t-p} + u_t - \beta_1 u_{t-1} - \cdots - \beta_q u_{t-q} \quad (3)$$

where x_t is the original series, δ is the mean of the series, u_t is a sequence of random variables with zero mean and constant variance, called *a white noise process*, and the β_j's and β_j's are constants. Equation (3) can be summarized as follows;

$$\left(1 - \alpha_1 L - \cdots - \alpha_p L^p\right) x_t = \delta + \left(1 - \beta_1 L - \cdots - \beta_q L^q\right) u_t \quad (4)$$

$$A(L) x_t = \delta + B(L) u_t \quad (5)$$

Where $A(L)$ is the autoregressive (AR) operator, given by $A(L) = 1 - \alpha_1 L - \cdots - \alpha_p L^p$ and $B(L)$ is the moving average (MA) operator, given by $B(L) = 1 - \beta_1 L - \cdots - \beta_q L^q$. For L denotes the backshift operator defined by $L^k x_t = x_{t-k}$. If $X_t = x_t - \delta$ and u_t is the shock at time t, then Equation (4) can be rewritten as presented in Equation (4) below;

$$A(L) X_t = B(L) u_t \quad (6)$$

The process is stationary if, with stochastic initial conditions – the stability conditions of the AR term are fulfilled, i.e. if $A(L)$ only has roots that are larger than 1 in absolute value. If, If the time series $\{X_t\}$ is nonstationarity due to the presence of one or several of five conditions: outliers, random walk, drift, trend, or changing variance, it is conventional that first or second differencing (d) is necessary to achieve stationarity. Hence, the original series is said to follow an autoregressive integrated moving average model or orders p, d and q denoted by ARIMA (p, d, q). For nonstattionary series, Equation (6) can be of the form

$$A(L) \nabla^d X_t = B(L) u_t \quad (7)$$

2.5 Estimation of ARIMA (P, D, Q)

The ARIMA model estimation can be performed using either the method of maximum likelihood estimator or conditional least squares.

2.5.1 Method of maximum likelihood

Estimation of ARIMA model is often performed by exact maximize likelihood assuming Gaussian innovations. The exact Gaussian likelihood function for an ARIMA model is given by

$$\begin{aligned} \log L\left(\mathrm{A}, \mathrm{B}, \sigma^2, d\right) &= -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log|\Omega| - \frac{1}{2} u' \Omega^{-1} u \\ &= -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log|\Omega| - S(\alpha, \beta, d) \end{aligned} \quad (8)$$

Where $X = (X_1, X_2, \cdots, X_T)'$ and $u = (u_1, u_2, , \cdots u_T)'$ where Ω is the symmetric Toeplitz covariance matrix for the T draws from the ARMA process for the unconditional residuals (Doornik and Ooms 2003) [4]. The Kalman filter can be used to efficiently evaluate the likelihood. The Kalman filter works with the state space prediction error decomposition form of the likelihood, which eliminates the need to invert the large matrix Ω. See Box, Jenkins, and Reinsel (2008, 7.4, p. 275) [3] for detail discussion.

2.5.2 Conditional least squares (CLS)

Box, Jenkins, and Reinsel (2008, Section 7.1.2 p 232.) point out that conditional on pre-sample values for the AR and MA errors, the normal conditional likelihood function may be maximized by minimizing the sum of squares of the innovations.

The methods for starting up the recursion by specifying pre-sample values of δ and u_t. Given these pre-sample values, the conditional likelihood function for normally distributed innovations is given by

$$\log L(\mathrm{A},\mathrm{B},\sigma^2,d) = -\frac{T}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T}u_t^2$$
$$= -\frac{T}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}S(\alpha,\beta,d) \qquad (9)$$

Notice that the conditional likelihood function depends on the data and the mean and ARIMA parameters only through the conditional least squares function $S(\alpha,\beta,d)$, so that the conditional likelihood may be maximized by minimizing $S(\alpha,\beta,d)$. The coefficients are estimated using an iterative algorithm that calculates least squares estimates. At each iteration, the back forecasts are computed and SSE is calculated. For more details

2.6 Model identification

The ACF of an MA (q) model cuts off after lag q whereas that of an AR (p) model is a combination of sinusoidals dying off slowly. On the other hand, the PACF of an MA (q) model dies off slowly whereas that of an AR (p) model cuts off after lag p. The AR and MA models are known to exhibit some duality relationships. Parametric parsimony consideration in model building entails the use of the mixed ARMA fit in preference to either the pure AR or the pure MA fit.

2.7 Model Comparison

There are several model selection criteria in literature such as; Schwarz Bayesian information criterion (SBIC), Aikaike information criterion (AIC), residual sum of squares and so on. If n is the sample size and RSS is the residual sum of squares, then, SBIC and AIC are given as follows;

$$SBIC = n\ln(RSS/(n-k)) + k\ln(n) \qquad (10)$$

$$AIC = 2k + n\ln(RSS/(n-k)) \qquad (11)$$

Where, n is the sample size, k is the number of estimated parameters (for the case of regression, k is the number of regressors) and RSS is the residual sum of squares based on the estimated model.

However, it is good to note that both Schwarz Bayesian information criterion (SBIC) and AIC are affected by the number of parameters included to be estimated in a model. For the case of BIC, it penalizes free parameters while AIC becomes smaller as the number of free parameters to be estimated increases. But for this study, sum of squares deviation forecast criterion introduced by Amaefula (2011) [1] will be used for model selection. And it is of the form;

$$SSDFC = \frac{1}{m}\sum_{i=1}^{m}(y_{t(l,i)} - \hat{y}_{t(l,i)})^2 \qquad (12)$$

Where l is the lead time, m is the number of forecast values to be deviated from the actual values (m should be reasonably large), $y_{t(l,i)}$ is the actual values of the time series corresponding to the i^{th} position of the forecast values and $y_{t(l,i)}$ is the forecast values corresponding to the i^{th} position of the actual values. In comparison, the model with the smallest value of SSDFC is the best fitted model. It is good to note that SSDFC is output based, it describes how the fitted model can produce forecast values to the closest precision.

2.8 Ng and perron (NP) Test

Ng and Perron (2001) [9] constructed four statistics that are based upon the GLS detrended data y_t^d. These test statistics are modified forms of Philips and Perron z_α and z_t statistics, the Bhargava (1986) [2] R_1 statistics, and the Elliot, Rothenberg, and Stock Point Optimal (ERS) (1996) [5] statistics. First, given the term;

$$K = \sum_{t=2}^{T}(y_{t-1}^d)^2/T^2 \qquad (13)$$

The modified statistics may then be written as

$$\left.\begin{aligned} MZ_\alpha^d &= (T^{-1}(y_T^d)^2 - f_0)/(2k) \\ MZ_t^d &= MZ_\alpha \times MSB \\ MSB^d &= (k/f_0)^{1/2} \end{aligned}\right\} \qquad (14)$$

$$MP_T^d = \begin{cases} (\bar{c}^2k - \bar{c}T(y_T^d)^2)/f_0, & if\ x_t = \{1\} \\ (\bar{c}^2k + (1-\bar{c})T^{-1}(y_T^d)^2)/f_0, & if\ x_t = \{1,t\} \end{cases} \qquad (15)$$

Where $y_T^d = y_t - x_t'\delta$, δ is the estimate from the auxiliary regression of equation (38) and

$$\bar{c} = \begin{cases} -7, if & x_t = \{1\} \\ -13.5, if & x_t = \{1,t\} \end{cases} \qquad (16)$$

Note that NP test requires a specification for x_t and a choice of method of estimating f_0.

3. Data Analysis and Results

This section presents the time series plot of monthly CPI data, results of NP unit root test, plots of ACF and PACF, model selection result, estimates of $ARIMA(p,d,q)$ model and diagnostic test for model adequacy.

The plot of the first natural log difference in Figure2 above shows changes in CPI. It is observable that there are more variations in the first half of the series than in the second half of the series. And 2012 to 2018 the observable changes are centred around zero

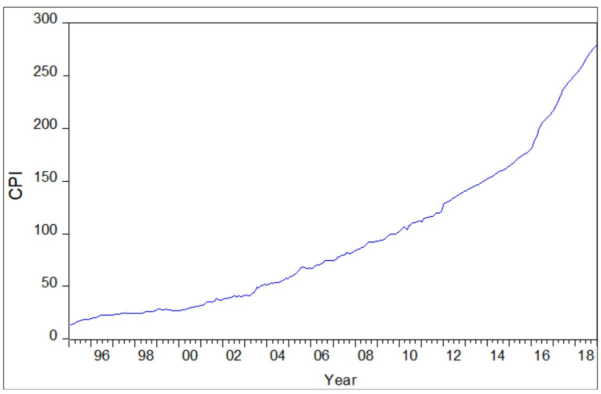

Fig 1: Time series plot of CPI in Nigeria (1995M1 – 2018M12)

The time series plot of CPI in Figure1 above exhibits a consistent upward trend for the period under study

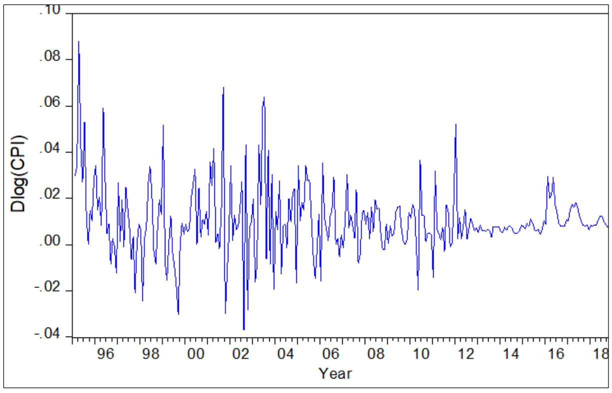

Fig 2: Time series plot of Dlog (CPI) in Nigeria (1995M1 – 2018M12)

The plot of the first natural log difference in Figure2 above shows changes in CPI. It is observable that there are more variations in the first half of the series than in the second half of the series. And 2012 to 2018 the observable changes are centred around zero

3.1 NP Unit Root Test

In order to check the order of integration of the variables under study, NP unit root tests was carried out and the result are presented in Table1and 2 below;

Table 1: NP unit root test analysis of CPI

Null Hypothesis: CPI has a unit root					
Exogenous: Constant, Linear Trend					
Lag length: 2 (Spectral GLS-detrended AR based on SIC, maxlag=15)					
Sample: 1995M01 2018M12					
Included observations: 288					
		MZa	MZt	MSB	MPT
Ng-Perron test statistics		1.75141	1.06763	0.60959	100.981
Asymptotic critical values*:	1%	-23.8000	-3.42000	0.14300	4.03000
	5%	-17.3000	-2.91000	0.16800	5.48000
	10%	-14.2000	-2.62000	0.18500	6.67000

*Ng-Perron (2001, Table 1)

The Ng-Perron test statistics are all greater than 1%, 5% and 10% critical values, hence, the CPI is integrated order one (I(1)) at its level series. This implies that the CPI is not stationary at the level series.

Table 2: NP unit root test analysis for first difference of CPI

Null Hypothesis: D(CPI) has a unit root					
Exogenous: Constant, Linear Trend					
Lag length: 1 (Spectral GLS-detrended AR based on SIC, maxlag=15)					
Sample (adjusted): 1995M02 2018M12					
Included observations: 287 after adjustments					
		MZa	MZt	MSB	MPT
Ng-Perron test statistics		-82.5417	-6.42271	0.07781	1.11033
Asymptotic critical values*:	1%	-23.8000	-3.42000	0.14300	4.03000
	5%	-17.3000	-2.91000	0.16800	5.48000
	10%	-14.2000	-2.62000	0.18500	6.67000
*Ng-Perron (2001, Table 1)					

The Ng-Perron test statistics in Table 2 are all less than 1%, 5% and 10% critical values, hence, the CPI at its first difference is integrated order zero (I (0)). This implies that the CPI is stationary at the first difference.

3.2 Correlogram

The correlogram presents the plots of autocorrelation function (ACF) and the partial autocorrelation function (PACF) of first difference in CPI variable for model identification as presented in Figure3 and Figure4.

The result of ACF in Figure3 shows a combination of sinusoidals dying off slowly. On the other hand, the PACF in Figure4 above cuts off after lag 2 thus, indicating that CPI follows an ARIMA (2, 1, 0) process. This tentative model identified will be compared with other possible ARIMA (p, d, q) models.

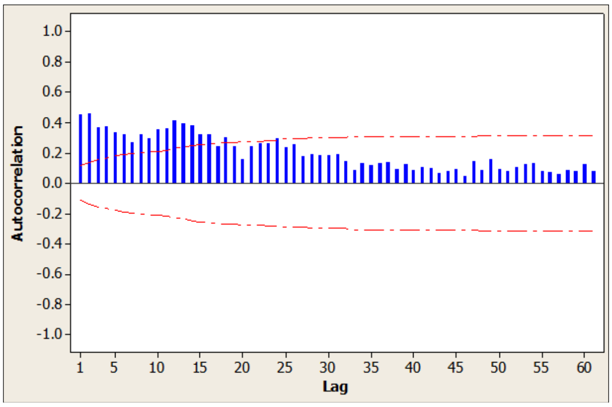

Fig 3: ACF plot for D (CPI) (with 5% significance limits for the autocorrelation)

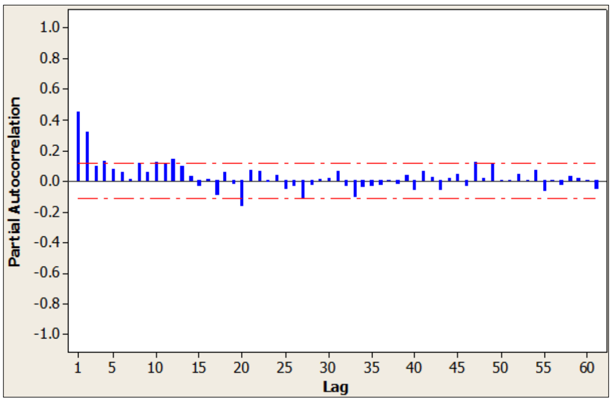

Fig 4: PACF plot for D (CPI) (with 5% significance limits for the partial autocorrelation)

The result of ACF in Figure3 shows a combination of sinusoidals dying off slowly. On the other hand, the PACF in Figure4 above cuts off after lag 2 thus, indicating that CPI follows an ARIMA (2,1,0) process. This tentative model identified will be compared with other possible ARIMA (p, d, q) models.

3.3 Model Comparison

This section presents compared 5 possible ARIM (p, 1, 0) models using SSDFC as presented in Table3 – Table5 below;

Table 3: Model selection using SSDFC with forecast period 100 and lead time 41

S/N	ARIMA Model	RSS	AIC	SBIC	SSDFC
1	*ARIMA*(1,1,0)	302.779	21.3675	28.6865	21.7293
2	*ARIMA*(2,1,0)	270.637	-7.83219	3.14625	20.2153
3	*ARIMA*(3,1,0)	267.696	-7.95573	6.68220	16.7370
4	*ARIMA*(4,1,0)	262.763	-10.2779	8.01954	13.4702
5	*ARIMA*(5,1,0)	260.963	-9.23112	12.7258	11.2795*

Table 4: Model selection using SSDFC with forecast period 237 and lead time 50

S/N	ARIMA Model	RSS	AIC	SBIC	SSDFC
1	*ARIMA*(1,1,0)	302.779	21.3675	28.6865	1501.39
2	*ARIMA*(2,1,0)	270.637	-7.83219	3.14625	1494.60
3	*ARIMA*(3,1,0)	267.696	-7.95573	6.68220	1492.40
4	*ARIMA*(4,1,0)	262.763	-10.2779	8.01954	1489.87
5	*ARIMA*(5,1,0)	260.963	-9.23112	12.7258	1483.10*

Table 5: Model selection using SSDFC with forecast period 40 and lead time 40

S/N	ARIMA Model	RSS	AIC	SBIC	SSDFC
1	*ARIMA*(1,1,0)	302.779	21.3675	28.6865	251.510
2	*ARIMA*(2,1,0)	270.637	-7.83219	3.14625	217.459
3	*ARIMA*(3,1,0)	267.696	-7.95573	6.68220	195.525
4	*ARIMA*(4,1,0)	262.763	-10.2779	8.01954	163.053
5	*ARIMA*(5,1,0)	260.963	-9.23112	12.7258	142.254*

The results of model selection using SSDFC at different forecast periods and lead times as shown from Table 3-5 consistently prefers ARIMA(5,1,0) as the best model among the five models compared. And the estimates of ARIMA (5, 1, 0) is presented in Table 6 below;

Table 6: Final Estimates of Parameters

Type	Coef	SE	Coef	T	P
AR	1	0.2510	0.0595	4.22	0.000
AR	2	0.2498	0.0609	4.10	0.000
AR	3	0.0473	0.0627	0.75	0.451
AR	4	0.1155	0.0610	1.89	0.060
AR	5	0.0830	0.0598	1.39	0.166
Constant		0.23949	0.05695	4.21	0.000

Differencing: 1 regular difference; Number of observations: Original series 288, after differencing 287; Residuals: SS = 260.963 (backforecasts excluded); MS = 0.929; DF = 281
The result of the model estimate in Table 6 showed that the coefficients of AR (1) and AR (2) are significant under 5% level. The implication is that the stationary CPI at current time is influenced by its lag1 and 2 values (and since the frequency of the data is monthly), it indicates that CPI is influenced by its past one and two month values.

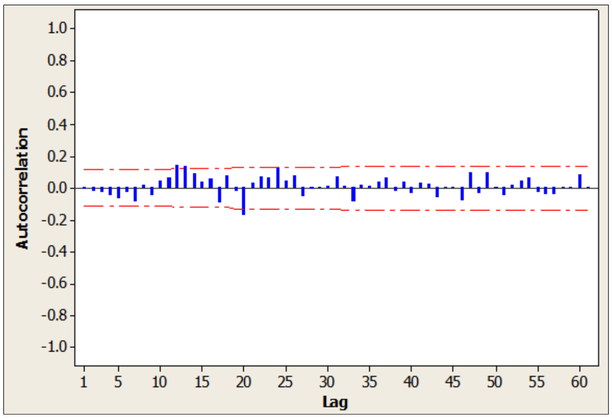

Fig 5: ACF of residuals for CPI (with 5% significance limits for the partial autocorrelation)

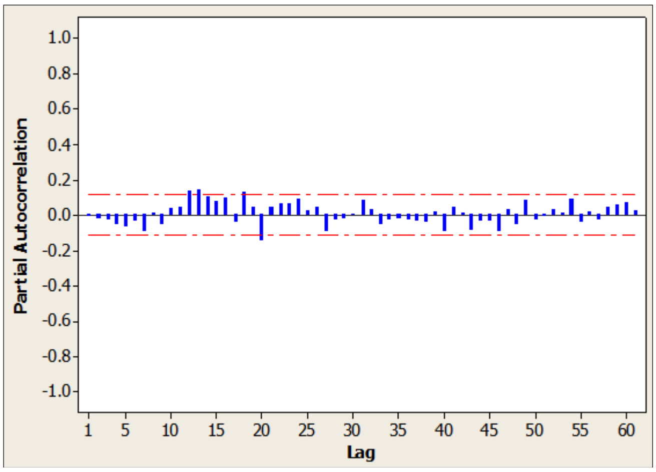

Fig 6: PACF of residuals for CPI (with 5% significance limits for the partial autocorrelation)

The ACF and PACF of residuals in Figure5 and Figure6 for the CPI data show non-significant spikes (the spikes are within the confidence limits) indicating that the residuals are uncorrelated. Therefore, the ARIMA (5, 1, 0) model appears to fit well and can be used to make forecasts

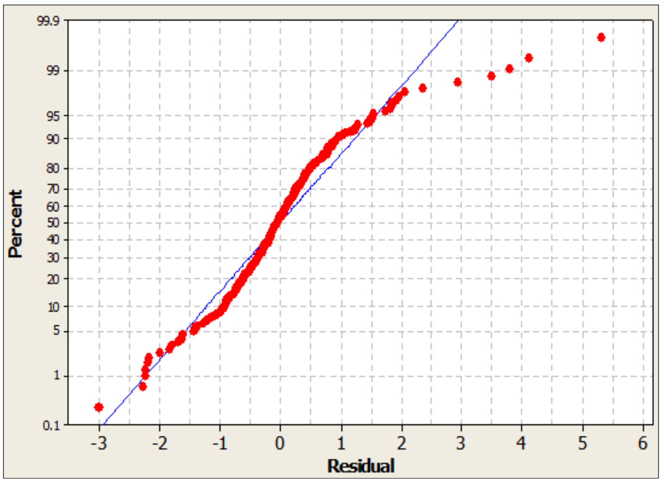

Fig 7: Probability plot of the Residuals

The diagnostic test using normal probability plot of residuals in Figure6 above indicates that the model residuals are normally distributed. Hence, the model fitted is adequate.

Table 7: Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12
Chi-Square	12.3
DF	6
P-Value	0.057

The Modified Box-Pierce (Ljung-Box) Chi-Square statistic in Table 7 is not significant under 5% up to 12^{th} lag indicating that the residuals of the fitted ARIMA (5, 1, 0) are not correlated. Hence the model is well specified and adequate.

Table 8: Forecasts from period 278

95% Limits Period	Forecast	Lower	Upper	Actual
279	255.151	253.262	257.040	255.569
280	256.806	253.781	259.832	257.741
281	258.372	254.144	262.601	260.576
282	259.911	254.541	265.282	263.810
283	261.381	254.809	267.952	267.046
284	62.784	254.966	270.603	269.716
285	264.134	255.067	273.202	272.039
286	265.440	255.130	275.750	274.116
287	266.709	255.171	278.247	276.394
288	267.940	255.186	280.695	278.501
289	269.140	255.186	283.094	
290	270.311	255.176	285.446	
291	271.457	255.163	287.751	
292	272.581	255.150	290.013	
293	273.685	255.139	292.232	

3.4 Discussion of result

The monthly CPI variable in Nigeria is integrated order one in its level series as reported by NP test and integrated order zero (I (0)) after the first difference. Hence, D (CPI) variable is stationary. The integrated order of CPI in Nigeria agrees with that of Kharimah *et al*. (2015) [8], Habimana *et al.* (2016) [6], Jere *et al*. (2016) [7] and Nyoni (2019) [10] for Malaysia, Rwanda, Zambia and Australia respectively. The SSDFC used as model selection criterion preferred ARIMA (5, 1 0) model as the best performing model with respect to prediction. This selected model is supported by RSS, indicating that SSDFC can be useful criterion to validate the best fitted model amongst two or more subclasses of ARIMA models or its likelihood.

The ARIMA (5, 1, 0) selected differs from that of Nyoni & Nathaniel (2019) [11] who fitted ARMA (1, 0, 2) model for inflation in Nigeria. Though, Nyoni and Nathaniel used different variable for inflation rather than CPI.

The Modified Box-Pierce (Ljung-Box) Chi-Square statistic in Table 7 is not significant indicating that the residuals of the fitted ARIMA (5, 1, 0) model are not correlated up to the 12th lag. The ACF and PACF of the model residuals in Figure 4 and Figure 5 respectively showed that the residuals are uncorrelated. The probability plot in Figure 6 reveals that the residuals are normally distributed; hence, the fitted model is adequate. And the forecast values of CPI exhibit a rising trend.

4. Conclusion

The Box and Jenkins ARIMA (p, d, q) process was used to model CPI in Nigeria. And five (5) different sub-classes (order) of ARIMA (5, 1, 0) models were identified and compared using model selection criterion, specifically the SSDFC introduced by Amaefula (2011) [1]. The model comparison showed that ARIMA (5, 1, 0) is preferred. This chosen model was also supported by RSS and all the diagnostic tests indicated that the model is adequate.

The finding reveals that CPI is significantly influenced by its past one and two month values.

And CPI in Nigeria showed an upwards trend over the forecast period. From the findings, it becomes imperative for policy makers to engage more on how to make monetary policies more proactive so as to fight such increase in inflation (CPI) as reflected in the forecasts. However, based on the output performance of the compared models, SSDFC revealed that ARIMA (5, 1, 0) is preferred and can be recommended for predicting the dynamics of CPI in Nigeria.

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