On contributions of Chandrakala Devi Dhananjaya to mathematics in Nepal

Sher Singh Raikhola, Dinesh Panthi and Kanhaiya Jha

Abstract
This article attempts to explore on contributions of Chandrakala Devi Dhananjaya and analyze her contributions in the field of Nepalese mathematics to materialize her book 'Shishubodha Tarangini'.

Keywords: History of Mathematics, Chandrakala Devi Dhananjaya, Shishubodha Tarangini, Lilavati, Gurukula, Wyakta Chandrika, Kul Debata

Introduction
Mathematics is a science of arts; it is an induction science and provides the way of thinking. The chronology is the backbone of history and mathematics is the backbone of science and technology. The history of mathematics is interrelated with the history of human civilization. Mathematics has become an integral part of the modern world in turn valuable to all other branches of our cultures. Every culture has developed some mathematics. In some cases, the mathematics has spread from one culture to another. It is claimed that the first mathematics or originatled around five thousand years ago along with the ancient Egyptian civilization, Babylonian civilization, Greek civilization, Chinese civilization, Hindu civilization. Thousands of years ago there were no numbers to represent two or three. Instead fingers, rocks, sticks, or eyes were used to represent numbers. There were neither clocks nor calendars to help keep track of time. Evidences of the fact that mathematics developed in culture and civilization spread into another culture and civilization can be found everywhere. Nowadays in the field of mathematics there are many works, which are sufficient to fulfill the demands of 21st century in the mathematical field (base 60). The history reveals that the number and counting system have developed before the recorded history of human civilization [9]. Ancient counting system was beginning with the concept of more and less. In primitive period, figure, pebbles, sticks, knots were used for keeping count. The history of mathematics teaching in Nepal initiated with Gurukula in ancient period [12]. The Gurukula education system continued for many years and there were no formal schools as we have today]. Sanskrit school was established in 1877 A.D. and Siddhanta Siromani, Lilawati, Bhaswati, Bij-Ganita referred as textbook of those schools [14]. Before 125 year ago there were no written mathematics books in Nepal [15]. Nepali students in those days used to go to Kashi (Banaras) to learn Sanskrit and they were taught famous books on mathematics like Siddhant Siromani of Bhaskaraacharya (II) [13].

Chadrakala Devi Dhananjaya had written a book named Shishubodha Tarangini in slokas form [10], which was published in 1933 A.D. She had not acquired any formal education. However, she wrote such a book, which is still considered most important to study the Nepalese history of mathematics. She is supposed to be the first Nepali woman writer of mathematics book and teacher in Nepal. Until now, no Nepali woman mathematician has been seen in writing mathematics book on single ownership. Therefore, we are interested to carry out a research work on her contribution in mathematics.

Statement of the Problem
Women do not seem to be more interested in mathematics till present in Nepalese context. However it is the great matter for Chandrakala Devi who used to live far away from the capital
city of the country and who was brought up in the atmosphere of village for writing the tough subject like mathematics even 80 years ago. She has presented the evidence that even the women can contribute in the field of mathematics by writing the book named ShishubodhaTarangini in the history of Nepalese mathematics. It is not a minor task to study, teach and write the tough subject like mathematics for those women who are living in the village in the contemporary time and even by overcoming the family situation of the woman. It is the great matter for the woman to write mathematics when even the male writers were found in very less number at that time. Without taking the formal education, she has written the mathematics subject as well as she has presented mathematical problems and their solution in poetic verse in her book. In her book, it is found lots of content in comparison with the other books written in Nepali language like Wyakta Chandrika, Bichitra Ganit, Ganitsagar, Gorkha Bilganit Shiksha, and Patti Ganit. Therefore, she is called the first woman mathematician of Nepal. While conducting the research of Nepali mathematics writer, so many male writers were found to be researched but regarding the women writers, they were not found to be researched so while conducting the research of female writer, it is conducted a research about the woman mathematician writer Chandrakala Devi Dhananjaya.

**Significance of the Study**

Contribution and Biography of First Nepalese woman mathematician in development of Nepalese mathematics is highly interesting and exciting research area. It promotes our interest in mathematics and focuses the contribution of Nepalese people to the development of mathematics, which is very important in every step of our daily life. The result of this study would help

1. To focus the contribution of Chadrakala Devi Dhananjaya in the development of mathematics.
2. To promote our interest in mathematics especially in woman.
3. To study the trends of mathematics education of Nepal.
4. To support the historical development of mathematics.
5. It helps to explore for how Nepalese mathematics contents developed and modified.

**Review**

Acharya and Pant [1] has written an article “Chandrakala Devi Dhananjaya and ShishubodhaTarangini in mathematical history” which basically concerned biography and effect of this book in this period. They have explained few parts of book “Shishubodha Tarangini” in the article. Bashyal [6] has conducted a research work on the title " A 1933 Nepali mathematics and Astrology Book Shishubodha Tarangini; Translation and commentary on mathematics chapters.” Researchers used content analysis approach where published literatures were kept as data and analysis was done through it. Documents study as of Chandrakala Devi Dhananjaya was said as data of this qualitative research.

The nature of this study was historical type based on document analysis. Area of this study is biographic and exploration of mathematical proof and explanation. The study was based on the book Shishubodha Tarangini of Chandrakala Devi Dhananjaya and her biography and contemporary mathematics of her period. The nature of this research was historical and qualitative basis.

There are many approaches for the qualitative and descriptive research to get the information. Thus the researcher has used primary data including in-depth interview and counseling with resource person. The secondary sources of information for the study was taken from the printed and collected documents like journals, books, documents, articles, and related literature.

Researchers visited libraries and studied classical documents like books, journals etc. to analyze mathematics book. The study is based on Historical documentation and description. Analysis of information required interpretation of them in a logical, analytical and descriptive manner.

Chandrakala was born in 1915 A.D. She was born as the fourth daughter of her father Himlal Paudel and mother Kunta Devi Paudel in Arghakhanchi district. She got married when she was the age of 7 years with Tikaram Dhananjaya. Her own family environment as well as the family environment of her husband influenced her since she had gone to her husband's home at her early age. Her husband had commenced writing poems at the age of 11 years. After the accomplishment of his study, he had started to teach in his own village and had initiated to write various books. Her husband encouraged her from recognizing the letters to book writing.

During that time one Nepali Lady Chadrakala Devi Dhananjaya had written Shishubodha Tarangini in slokas (verse) form which was published in 1933 A.D. She has been supposed to be the first Nepali Lady writer of mathematics book and teacher in Nepal who had a sound knowledge of mathematics at that time.

Masterpieces published by Chadrakala Devi Dhananjaya [16]

1. Upadesh Chandrakala (Part I); Published date: 1934 A.D
2. ShishubodhaTarangini (Part I & III) (Unavailable)
3. ShishubodhaTarangini (Part II); Published date::1933 A.D

**Shishubodh Tarangini**

Shishubodha Tarangini is based on mathematics and astronomy. Some content were cited from aryabhattiya, lilavati and Wyakta Chandrika [16]. It was published for teaching mathematics and Jyotish to children. It contained 80 pages in total. In this book, many mathematical problems related to unitary method, profit and loss, square root and cub root, HCF and LCM, fractions, decimal, simple interest, and sequence etc. are given with suitable examples. The first verse of this book is an invocatory verse on Lord Ganesha, Kuldeba, and other gods. Firstly, she defines the various units of money, which were in vogue during those days. Shishubodha Tarangini [10] mainly deals with what we call as ‘Arithmetic’ in today's mathematical parlance. It consists of 134 verses written in Nepali as well Sanskrit in poetic form (verse). There are certain verses which deal with number system, fundamental operation of mathematics, decimal and its operation, fraction and its operation, interest, HCF, LCM, annuity, Profit and loss, unitary Method, sequence and series etc. Moreover, it also includes certain elements of algebra such as finding an unknown quantity subject to certain constraints using the method of supposition. The content of Shishubodha Tarangini has -Fundamental operation of mathematics, HCF and LCM, Fraction and Decimal, Square and Square root, Cube and Cube root, Unitary method, Sequence and Series.

**Some Examples**

In this book, we can see many mathematical problems related to unitary method, profit and loss square root and cub root, HCF LCM, Fractions, Decimal, Simple Interest, and Sequence etc. On the concern of other feature of the book, the researchers see that this book started with
Shree Ganeshaya Namah | Shree Barahayya Kuldevataiy Namah | Shree Paticharanara Bindabhyan Namah |

The first verse of this book is an invocatory verse on Lord Ganesh, Kaldevata, and other gods. It was customary in those days to invoke the beginning of any auspicious event. We can see this type of customs practiced in Lilavati, most popular book of Hindu Mathematics written in eleventh century of Bhaskaracharya II and Wyakta Chandrika, renowned book of Nepal as well of India written by Gopal Padey that were practiced in ninth century.

Firstly, she defines the various units of money which were in vogue during those days.

Chaadi Aath Lalako Eka Masha Bahra Mashako Eka Tola Hunchha | Sun Dash Lalako Eka Masha Dash Mashako Eka Tola Hunchha [ ]

The above verse indicates the different measurement system in the past like measures of gold. Here, although the weight is equal but it is said that silver is 1 masa equals to 8 lal and 1 tola equals to 12 masa and on the other hand gold is 1 masa equals to 10 lal and 1 tola equals to 10 masa. Likewise it is found that there is less silver in lal in comparison to gold and masa less gold in comparison to lal.

Likewise, there are so many other units of measurements like the units of length, measures of grain in volume and lastly the measure of time. This indicates that the text is quite formal in treatment. It sets the character of the work not as an abstract piece but rather one of practical significance in day-to-day applications moreover, she justifies use of mathematics in everyday life. It clearly states that the value of digits increase by a factor ten from right to left.

Fundamental Operation of Mathematics

Chandrakala Devi has presented the subject matter of her book from simple to complex, problems are designed to explore and developed creative and critical thinking, problems are based on the surrounding and tried to connect mathematics to real life context. To justify it broadly firstly she has generally introduced number counting, addition and subtraction, multiplication and division as well as she has given the problems having brackets. Like this

+ This is the sign for addition. This is called (dhaana) sign.
= This is the sign for equal. This is called (Sama) sign.
- This is the sign for subtraction. This is called (rina) sign,
× This is the sign for multiplication. This is called (gunan) sign.
÷ This is the sign for division. This is called (bhaag) sign.
0 [ ] [ ] these are brackets, in which the first sign is called dirghamatra. They show the unity in a group.

For example (7+4)-(5-2) or (7+4)-(5-2) or \( \frac{7+4}{5-2} = 2 \) or \( \frac{7+4}{5-2} = 8 \).

These all equal 8. All of them say add 7 and 4 to get 11, and subtract 5-2 to get 3, and subtract (it) from 11. All of these (notations) refer to the same (expression).

She has justified the algorithm of addition after the function of brackets and after that; she has clarified the process with the instances. She has not only applied this process in addition but also in subtraction, multiplication and division that makes the students or the readers easier to gain the mathematical knowledge. However she has included the significant process like simple to complex, chaining process in her book that made the development of the eastern mathematics powerful independently like the western mathematics.

HCF and LCM

Her book includes the topics related to HCF and LCM after addition, subtraction, multiplication and division and the same process has existed even in this era. It means to say that the book Shishabodh Tarangini's syllabus has been considered to arrange its scope and sequence as well as it includes the psychological aspects very well since the arrangement of the syllabus on the basis of its scope and sequence is time relativistic in modern time too. Let's discuss what sorts of problems are presented about HCF and LCM:

Chandrakala, under the topic of HCF and LCM, has defined in Nepali language as follows:

1 – Mahattamapawartan Bhaneko | Dui Wa Adhik Rasi Lai Sun sanksha Le Bhag Dida nishehsa Huncha Tesha Lai Apwartan Bhandachhan | Tee Apawartak Haruma Jo Sabai Bhanda Thulo Chha Tyashalai Tee Rasiko Mahattamapawartan Bhandachhan

If two or more than two variables are divided by a number without remaining is called refraction and the highest number among that refraction is called HCF.

-Jo Dui Athawa Adhik Rasi Le Jati Rasilai Nishehsa Garda Chhan Uti Rasi Ma Jun Sabai Bhandasan Chha Usailai Tee Dui Athawa Aadhik RasikoLagutamapawartyan Bhandachhan

If two or more than two variables divide any other variables without remaining, the smallest number among those variables is called LCM.

After the definition, she has literally presented the methods of computing HCF and LCM in poetic form. Since there are various ways to compute HCF and LCM included in her book. She has presented the computation of HCF and LCM as follows:

(34)Chauchha Chaustīth PheBarha Chhattīsth Enko Pani | Mahattamapawartan Sab Bhanlaukati Bho Gani || 2 ||

It means that:

14 and 34 again 12 and 36 too

What is the HCF of that after counting?

Ans. HCF of 14 and 34

14 34 2

28

6 14 2

12

2 6 3

6

Here divided by 2 without remaining so 2 is the HCF

In the same way to calculate the HCF of 12 and 36, so 12 is the HCF.

Fraction and Decimal

After HCF and LCM Chandrakala has included fraction and decimal with their operations like addition, subtraction, multiplication and division in her masterpiece. In the book Wyakta Chandrika of Pd. Gopal Pande defined fraction as:

when a quantity is composed of one or more equal parts of the unit, its measure is called a fractional number and simply fraction [11]. One of the examples is given like this:

(57) Tinwate Chagle Nani | Chutarwate Tinlauguna |
It means to say that \( \frac{3 \times 4}{12} = \frac{12}{12} = \frac{1}{1} \) and \( \frac{2 \times 4}{2} = \frac{8}{2} = \frac{4}{3} \).

**Next problem:** Example (67) Sat, Phertin Binduchaubis, PherBindu Pachahattar]

Bindusunya Yakawanna Esabakonani | yogara || I ||
The above verse means to say, what is the sum of 7, 3.24, 0.75 and 0.051?
The verses are also quite poetic and beautiful. The use of poetic language typically involves the use of such adjectives as, O! Auspicious girl, Oh Friend! Oh Brother! My beloved, Nani etc. This clever use of language is partly teasing but also engaging and challenging the students intellectually. This is quite contrasting to modern mathematical text books, which are always in prose form and quite dry.

**Square, Square Root and Cube, Cube Root**
The first mathematics book written in Nepali language by Gopal Pande named Wyakta Chandrika involves the arrangement of the contents and since it was widely used and it was popular in the schools at contemporary age so it is seen some influence in her book too. General introduction and the methods of calculating square, square root, cube, cube root is mentioned in Shishubodha Tarangini like Wyakta Chandrika. Among them one is presented below.

(11) Barha Keri Duitsau Satanwe | Dashajar Teen, Pheri Bayanabe ||
Hunchha Bargd Eni Ankako kati | Jandachhau Bhani Bataau Jhatahatoi || 7 ||
The square of 12 is 12 x 12 = 144, square of 297 = 88201, Square of 10003 is 100060009 and square of 92 = 8464.
The above given verse indicates: What is the square of 12, 297, 10003 and 92? As similar to square and square root, Chandrakadal presented problems related to cube and cube root in her book.

(23) Ghananauko Bhan Ani TinakaGhanako Ghana | JanekachhauGhanabhag PanchakaGhanako Bhana || 13 ||
The cube of 9 is 9x9x9= 729. The cube of 3, i.e., 27, is 19683. The cube of 125 is 1953125.

**Unitary Method**
Generally, unitary method means technique of carrying out a calculation to find the value of a number of items by first finding the cost of one of them. It sets the tone of the work not as an abstract piece but rather one of practical significance in day-to-day applications. Among the so many mathematics books of eastern Hindu Philosophy, some are Aryabhattiya of fifth century, Brahasmaddhanta of seventh century, and Lilavati of Bhaskaracharya of eleventh century. Among these aforementioned books Bhaskaracharya has talked about unitary method through Lilawati. Unitary method is equally given the significant place in the modern time as it was given in the ancient time \(^3\). Shishubodha Tarangini is no more an exception. One of the examples is given as:

(127) Ramale Aathha Dimna Garadaluchha Ulikam Somrele Chhadima ||
Tehidubai Janale Sangha Mili Garada Hunchha Kam Kati Dimna || 13 ||
The verse says that Ram can do a piece of work in 8 days; so can Som in 6 days.
In what time will they do it if they work together?

Although she has not included about the separate chapter of simple interest and compound interest in her book Shishubodha Tarangini, She seemed to have adequate knowledge about simple and compound interest. She has clearly presented the problems and solving methods of those problems regarding the interest. One of the instances is presented as follows:

(52) Kohi Eka Janale Diyechha Rupiya Aatha Saikada Barsaka ||
Darale Teensaye Pachasa, Teesabai Liyer Yai Reetaka ||
Chakraibiridhi Biyajama Teen Barsa Sammana Teeniki Kati ||
Hundo Bho Bhan Sudalau Timi Yaha Junne Bhyae Jhatuhati || 2 ||
The above verse says: - What is the compound interest of the principal 350 with 8% interest rate in 3 years? To study the book Shishubodha Tarangini \(^10\), with deep inspection it is found that there is not used the formula of calculating profit and loss as mentioned in present mathematical text book of secondary level. Furthermore, in her book, the content like profit and loss, use of percentage, concept of cost price is not mentioned but her book includes the practical problems of profit and loss. The mathematical books that were published during the contemporary period in Nepal primarily focused on solving the practical problems instead of developing mathematical knowledge, which is also mentioned in \(^12\). \(^3\).

**Sequence and Series**
The concept of sequence and series are found to be mentioned in her book. During the early thirties of the twentieth century, the books that were taught in Nepal in Nepali language like Wyakta chandrika, Bichitra Ganit, Ganiit Sagar, Saral Ganiit, Gorkha Bijnaiti Kshikhya, Arithmetica, and Patigantals also do not include the concepts like sequence and series. However, the book Shishubodha Tarangini is found to be one step ahead in comparison with the books aforementioned regarding the matter of sequence and series. She has presented the method of calculating sum of natural number having the \(n\) terms in a poetic form verbally.
She presented different formulas to compute sum of arithmetic series and those formulas are arranged simple to complex manner. Firstly, she wrote the rules of addition in Nepali language as:

(128) Eka Joder Padale Padardhakan Gunana ||
Bho Ankako Yoga Yahi Sankalita Bhanit Janana || 1 ||
Here, we cannot see any mathematical symbols and any numbers however; it gives very appropriate formula to compute the sum of first \(n\) natural number of a series. It means:
Add one in the numbers of term and multiply by half of number of terms and it is the sum of that series.
Mathematically it means:
If \(1+2+3+4+\ldots\ldots\ldots+n\) is a series of first \(n\) natural number.
Therefore, here total number of terms = \(n\)
Add one to the number of terms \(n\), so it becomes \((n+1)\) and need to multiply this term \((n+1)\) with half of total term i.e. \(n/2\).
Accoding to the above sum of this series
\[
\frac{n}{2} = \frac{n(n+1)}{2}
\]
which is sum of first \(n\) natural number
Most common formula to compute the sum of natural number is also even exactly appropriate nowadays. Moreover, she posed an example of series where such formula can be used that means she wrote as follows:

~46~
It means; A person gives four rupees in a first day to his priest the than he increased five rupees per days. Calculate how much money did he gives up to fifteen days?
Moreover, she solves this problem as follows;
Here, first term = 4
Common difference = 5
Number of term = 15
Here, \((15 - 1) = 14 \times 5 = 70 + 4 = 74\) is this last term (l)
Again, \(74 + 4 = 78\) \(= \frac{78}{2} = 39\) this is midterm.
Now \(39 \times 15 = 585\)
also using above formula
\[ S_n = \frac{n}{2} \left[ l + a \right] = \frac{15}{2} \left[ 74 + 4 \right] = \frac{15}{2} \times 78 = 15 \times 39 = 585 \]
Most common formula to compute the sum of natural number which is almost same as now days. Moreover, in current textbook of secondary mathematics of Nepal as well other SAARC countries we can see such mathematical problems related to sequence and series as similar manner of Chandrakala's Shishubodha Tarangini, \(^{10}\). This provided that mathematical knowledge is discovered not invented because how it is possible to remains same before 100 years and now a days.
The book has great significance in the present time because it provides the basis as well as reference for the present day's teachers to devise mathematical books. The books had been undertaken in order to supply a demand for an easy introduction to elementary mathematical concepts for Nepalese students. At that time in Nepal, few mathematics books in Nepali languages had been published on Mathematics and desired to prepare new treatise on Mathematics which contained the mathematical problems almost in verbal forms. Despite being the old book, it had got the content of present time textbook. Nepali words have been used for questions in examples and practice. For variables, we can see Nepali sounds and alphabets. This shows that the Nepali-speaking students can benefit from the book. In the book, there are Nepali units of measurement (Dam, Paisa, Lal, Masa, Mana, Pathi, Aana, Ropani, Mann, Auns Angul, and Bittaetc.) and practical problems, which show the formation of original questions by the author.
The book is written in a very simple way. It includes the subject matter presented in it from simple to complex way. Arithmetic and Algebra related subject matters are included in this book. Before solving any problems, the rules for solution are given in the chapter and after that, the problems are solved respectively. Another good aspect of the book is that the same problem is solved by applying various methods. This book includes both general and specific questions. This means it is not easy to answer for which level of students the book is prepared. This is also not clear for which grade the book is for.
She used verbal problems in slokas form, which were based on mathematical formulas. She used an advanced learning theory induction like to find the sum of natural numbers. Therefore, she has better knowledge of teaching and learning theory. Thus, she has advanced pedagogical concept for learning and expressing such concept through her book Shishubodha Tarangini.

**Results**

On the study, following results are obtained.
Chandrakala Devi used the local terms in measurement, like Mana, Pathi, Aana, Ropani, Auns, Paisa, Gadda, Tola, Angul, Bitta etc., are used which has also provided evidence the use of entho-mathematical concepts.

Inductive method was taken into consideration on her book Shishubodha Tarangini.

In 1933s, practical mathematics was practiced which was mainly based on daily activities for measurements. She expressed her mathematical problems- answer in poetic form.

Inspiration and encouragement can be found to learn mathematics in the book of Chandrakala Devi Dhananjaya.

The book contains the mathematical problems which are still in modern curriculum of six to ten classes of Nepal.

Conclusion

Chandrakala Devi Dhananjaya had written a book named Shishubodha Tarangini in poetic form, which was published in 1933 A.D. She had not acquired any formal education. However, she wrote such a book, which is still considered most important to study the Nepalese history of mathematics. She is supposed to be the first Nepali woman writer of mathematics book and teacher in Nepal. Until now, no Nepali woman mathematician has been seen in writing mathematics book on single ownership. Therefore, we are interested to carry out a research work on her contribution in mathematics.

The focus of the study is to highlight the contributions of Chandrakala Devi Dhananjaya in Nepalese mathematics through her book named Shishubodha Tarangini. The book Shishubodha Tarangini is no more in practice these days, however, the contents of the book are still being practiced in a reformed manner. The letters inside the books are visible, large enough to see, understandable, compressive and written in Nepalese languages with few Sanskrit languages as well. Some vocabularies are not today's fashion however, these can be understood. Moreover, Shishubodha Tarangini mainly deals with what we call as 'Arithmetic' in today's mathematical fashion. It consists of 134 verses written in Nepali in poetic form (terse verses). Especially it is written for children or for beginners of concerned subject matter sharing them the knowledge of mathematics and astrology. It contains the problems to modern curriculum of classes from six to ten in arithmetic.

References

6. Basyal D. has conducted a research (for PhD. Award) is titled “A 1933 Nepali mathematics and Astrology Book Shishubodha Tarangini. Translation and commentary on mathematics chapters.”(PhD Desertation), New Mexico State University, U.S.A, 2015.