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New class of fuzzy topological spaces

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Abstract
In this paper, we introduce new class of fuzzy sets called fuzzy generalized semi-closed sets and fuzzy generalized semi open sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Also we introduce fuzzy generalized semi-closed sets and fuzzy generalized semi open sets in fuzzy topological spaces and study some of their properties.

Keywords: Fuzzy gs-closed sets, Fuzzy gs-open sets

Introduction

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, L.A.Zadeh [1] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L.Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces. Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function

$\mu_A : X \longrightarrow [0,1]$ of A, defined by
$\mu_A(x) = 1$, if $x \in A$.
$= 0$, if $x \in A$.

Thus an element $x \in X$ isin A if $\mu_A(x) = 1$ and is not in A if $\mu_A(x) = 0$. In general if X is a set and A is a subset of X then A has the following representation. $A = \{ (x, \mu_A(x)): x \in X\}$, here $\mu_A(x)$ may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A.Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1 ; he called such a class as fuzzy subset. A fuzzy subset A in X is characterized as a membership function
$\mu_A : X \longrightarrow [0,1]$, which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.
The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy gs-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy gs-closed but not conversely. Also we introduce fuzzy gs-open sets in fuzzy topological spaces and study some of their properties.

1. Preliminaries

1.1 Definition: [1] A fuzzy subset A in a set X is a function $A : X \rightarrow [0, 1]$. A fuzzy subset in X is empty iff its membership function is identically 0 on X andis denoted by 0 or μ_ϕ.The set X can be considered as a fuzzy subset of X whosemembership function is identically 1 on X and is denoted by $\mu_x$or I_x.

In fact everysubset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzysubset is a generalization of the concept of a subset.

1.2 Defnition: [1] If A and B are any two fuzzy subsets of a set X, thenA is said to be included in B or A is contained in B iff $A(x) \leq B(x)$ for all x in X. Equivalently, $A \leq B$ iff $A(x) \leq B(x)$ for all x in X.

1.3 Defnition: [1] Two fuzzy subsets A and B are said to be equal if $A(x) = B(x)$ for every x in X.
Equivalently $A = B$ if $A(x) = B(x)$ for every x in X.

1.4 Defnition: [1] The complement of a fuzzy subset A in a set X, denoted by A' or $1 - A$, is the fuzzy subset of X defined by $A'(x) = 1 - A(x)$ for all x in X. Note that$(A')' = A$.

1.5 Defnition: [1] The union of two fuzzy subsets A and B in X, denoted by $A \vee B$, is a fuzzy subset in X defined by $(A \vee B)(x) = \text{Max}\{\mu_A(x), \mu_B(X)\}$ for all x in X.

1.6 Defnition: [1] The intersection of two fuzzy subsets A and B in X, denoted by $A \wedge B$, is a fuzzy subset in X defined by $(A \wedge B)(x) = \text{Min}\{A(x), B(x)\}$ for all x in X.

1.7 Defnition: [1] A fuzzy set on X is 'Crisp' if it takes only the values 0 and 1 on X.

1.8 Defnition: [2] Let X be a set and be a family of fuzzy subsets of (X,τ) is called a fuzzy topology on X iff τ satisfies the following conditions.

(a) (i)μ_ϕ ; $\mu_X \in \tau$: That is 0 and $1 \in \tau$
(b) (ii) If $G_i \in \tau$ for $i \in I$ then $\vee G_i \in \tau$
    $i \in I$
(c) If $G,H \in \tau$ then $G \wedge H \in \tau$

The pair (X, τ) is called a fuzzy topological space (abbreviated as fts). The membersof $\tau$are called fuzzy open sets and a fuzzy set A in X is said to be closed iff $1 - A$ is an fuzzy open set in X.

1.9 Remark: [2] Every topological space is a fuzzy topological space but not conversely.

1.10 Defnition: [2] Let X be a fts and A be a fuzzy subset in X. Then $\wedge$ {B : B is a closed fuzzy set in Xand $B \geq A$} is called the closure of A and is denoted by A or cl(A).

1.11 Defnition: [2] Let A and B be two fuzzy sets in a fuzzy topological space $(X,\tau)$and let $A \geq B$. Then B is called an interior fuzzy set of A if there exists $G \in \tau$ suchthat $A \geq G \geq B$, the least upper bound of all interior fuzzy sets of A is called theinterior of A and is denoted by A^0.

1.12 Definition: [3] A fuzzy set A in a fts X is said to be fuzzy semiopen if and only if there exists a fuzzy open set V in X such that $V \leq A \leq cl(V)$.

1.13 Definition: [3] A fuzzy set A in a fts X is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set V in X such that $int(V) \leq A \leq V$. It is seen that a fuzzy set A is fuzzy semiopen if and only if 1-A is a fuzzy semi-closed.

1.14 Theorem: [3] The following are equivalent:

a) $\mu$is a fuzzy semiclosed set,
b) μ^C is a fuzzy semiopen set,
c) $int(cl(\mu)) \leq \mu$.
d) $int(cl(\mu)) \geq \mu^c$

1.15 Theorem: [3] Any union of fuzzy semiopen sets is a fuzzy semiopen set and (b) any intersection of fuzzy semi closed sets is a fuzzy semi closed.

1.16 Remark [3]

(a) Every fuzzy open set is a fuzzy semiopen but not conversely.
(b) Every fuzzy closed set is a fuzzy semi-closed set but not conversely.
(c) The closure of a fuzzy open set is fuzzy semiopen set
(d) The interior of a fuzzy closed set is fuzzy semi-closed set

1.17Definition: [3] A fuzzy set $\mu$of a fts X is called a fuzzy regular open set of X if $int(cl(\mu).= \mu$.

1.18 Definition: [3] A fuzzy set $\mu$of fts X is called a fuzzy regular closed set of X if $cl(int(\mu)) = \mu$.

1.19 Theorem: [3] A fuzzy set $\mu$of a fts X is a fuzzy regular open if and only if μ^C fuzzy regular closed set.

1.20 Remark: [3]
(a) Every fuzzy regular open set is a fuzzy open set but not conversely.
(b) Every fuzzy regular closed set is a fuzzy closed set but not conversely.

1.21 Theorem: [3]
(a) The closure of a fuzzy open set is a fuzzy regular closed.
(b) The interior of a fuzzy closed set is a fuzzy regular open set.

1.22 Definition: [4] A fuzzy set αin fts X is called fuzzy rw closed
if cl(α)$\leq$ μwhenever $\alpha \leq$ $\mu$and $\mu$is regular semi-open in X.

2. Fuzzy generalized semi closed sets and fuzzy generalized semi open sets.
Definition 2.1: Let (X, T) be a fuzzy topological space. A fuzzy set α of X is called fuzzy
generalized semi-closed (briefly, fuzzy gs-closed) if scl(α) $\leq$ σ whenever $\alpha \leq$ σ and σ is fuzzy-open in fts X.
We denote the class of all fuzzy generalized semi closed sets in fts X by FGSC(X).

Theorem 2.2: Every fuzzy closed set is a fuzzy gs-closed set in a fts X.
Proof: Let α be a fuzzy closed set in a fts X. Let β be a fuzzy open set in X such that
$\alpha \leq$ β. Since α is fuzzy closed, s-cl(α)= α. Therefore s-cl(α) $\leq \beta$. Hence α is fuzzy gs-closed in fts X.
The converse of the above Theorem need not be true in general as seen from the following example.

Example 2.3: Let X= {a, b, c}. Define a fuzzy set α in X by
α(x) = 1 if x = a
0 othewise
β (x) = 1 if x = a,b
0 otherwise.
Let T= {1, 0, β}. Then (X, T) is a fuzzy topological space. Define a fuzzy
set μ in X by

μ(x) = 1 if x = b
0 otherwise.
Then μ is a fuzzy gs-closed set but it is not a fuzzy closed set in fts X.

Corollary 2.4: By K.K.Azad we know that, every fuzzy regular closed set is a fuzzy closed set but not conversely. By Theorem 2.2 every fuzzy closed set is a fuzzy gs-closed set but not conversely and hence every fuzzy regular closed set is a fuzzy gs-closed set but not conversely.

Remark 2.5: Fuzzy gs closed sets and fuzzy semi-closed sets are independent.

Example: (i) Consider Let X= {a, b, c} and the functions α, β: X $\longrightarrow$ [0, 1] be defined as

α(x) = 1 if x = a
0 otherwise

β(x) = 1 if x = a, b
0 Otherwise

Consider T= {1, 0, α, β}. Then (X, T) is a fuzzy topological space. Then the fuzzy set

μ(x) = 1 if x = b
0 otherwise

is a fuzzy gs-closed set but it is not a fuzzy semi-closed set in fts X.

Example: (ii) Let X= {a, b, c} and the functions α, β, γ: X $\longrightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b
0 therwise

γ(x) = 1 if x = a,b
0 otherwise

Consider T= $\{1,0,\alpha,\beta,\gamma\}$. Then (X, T) is a fuzzy topological space.

Then the fuzzy set

α (x) = 1 if x = a
0 otherwise.

is a fuzzy semi-closed set but it is not a fuzy gs-closed set in fts X.

Remark 2.6: Fuzzy gs-closed sets and fuzzy rw-closed sets are independent.
Example (i): Let X= {a, b, c,d} and the functions $\alpha, \beta, \gamma, \delta$: X $\longrightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b
0 Otherwise

γ(x) = 1 if x = a,b
0 otherwise
δ(x) = 1 if x = a,b,c
0 Otherwise

Consider T= $\{1,0,\alpha,\beta,\gamma,\delta\}$. Then (X, T) is a fuzzy topological space.

Then (X, T) is a fts then the fuzzy set

μ(x) = 1 if x = a, d

0 otherwise.

is a fuzzy gs-closed set but it is not a fuzzy rw closed set in fts X.

Example: (ii) Let X= {a, b, c} and the functions α: X $\longrightarrow$ [0, 1] be defined as
α (x) = 1 if x = a
0 otherwise
Consider T= $\{1,0,\alpha\}$. Then (X, T) is a fuzzy topological space. Then the fuzzy set
α (x) = 1 if x = a
0 otherwise.
is a fuzzy rw-closed set but it is not a fuzzy gs closed set in fts X.

Theorem 2.7: If α and β are fuzzy gs-closed sets in fts X, then $\alpha \vee \beta$
fuzzy gs-closed set in fts X.

Proof: Let σ be a fuzzy open set in fts X such that $\alpha \vee \beta \leq \sigma$. Now $\alpha \leq \sigma$ and $\beta \leq \sigma$.Since $\alpha$and $\beta$are fuzzy gs-closed sets in fts X, s-cl(α) $\leq \sigma$ and s-cl(β) $\leq \beta$.
Therefore s-cl(α) ∨ s-cl(β) $\leq \sigma$. But s-cl(α) ∨ s-cl(β)= s-cl($\alpha \vee \beta$).Thus s-cl($\alpha \vee \beta$)$\leq \sigma$. Hence $\alpha \vee \beta$ is a fuzzy gs-closed set in fts X.

Remark 2.8: If $\alpha$and β are fuzzy gs-closed sets in fts X, then $\alpha \wedge \beta$ need not be a fuzzy gs-closed set in general as seen from the following example.

Example 2.9: Consider the fuzzy topological space (X, T) defined
Let X= {a, b, c,d},T= $\{1,0,\alpha,\beta,\gamma\}$in this fts X, The fuzzy sets $\delta_1\delta_2$: X $\longrightarrow$ [0, 1] are defined by

δ_1(x) = 1 if x = {a,c,d}
0 otherwise.

δ_2(x) = 1 if x = {a,b,c}
0 otherwise

and α, β, γ: X $\longrightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = c,d
0 otherwise

γ(x) = 1 if x = a,c,d
0 otherwise

Then $\delta_1 and\ \delta_2$ are fuzzy gs closed sets in fts X.
Let μ = $\delta_1 \wedge \delta_2$, then

μ (x) = 1 if x = a, c
0 otherwise.

μ = $\delta_1 \wedge \delta_2$, is not a fuzzy gs-closed set in fts X.

Theorem 2.10: If a fuzzy set α of fts X is both fuzzy regular open and fuzzy gs-closed, then α is a fuzzy pre-closed set in fts X.

Proof: Suppose a fuzzy set α of fts X is both fuzzy regular open and fuzzy gs-closed. As every fuzzy regular open set is a fuzzy open set and $\alpha \leq \alpha$ we have s-cl(α) $\leq \alpha$.
Also $\alpha \leq s -$cl(α).Therefore s-cl(α) $= \alpha$. That is α is fuzzy closed. Since α is fuzzy regular open, int(α)= α. Now cl(int(α))=cl(α) = α. Therefore α is a fuzzy semi- closed set in fts X.

Theorem 2.11: If a fuzzy set α of a fts X is both fuzzy open and fuzzy gs closed, then α is a fuzzy closed set in fts X.

Proof: Suppose a fuzzy set α of a fts X is both fuzy open and fuzzy gs-closed. Now, $\alpha \leq \alpha$, we have $\alpha \leq s -$cl(α). Therefore s-cl(α) = α and hence $\alpha$is a fuzzy closed set in fts X.

Theorem2.12: If a fuzzy set α of a fts X is both fuzzy open and fuzzy generalized semi-closed, then α is a fuzzy gs-closed set in fts X.

Proof: Suppose a fuzzy set α of a fts X is both fuzzy open and fuzzy generalized semi closed. Now
$\alpha \leq \alpha$, by hypothesis we have s-cl(α) $\leq \alpha$.Also $\alpha \leq s -$cl(α)Therefore s-cl(α)= α. That is α is a fuzzy closed set and hence $\alpha$is a fuzzy gs-closed set in fts X, as every fuzzy closed set is a fuzzy gs-closed set.

Remark2.13: If a fuzzy set γ is both fuzzy open and fuzzy gs-closed set in a fts X, then γ need not be a fuzzy generalized semi closed set in general as seen from the following example.

Example 2.14:
Let X= {a, b, c,d} and the functions α, β, γ: X ⟶ [0, 1] be defined as
α (x) = 1 if x = a
0 otherwise
β(x) = 1 if x = c,d
0 otherwise
γ(x) = 1 if x = a,c,d
0 otherwise

Consider T= {1,0,α,β, γ}. Then (X, T) is a fuzzy topological space.In this fts X,$\gamma$is both fuzzy open and gs closed.

Theorem 2.15: Let αbe a fuzzy gs-closed set of a fts X and suppose $\alpha \leq \beta \leq s -$cl($\alpha$). Then β is also a fuzzy gs-closed set in fts X.

Proof: Let $\alpha \leq \beta \leq s -$cl($\alpha$) and αbe a fuzzy gs-closed set of fts X. Let σbe any fuzzy open set such that $\beta \leq \sigma$.Then $\alpha \leq \sigma$ and $\alpha$is fuzzy gs-closed, we have s-cl(α)$\leq \sigma$.
But s-cl(β) $\leq s -$cl(α) and thus s-cl(β) $\leq \sigma$. Hence $\beta$is a fuzzy gs-closed set in fts X.

Theorem 2.16: In a fuzzy topological space X if FO(X) = {1, 0}, where FO(X) is the family of all fuzzy open sets then every fuzzy subset of X is fuzzy gs-closed.

Proof: Let X be a fuzzy topological space and FRGα(X) = {1, 0}. Let α be any fuzzy subset of X. Suppose α= 0. Then 0 is a fuzzy gs-closed set in fts X. Suppose $\alpha \neq 0$. Then 1 is the only fuzzy open set containing $\alpha$and so s-cl(α) ≤ 1. Hence $\alpha$is a fuzzy gs-closed set in fts X.
Theorem 2.17: If $\alpha$is a fuzzy gs-closed set of fts X and s-cl(α)∧(1- s-cl(α))=0, then s-cl(α)- αdoes not contain any non-zero fuzzy openset in fts X.

Proof: Suppose α is a fuzzy gs-closed set of fts X and s-cl(α)∧ (1-s-cl(α))=0,.We prove the result by contradiction. Let β be a fuzzy open set such that s-cl(α)- $\alpha \geq \beta$ and $\beta \neq 0$.
Now $\beta \leq$s-cl(α)-α i.e $\beta \leq$1-α which implies $\alpha \leq$1-β, since $\beta$is fuzzy open in fts X,then 1- β is fuzzy open in X; Since α is a fuzzy gs-closed set in fts X, by definition
s-cl(α)≤1-βSo $\beta \leq \mathrm{s}-\mathrm{cl}(\alpha)$ Therefore, $\beta \leq$s-cl(α)∧ (1-s-cl(α))=0, by hypothesis. This shows that β=0 which is a contradiction. Hence s-cl(α)-αdoes not contain any non-zero fuzzy$-open$ set in fts X.

Corollary 2.18: If α is a fuzzy gs-closed set of fts X and s-cl(α)∧ (1-s-cl(α))=0,
then s-cl(α)- α does not contain any non-zero fuzzy open set in fts X.

Proof: Follows form the corollary 2.18 and the fact that every fuzzy regular
open set is a fuzzy openset in fts X.

Corollary 2.19: If α is a fuzzy gs-closed set of a fts X and s-cl(α)∧ (1-s-cl(α))=0,
then s-cl(α)- αdoes not contain any non-zero fuzzy regular closed set in fts X.

Proof: Follows form the Theorem 2.18 and the fact that every fuzzy regular open set is a
Fuzzy open set in fts X.

Theorem 2.20: Let α be a fuzzy gs-closed set of fts X and s-cl(α)∧ (1-s-cl(α))=0,Then α is a fuzzy closed set if and only if s-cl(α)- $\alpha$is a fuzzy $-$openset in fts X.

Proof: Suppose α is a fuzzy closed set in fts X. Then s-cl(α)= $\alpha$and so s-cl(α)- $\alpha = 0$, which is a fuzzy open set in fts X. Conversely suppose s-cl(α)- α is a fuzzy $open$ set in fts X. Since $\alpha$is fuzzy gs-closed, by corollary 2.18 s-cl(α)- α does not contain any non-zero fuzzy regular open set in fts X then That is s-cl(α)= $\alpha$and hence $\alpha$is a fuzzy closed set in fts X.

We introduce a fuzzy gs-open set in fuzzy topological space X as follows.

Definition 2.21: A fuzzy set α of a fuzzy topological space X is called a fuzzy generalized semi-open (briefly, fuzzy gs-open) set if its complement $\alpha^{C}$is a fuzzy gs-closed set in fts X.

We denote the family of all fuzzy gs-open sets in fts X by FO(X).

Theorem 2.22: If a fuzzy set $\alpha$of a fuzzy topological space X is fuzzy open, then it is fuzzy gs-open but not conversely.

Proof: Let αbe a fuzzy open set of fts X. Then $\alpha^{C}$is fuzzy closed. Now by Theorem 2.2, α^{C} is fuzzy gs-closed. Therefore $\alpha$is a fuzzygs-open set in fts X.

The converse of the above Theorem need not be true in general as seen from the following example.

Example 2.23
(i) Let X= {a, b, c} and define the fuzzy set α in X by α: X $\longrightarrow$ [0, 1] be defined as
α (x) = 1 if x = a
0 otherwise

Let T= {1,0,α}Then (X, T) is a fuzzy topological space. Then the fuzzy set β in X by
β (x) = 1 if x = a,c
0 otherwise.
Then β is a fuzzy gs-open but it is not fuzzy open set in fts X.

Corollary 2.24: By K.K.Azad, we know that, every fuzzy regular open set is a fuzzy open set but not conversely. By Theorem 2.23, every fuzzy open set is a fuzzy gs-open set but not conversely and hence every fuzzy regular open set is a fuzzy gs-open set but not conversely.

Theorem 2.25: A fuzzy set $\alpha$of a fuzzy topological space X is fuzzy gs-open if and only if $\delta \leq s-\mathrm{int}(\alpha)$ whenever and $\delta \leq \alpha$ and δ is a fuzzy $open$set in fts X.

Proof: Suppose that $\delta \leq s-\mathrm{int}(\alpha)$ whenever $\delta \leq \alpha$and δ is a fuzzy $open$set in fts X. To prove that $\alpha$is fuzzy gs-open in fts X. Let $\alpha^{C} \leq \beta$and $\beta$is any fuzzy $open$ set in fts X. Then $\beta^{c} \leq \alpha$, since β is fuzzy open in fts X; then 1- $\beta$is also fuzzy-open set in fts X.then β^{c} is also fuzzy-open in fts X By hypothesis $\beta^{c} \leq$s-int(α)Which implies [s-int(α)]$^{c} \leq \beta$ that is s-int(α^{c})≤ β^{c} since s-cl(α^{c})= $[\mathrm{s}-\mathrm{int}(\alpha)]$c,Thus$\alpha^{C}$ is a fuzzy gs-closed and hence α is fuzzy gs-open in X.
Conversely, suppose that α is fuzzy gs-open. Let $\beta \leq \alpha$and $\beta$is any fuzzy $open$ in fts X. Then$\alpha^{C} \leq \beta^{c}$since β is fuzzy open in fts X,then 1- β is fuzzy -open in X;then β^{c} is also fuzzy open in fts X.since α^{C} is fuzzy gs-closed,we have s-cl(α^{C}) ≤ $\beta^{c}$and so,$\beta \leq$s-int(α), since s-cl(α^{c}) = $[\mathrm{s}-\mathrm{int}(\alpha)]$c.

Theorem 2.26: If $\alpha$and $\beta$are fuzzy gs-open sets in a fts X, then $\alpha \wedge \beta$ is also a fuzzy gs-open set in fts X.

Proof: Let $\alpha$and β be two fuzzy gs-open sets in a fts X. Then α^C and $\beta^C$are fuzzy gs-closed sets in fts X. By Theorem 2.8, $\alpha^C \vee \beta^C$is also a fuzzy gsclosed set in fts X.
That is $\alpha^C \vee \beta^C = \alpha^C \wedge \beta^C$is a fuzzy gs-closed set in X. Therefore $\alpha \wedge \beta$is also a fuzzy gs-open set in fts.

Example 2.27
Consider the fuzzy topological space (X, T) defined in 2.6
Let X= {a, b, c,d},T= $\{1,0,\alpha,\beta,\gamma,\delta\}$in this fts X, The fuzzy sets $\delta_1\delta_2$:X ⟶ [0, 1] are defined by

δ_1(x) = 1 if x = a
0 otherwise.

δ_2 (x) = 1 if x = c
0 therwise

Then $\delta_1 and\ \delta_2$ are fuzzy gs open sets in fts X.

Let μ = $\delta_1 \vee \delta_2$, then

μ (x) = 1 if x = a, c
0 otherwise.

μ = $\delta_1 \vee \delta_2$, Is not a fuzzy gs-closed set in fts X.

Theorem 2.28: If s-int(α)$\leq \beta \leq \alpha$and α is a fuzzy gs-open set in a fts X, then β is also a fuzzy rw-open set in fts X.

Proof: Suppose s-int(α)$\leq \beta \leq \alpha$and α is a fuzzy gs-open set in a fts X. To prove that β is a fuzzy gs-open set in fts X. Let σbe any fuzzy rgα-openset in fts X such that $\sigma \leq \beta$.Now $\sigma \leq \beta \leq \alpha$That is $\sigma \leq \alpha$.
Since α is fuzzy gs-open set of fts X, $\sigma \leq S - int(\alpha)$by Theorem 2.26. By hypothesis s-int(α)$\leq \beta$.Then int [s-int(α)] ≤ s-int (β)That is s-int(α) ≤s-int(β).
Then $\sigma \leq$ s-int(β) Again by Theorem 2.26 β is a fuzzy gs-open set in fts X.

Theorem 2.29: If a fuzzy subset $\alpha$of a fts X is fuzzy gs-closed and s-cl(α) ∧ (1-s-cl(α)) = 0, then s-cl(α)- $\alpha$is a fuzzy gs-open set in fts X.

Proof: Let αbe a fuzzy gs-closed set in a fts X and Let s-cl(α) ∧ (1-s-cl(α)) = 0,
Let β be any fuzzy open set of fts X such that β ≤s-int(α), Then by corollary 2.18 s-cl(α)- α does not contain any non-zero fuzzy open set and so β= 0.
Therefore β ≤s-int(α)(cl(α)- α. By theorem 2.26 p-cl(α)- $\alpha$is a fuzzy gs-open set in fts X.

Theorem 2.30: Let $\alpha$and βbe two fuzzy subsets of a fts X. If $\beta$is a fuzzy gs-open set and α ≥s-int(β), then$\alpha \wedge \beta$is a fuzzy gs-open set in fts X.

Proof: Let β be a fuzzy gs-open set of a fts X and α ≥s-int(α),That is s-int(B) ≤ $\alpha \wedge \beta$.
Also s-int(B) ≤ $\alpha \wedge \beta$ and $\beta$is a fuzzy gs-open set. By Theorem 2.29,$\alpha \wedge \beta$is also a fuzzy gs-open set in fts X.

References

1. Zadeh LA. Fuzzy sets, Information and control, 1965; 8:338-353.
2. Chang CL. Fuzzy topological spaces, JI. Math. Anal. Appl., 1968; 24:182-190.
3. Azad KK. On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity. JI. Math. Anal. Appl. 1981; 82(1):14-32.
4. Wali RS. on some topics in general and fuzzy topological spaces;ph.d thesis, Karnatak university Dharwad, 2007.
5. Thakur SS, Bajpai Pandey Jyoti. “Intuitionistic Fuzzy rgα-closed sets”, International Journal of Fuzzy system and Rough System 4(1):67-73.
6. Fukutake T, Saraf RK, Caldas M, Mishra S. Mappings via Fuzzy gp-closed sets, Bull. of Fukuoka Univ. of Edu. 2003; 52(III):11-20.
7. Arya SP, Nour TM. Characterizations of S-normal spaces,Indian J pure APP.Math. 1990; 21:717-719.
8. Wali RS, Vivekananda Dembre. on pre generalized pre regular weakly closed sets in topological spaces, to appear in Journal of Computer and Mathematical Science, 2015.