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Research on the mutual influence between the population of deer and grass quantity based on difference method

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Abstract
By establishing different difference equation models, this paper analyzes the relationship between the population of deer and grass quantity in the food chain in detailed. It is found that when the maximum quantity of grass changes the trend of the population of the two species is likely to be different: when the maximum quantity of grass is small, the grass quantity will decrease first and then increase, and when the grass quantity is large, it will increase first and then decrease. The trend of the deer population is opposite to the trend of the grass quantity.

Keywords: Population, difference equation, stability, MATLAB

1. Introduction

A creature in nature often feeds on another one, and at the same time it provides for one or more other species as food, which forms food chains in the ecosystem[1,2]. The interaction between two adjacent species in a food chain has greatly affected the survival and development of this two biological populations[3-8]. In this case, the method of difference equation has been applied to analyze this problem, but generally speaking, it is not comprehensive enough [9-13]. Based on previous studies, this paper takes two common species in the ecosystem-deer and grass as an example to analyze the trend of two populations with different initial quantities and in different environmental conditions to push the research in this field further and get more valuable results.

2. The difference equation model of the population of two species

2.1 The difference equation of the population of two species

Firstly, according to the ecological knowledge [1-4], the basic assumptions are made as follows:

1. The population of deer and grass quantity are only affected by grass or deer themselves, and other external factors are not considered;
2. The growth of grass follows the law of discrete form logistic;
3. Grass is the prey and deer is the predator, so they follow the Prey-Predator model.

Now supposing that r represents the annual intrinsic growth rate of grass, N represents the maximum quantity of grass, a represents the quantity of the deer's annual consumption of grass, b represents the annual mortality of the deer herd, d represents the deer's annual compensation rate, x_k represents the grass quantity in the kth year , and y_k represents the population of deer in the kth year, then according to the hypothesis, the relationship between the grass quantity and the population of deer obeys the following difference equations [5-12]:

$$\begin{cases} x_{k+1} = x_k + r\cdot(1-\frac{x_k}{N}) - \frac{a y_k x_k}{N} \\ y_{k+1} = y_k\cdot(1-d+\frac{b\cdot x_k}{N}) \end{cases} \tag{1}$$

The initial value is (x_0, y_0).

2.2 The calculation of the equilibrium point and its stability judgment

Let $x_k = x_{k+1}, y_k = y_{k+1}$ in equation (1) solve the equilibrium point, and we get the point:

$$p = \frac{Nd}{b}, q = \frac{rN}{a}(1 - \frac{d}{b}) \tag{2}$$

The stability of this equilibrium point will be analyzed below. The equivalent form of difference equations (1) is as follows:

$$\begin{cases} \frac{x_{k+1} - x_k}{k+1-k} = r \cdot (1 - \frac{x_k}{N}) \cdot x_k - \frac{a y_k x_k}{N} \\ \frac{y_{k+1} - y_k}{k+1-k} = -y_k \cdot d + y \cdot \frac{bx}{N} \end{cases} \tag{3}$$

Therefore, the difference equations (3) are discrete forms of differential equations (4):

$$\begin{cases} \dot{x} = r \cdot (1 - \frac{x}{N}) \cdot x - \frac{ayx}{N} \\ \dot{y} = -y \cdot d + y \cdot \frac{bx}{N} \end{cases} \tag{4}$$

It can be verified that the equilibrium point (2) of the difference equation (1) is also the equilibrium point of the differential equations (4), which means that the stability of the equilibrium point (2) is the same in two groups of equations. Therefore, we will analyze the stability of the equilibrium point (2) of the difference equations (1) according the differential equations (4).

According to the relevant knowledge of the stability theory of differential equations, the Jacobian matrix of the differential equations (4) at the equilibrium point (2) is calculated as:

$$A = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{pmatrix} = \begin{pmatrix} -\frac{rd}{b} & -\frac{ad}{b} \\ \frac{br}{a}(1 - \frac{d}{b}) & 0 \end{pmatrix} \tag{5}$$

Then the characteristic equation $|A - \lambda E| = 0$ of matrix A is calculated and we can obtain a quadratic equation of one variable as follows:

$$\lambda^2 + \frac{rd}{b}\lambda + br(-\frac{d}{b} + 1) = 0 \tag{6}$$

$$\Delta = (\frac{rd}{b})^2 - 4br(1 - \frac{d}{b}) \tag{7}$$

$$\lambda_{1,2} = \frac{-\frac{rd}{b} \pm \sqrt{\Delta}}{2} \tag{8}$$

Because $-\frac{rd}{b} < 0$ is always true, the stability of the equilibrium point depends on the value of the discriminant Δ. According to the discussion in the stability theory of the differential equations, we can obtain that:

1. When $\Delta < 0$, the equilibrium point is a stable focus;
2. When $\Delta \geq 0$, $b \geq d$, the equilibrium point is a stable node, when $\Delta \geq 0$, $b < d$ the equilibrium point is an unstable node.

3. The cases that the initial values of the population of two species are changed

3.1 Equilibrium point and its stability of the population of two species

Now we consider two cases that the initial values of the population of deer and grass quantity take two different values under a fixed environmental condition. The first case is that the initial population of deer is 100 and the initial quantity of grass is 1000; the second case is that the initial population of deer is 100 and the initial grass quantity is 3000. We suppose that the annual intrinsic growth rate of grass is 0.8, and the maximum quantity of grass is 3000. When the grass is flourishing, each deer consumes 1.6 quantity of grass annually. Without grass, the annual mortality of deer is as high as 0.9 percent. The existence of grass can compensate 1.5 deer for deer's death when the grass quantity is the most.

According to formula (2) (7), the equilibrium points of the number of two populations are same when the above initial values are different. We suppose that

$r = 0.8$, $N = 3000$, $a = 1.6$, $d = 0.9$, $b = 1.5$

to find the equilibrium point and get it as follows:

$(p, q) = (1800, 600)$

$\Delta \approx -1.636 < 0$

According to the analysis of the stability of the equilibrium point above, it can be found that this equilibrium point is a stable focus. And when time tends to infinity, the grass quantity gradually stabilizes at about 1800, and the population of deer gradually stabilizes at about 600.

However, the change trend of the number of two populations is still unclear. Therefore, in order to make the change process of two populations more detailed, two cases of different initial values are discussed and explained in the next.

3.2 The case that the initial grass quantity is 1000 and the initial population of deer is 100

When the initial value is $(x_0, y_0) = (1000, 100)$, we use MATLAB to draw the images of the populations of two species, as shown in Figures 1, 2 and 3:

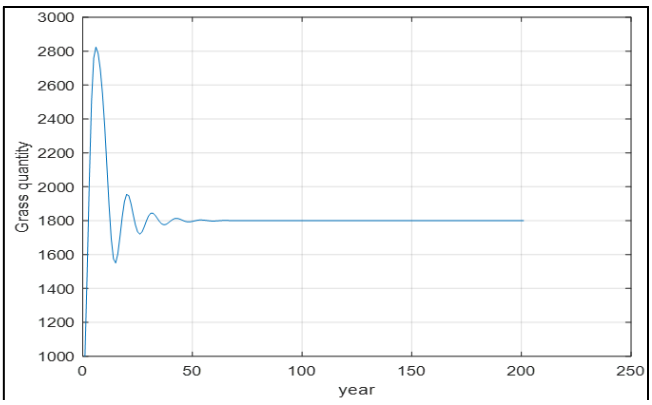

Fig 1: The grass quantity with the initial grass quantity1000

It can be seen from Figure 1 that the grass quantity increases in the initial period of time. The grass quantity reaches its maximum 2824.5, then decreases sharply, and then increases again. After repeating this process, it will be stable at about 1800 after about 50 years.

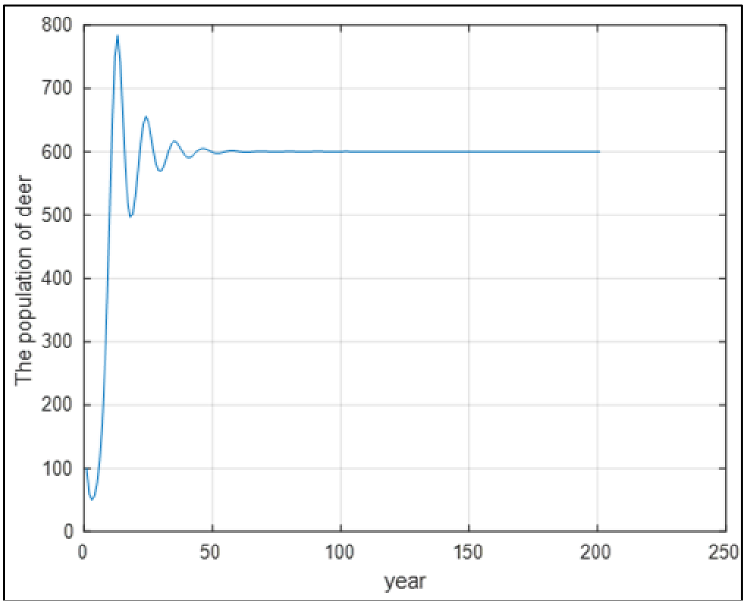

Fig 2: The population of deer with the initial grass quantity 1000

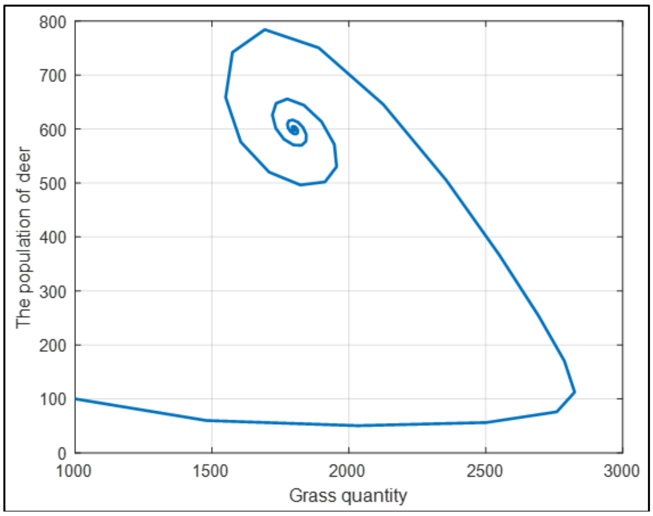

Fig 3: The population of two species when the initial grass quantity is 1000

It can be seen from Figure 2, the population of deer decreases slightly in the initial period of time, and then increases continuously. The maximum population of deer reaches 655, then decreases again, and then rises again. After repeating this process, it will be stable at about 600 after about 50 years.

It can be seen from Figure 3 that the increase of the grass quantity can promote the the population of deer to increase, while the increase of the population of deer will inhibit the grass quantity from increasing. The interaction between the two species makes the population of two species adjust with fluctuation, and finally stabilize at the equilibrium point $(p,q)=(1800,600)$ after 50 years.

3.3 The case that the initial grass quantity is 3000 and the initial population of deer is 100

When the initial value is, $(x_0, y_0)=(1000,100)$ we use MATLAB to draw the images the populations of two species again, as shown in Figures 4, 5 and 6.

It can be seen from Figure 4 that the grass quantity decreases in the initial period of time. The grass quantity reaches its minimum 1792, then increases again, and then decreases again. After repeating this process, it will be stable at about 1800 after about 50 years.

It can be seen from Figure 5 that the population of deer increases greatly in the initial period of time. The maximum number reaches 785, then decreases, and then rises again. After repeating this process, and it is stable at about 600 in about 50 years.

It can be seen from Figure 6 that the grass quantity is high at the beginning, which makes the population of deer increases greatly. However, because the number of deer is too large, the grass quantity decreases sharply, making the population of deer decreases next. The interaction between the two species makes the population of two species change with fluctuation, and they finally stabilize at the equilibrium point $(p,q)=(1800,600)$ after about 50 years.

Comparing the above two cases, the different condition is different initial grass quantity. When the grass quantity is higher, the grass quantity first decreases and then increases, and the population of deer first increases and then decreases; when the grass quantity is small, the grass quantity first increases and then decreases, and the population of deer first decreases and then increases. In both cases, the population of two species tends towards the point $(1800,600)$ at last.

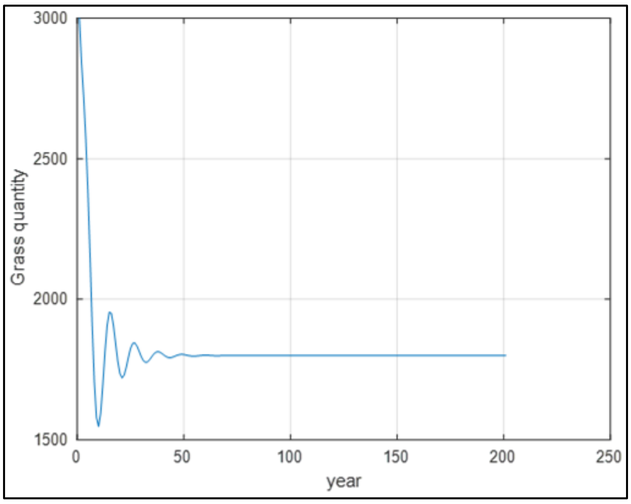

Fig 4: The grass quantity with the grass initial quantity 3000

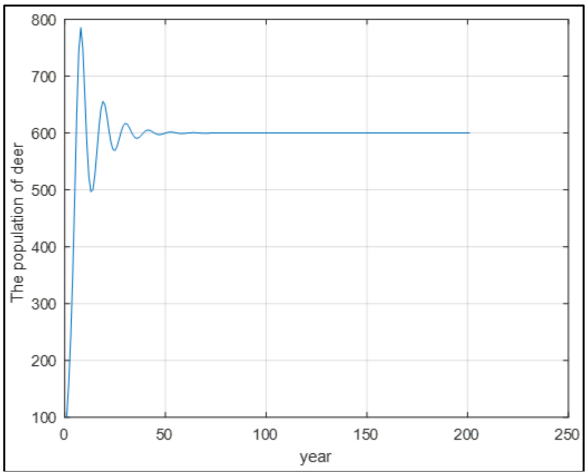

Fig 5: The population of deer with the grass initial quantity 3000

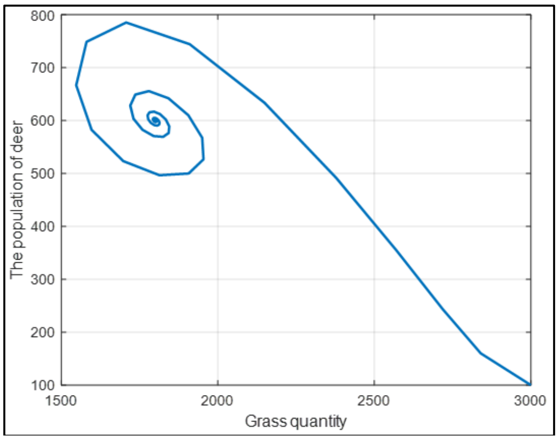

Fig 6: The population of two species when the initial grass quantity is 3000

4. Cases that the environmental conditions change

In order to analyze how the grass quantity and the population of deer in different conditions of real environment change, we change the intrinsic growth rate of grass *r*, the maximum quantity of grass *N*, the annual grass consumption quantity *a*, the annual mortality rate *d* and compensation rate *b* of deer in an appropriate range, and draw the change images of their populations to analyze the characteristics of the changing trend under different conditions.

4.1 Cases that the intrinsic growth rate of grass *r* is changed

When the parameters $N = 3000$, $a = 1.6$, $d = 0.9$, $b = 1.5$ are fixed, we only change the intrinsic growth rate of the grass *r*, and draw the images of the population of two species when *r* = 0.8, 0.5 and 0.2, as shown in Figure 7.

By using formula (2), the stable point can be calculated as $(1800, 375)$ when $r = 0.5$ and $(1800, 148.2)$ when $r = 0.2$.

It can be seen from Figure 7 that when the intrinsic growth rate of grass is 0.5, the fluctuation time of the population of deer and grass quantity is slightly longer than the case that $r = 0.8$, the fluctuation range of deer population is larger, and the grass quantity is less when the population of two species is stable; when the intrinsic growth rate of grass is 0.2, the population of deer and grass quantity tend to be stable after about 150 years, which is longer than the case that $r = 0.8$, and the fluctuation amplitude of the populations becomes larger. Additionally, the final grass quantity is smaller when the population of deer and grass quantity becomes stable in the latter case

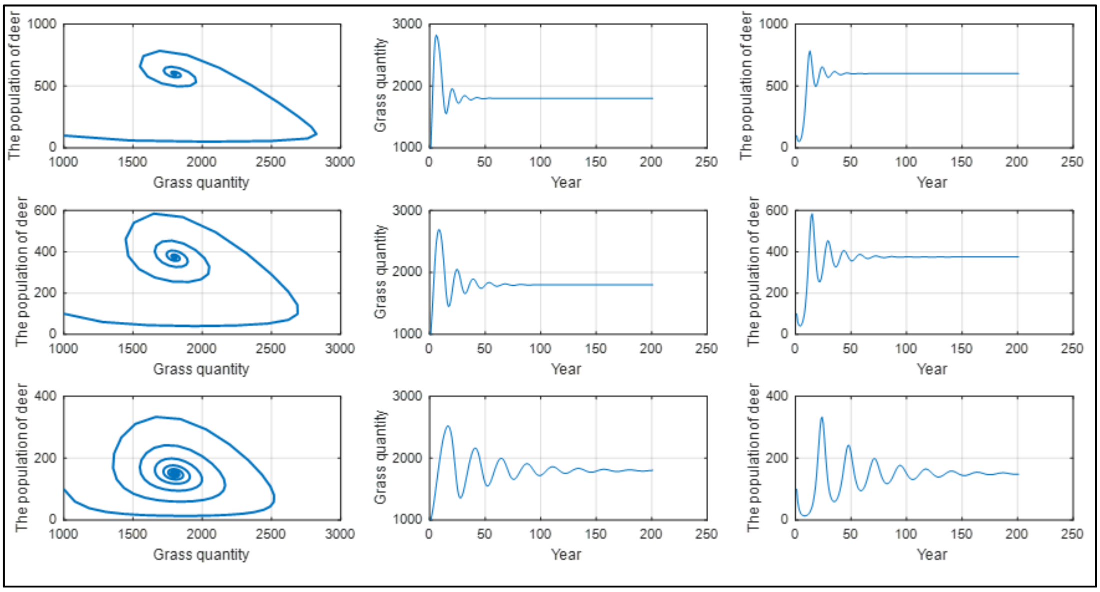

Fig 7: The population of two species with different intrinsic growth rate

4.2 Cases that the maximum quantity of grass *N* is changed

When the parameters $r = 0.8$, $a = 1.6$, $d = 0.9$, $b = 1.5$ are fixed, we only change the maximum quantity of grass *N* to be 3000, 1000, 5000, and draw the images of the population of two species, as shown in Figure 8.By using formula (2), the stable point can be calculated as $(600, 200)$ when $N = 1000$ and $(3000, 1000)$ when $N = 5000$.

It can be seen from figure 8 that when the maximum quantity of grass is 1000, the grass quantity first decreases and then increases in the initial period of time, and becomes smaller than the previous case that $N = 3000$ when the population of two species is stable; when the maximum quantity of grass changes to 5000, the grass quantity first increases and then decreases, and finally the population of deer and grass quantity are larger than the case that $N = 3000$ when they become stable

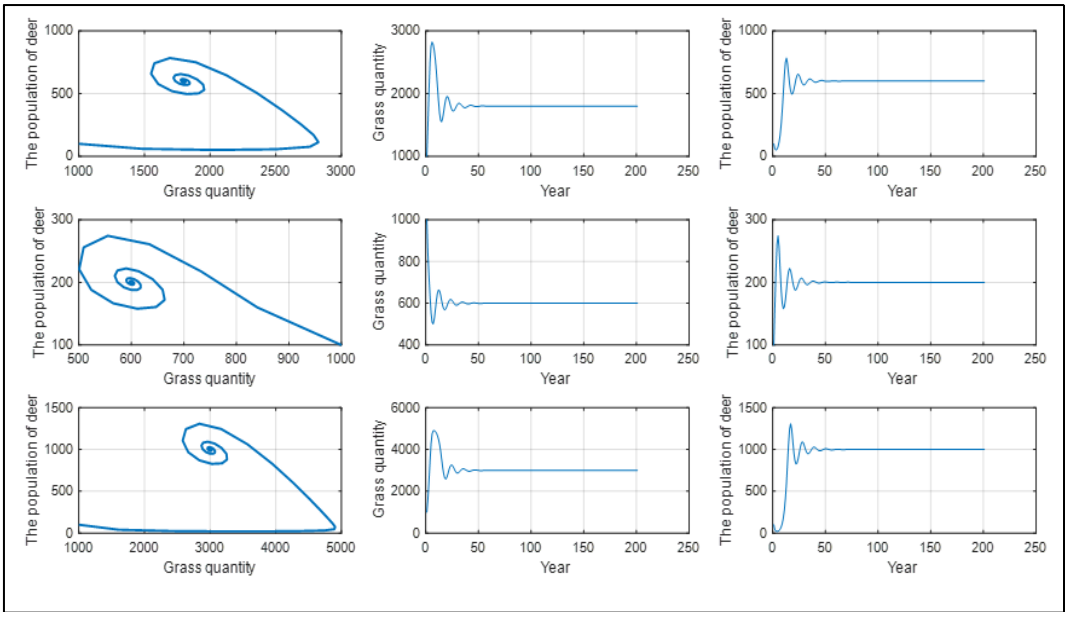

Fig 8: The population of two species with different maximum quantity of grass

4.3 Cases that the deer's annual consumption of grass *a* is changed

When the parameters $r=0.8$, $N=3000$, $d=0.9$, $b=1.5$ are fixed, we only change the deer's annual consumption of grass *a* to be 1.6, 1, 2, and draw the images of the population of deer and grass quantity, as shown in Figure 9.

By using formula (2), the stable point can be calculated as $(1800{,}960)$ when $a=1$ and $(1800{,}500)$ when $a=2$.

It can be seen from Figure 9 that when the deer's annual consumption of grass is 1, the grass quantity and the population of deer will remain at 1800 and 960 after they become stable finally. The fluctuation trend is similar to the case that $a=1.6$. When the grass quantity is 2, the grass quantity is still 1800 after stabilization, while the population of deer reduces to 500, and the fluctuation trend is also similar to the initial case.

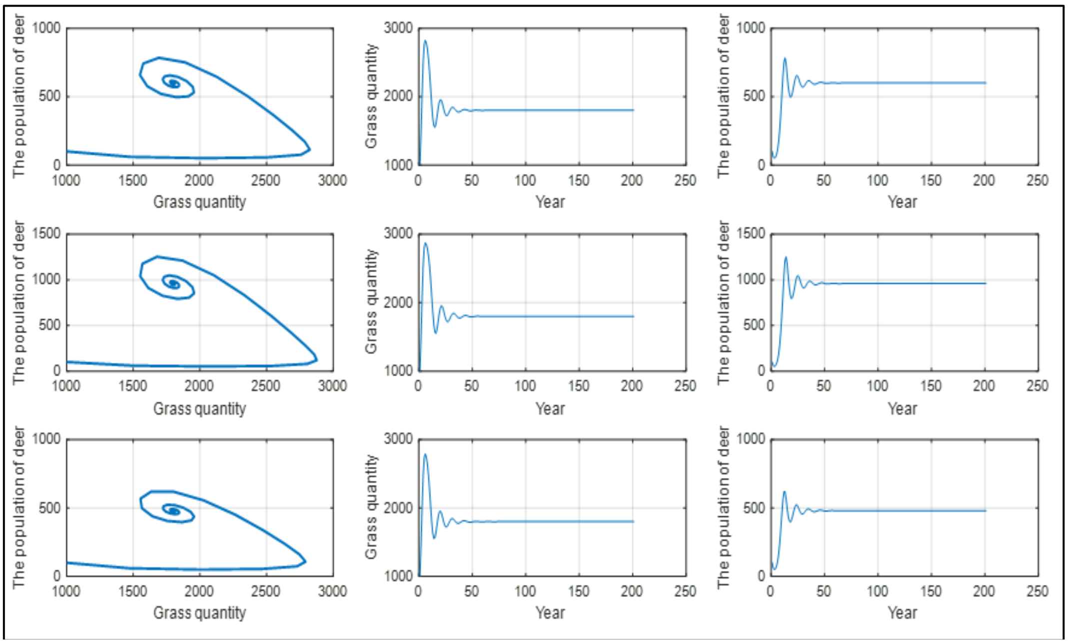

Fig 9: The population of two species with different deer's annual consumption of grass

4.4 Cases that the annual mortality rate *d* of deer is changed

When the parameters $r=0.8$, $N=3000$, $a=1.6$, $b=1.5$ are fixed, we only change the annual mortality rate of deer *d* to be 0.9, 0.7, 0.95, and draw the images of the population of deer and grass quantity, as shown in Figure 10:

By using formula (2), the stable point can be calculated as $(1400{,}800.2)$ when $d=0.7$ and $(1900{,}550)$ when $d=0.95$.

It can be seen from Figure 10 that when the annual mortality rate of the deer is 0.7, the fluctuation time of the population of two species lasts for about 100 years which is longer than the case $d=0.9$ and the population of the two species reaches a stable state after more mutual restrictive processes; when the annual mortality rate of deer is 0.95, the population of two species fluctuates slightly shorter which only lasts for about 40 years.

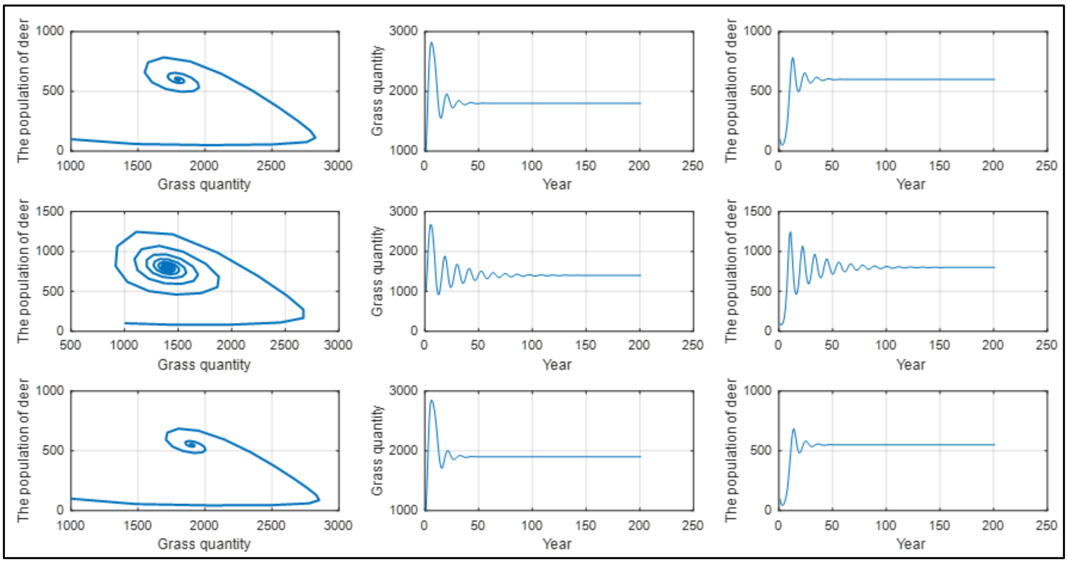

Fig 10: The population of two species with different annual mortality rate

4.5 Cases that the annual compensation rate of deer *b* is changed

When the parameters $r=0.8$, $N=3000$, $a=1.6$, $d=0.9$ are fixed, we only change the annual compensation rate of deer *b* to be 1.5, 1.2, 2, and draw the images of the grass quantity and the population of deer again, as shown in Figure 11.

It can be seen from Figure 11 that when the annual compensation rate of deer is 1.2, the fluctuation time of the grass quantity and the population of deer become shorter than the case $b=1.5$ which is about 30 years, and the number of fluctuations is fewer which is only two fluctuations. When the annual compensation rate of deer is 2, the population of two species has not been stable yet in 500 years, which means it fluctuates for a longer time than the cases that $b=1.5$ and $b=1.2$, and the number of fluctuations is also more. Therefore, this equilibrium point looks like a center point

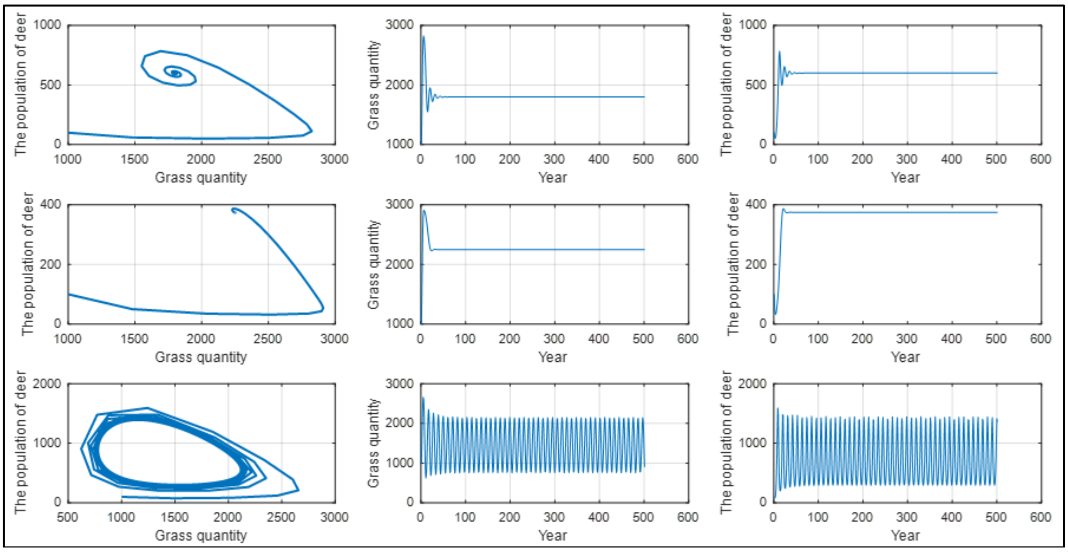

Fig 11: The population of two species with different annual compensation rate of deer

5. Conclusion

Different values of the above parameters represent different initial values of the population of species and different environmental conditions. Therefore, the following laws can be obtained through the above analysis:

1. Under the same environmental conditions, if the initial values of the grass quantity and the population of deer take different sets of values, the final stability of the population of the two species is the same, and if both are stable, then their stable points can be calculated the same;
2. When the intrinsic growth rate of grass is large, the fluctuation time of the grass quantity and the population of deer is shorter and the fluctuation range is smaller. While the intrinsic growth rate of grass is small, the fluctuation time is longer and the fluctuation range is larger;
3. Both the trends of the grass quantity and the population of deer may change when the maximum quantity of grass changes. When the maximum grass quantity is small, the grass quantity will first decrease and then increase. While the maximum quantity of grass is large, grass quantity will first increase and then decrease. Correspondingly, the trend of the population of deer is opposite to the trend of grass quantity;
4. When the annual mortality rate of deer is large, the fluctuation time of the grass quantity and the population of deer is short, while the death rate of the deer is small, the fluctuation time is long;

5. When the annual compensation rate of deer is large, the fluctuation time of the grass quantity and the population of deer is long, while the annual compensation rate of deer is small, the fluctuation time is short.

In conclusion, to achieve the desired effect of the population of these two species, humans can control the initial population of and deer and grass quantity, the growth of grass, and the death and birth of deer etc.

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